Vol. XXIV No. 12 SCIENTIA SINICA December 1981

THE "BOLUS FLOW" SOLUTION OF THE PLASMA BETWEEN THE RED BLOOD CELLS FLOWING THROUGH A CAPILLARY

YAN ZONGYI (严宗毅)

(*Institute of Mechanics, Academia Sinica*)

Received May 7, 1981.

ABSTRACT

In this paper, we have derived an approximate "Bolus Flow" solution between two adjacent red blood cells (RBC) in a capillary, using the Galerkin method and numerical smoothing technique, and then examined in detail the effects of the ratio λ of the diameter of the RBC to that of the capillary and those of the leakback Q in the thin lubrication layer on the Bolus Flow. Because of the simplicity of the method employed here, only a small amount of numerical calculation is involved. Furthermore, we have rectified somewhat unreasonable formulation of the boundary conditions presented previously so as to match the Bolus Flow with the flow in the lubrication layer. The results are reasonably in agreement with other theories, providing correct boundary conditions for the determination of the deformation of RBC.

I. INTRODUCTION

A capillary is the very place where the process of metabolism between blood and tissue takes place. The investigation of the blood flow in it is of physiological and pathological significance. A lot of difficulties may occur in quantitative observations owing to its tiny construction. On the other hand, model experiments without proper simulation of the real flow in the capillary could only provide a qualitative sketch of it. For this reason, various kinds of idealized mathematical models have been suggested abroad during the recent decade.

The diameter of a capillary is usually slightly larger or even smaller than that of the RBC, which has to squeeze into the capillary one by one in a single line. With large deformation, the RBC nearly fills up the whole vessel lumen. Microcinematograph and large-scale modeling have shown that no matter how close to that of the vessel the diameter of the RBC is, there is always a rather thin lubrication layer of the plasma, and that the plasma between two adjacent RBC's is making circulation with respect to the coordinate system fixed on the RBC, known as "Bolus Flow".

The exposition of the relations between pressure drop and flux is an essential topic in microcirculation mechanics. Some authors regarded RBC as rigid or undeformable particles. With their real shape unknown, one fails to yield an exact relation between them. In 1968, Lighthill[1] assumed that the RBC was an elastic pellet and he took into account the interaction between the deformation of the pellet and the plasma flow in the lubrication layer, and thus the shape of the RBC as well as the pressure drop could be determined. In reality, the elastic model of the cell membrane is much more complicated than that postulated in reference [1]. Moreover, if we only deal with the lubrication layer, errors may also stem from the neglect of the Bolus Flow on both sides of the RBC. Consequently, the results that merely qualitatively conform to those in model experiments are unable to explain convincingly the reason why the parachute shape on the backside of the RBC is observed *in vivo* experiments and large-scale modeling. Therefore, it seems necessary to examine the Bolus Flow closely.

By expanding a series of eigenfunctions, Lew and Fung[2] worked out a semi-analytic Bolus Flow solution for the first time in 1969. They reduced the RBC to a cylindrical disk with the same diameter as that of the vessel. Then the method was extended to the cases with the diameter smaller than that of the vessel by adopting infinitely thin disk model with RBC equidistantly distributing along the symmetrical axis[3]. In 1970, Bugliarello and Hsiao[4] obtained a numerical Bolus Flow solution for the cases with the diameter larger or smaller than that of the vessel. In those papers, the physical picture of the Bolus Flow is depicted, physiological meaning of it is described, and several conclusions of great interest are drawn.

The common disadvantage of the above mathematical models consists in the fact that the Bolus Flow solution does not match with the velocity profile in the thin lubrication layer. As is known, the thickness of the layer varies and the velocity of the RBC with respect to the vascular wall is higher than the average velocity of the plasma and the RBC, making the leakback in the vessel inevitable. Both of them will exert a certain influence on the Bolus Flow between the RBC's. The problem was not formulated in a reasonable way, either by neglecting the thickness of the lubrication layer and the leakback, or by not matching the Bolus Flow with the velocity profile of the layer. In 1972, Fitz-Gerald[5] tried to treat them properly and produced a series of solutions for the stream function. However, the expressions for the pressure and velocity turned out to be divergent.

In this paper, we have succeeded in application of the Galerkin method to obtain a semi-analytic Bolus Flow approximation which strictly satisfies the match condition. And, the effects of the lubrication layer thickness and the leakback on the Bolus Flow are then discussed.

II. The Formulation of the Problem

Supposing that the capillary is a straight circular tube with radius r_0^*, in which there are axisymmetrically suspending cylindrical disk-shaped RBC with radius λr_0^* at an interval $2L$, and that the plasma (regarded as the Newtonian fluid) with viscosity μ^* fills up the space between them, we put the origin of the coordinate system (fixed on the RBC moving at a speed U^*, i.e. the speed with respect to the wall) at the midpoint between two RBC's on the axis, r^*, x^* being radial and axial coordinates respectively. (Fig. 1)

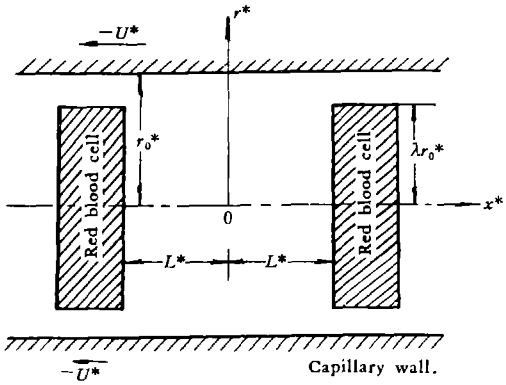

Fig. 1

Next, we try to derive dimensionless coordinates and physical variables concerned as follows:
$L = L^*/r_0^*$, $r = r^*/r_0^*$, $x = x^*/r_0^*$, $\bar{x} = x/L = x^*/L^*$, axial velocity $u = u^*/U^*$, radial velocity $v = v^*/U^*$, pressure $p = p^*r_0^*/\mu^*U^*$, shear stress $= \tau^*r_0^*/\mu^*U^*$, stream function $\varphi = \varphi^*/U^*r_0^{*2}$. Clearly, $r = 0, 1$ indicate the symmetric axis and the wall of the vessel, respectively; $x = \pm 1$, $0 \leqslant r \leqslant \lambda$ the surfaces of the RBC; $x = \pm 1$, $\lambda \leqslant r \leqslant 1$ the exit and entrance, where λ is the ratio of the diameter of RBC to that of the vessel.

To derive the boundary conditions at the exit and entrance of the lubrication layer with axial pressure gradient and axial velocity omitted, we obtain the momentum equation and the boundary conditions for the plasma flow in the lubrication layer:

$$\frac{dp}{dx} = \frac{1}{r}\frac{d}{dr}\left(r\frac{du}{ur}\right),$$

$$u|_{r=\lambda} = 0, \quad u|_{r=1} = -1,$$

$$\int_\lambda^1 urdr = -Q, \tag{1}$$

where $Q = Q^*/U^*r_0^*$, $2\pi Q^*r_0^*$ denotes the peripheric leakback around the vessel in the lubrication layer. Integrating Eq. (1), we obtain the velocity profile in this layer

$$u = g_1r^2 + g_2\ln r - g_1 - 1, \tag{2}$$

where

$$g_1 = \frac{1}{4}\frac{dp}{dx} = \frac{4Q - 2 - \dfrac{1-\lambda^2}{\ln\lambda}}{(1-\lambda^2)\left(1 + \lambda^2 + \dfrac{1-\lambda^2}{\ln\lambda}\right)}, \quad g_2 = \frac{1}{\ln\lambda}[g_1(1-\lambda^2) + 1].$$

If the stream function is defined as:

$$u = \frac{1}{r}\frac{\partial\phi}{\partial r}, \quad v = -\frac{1}{r}\frac{\partial\phi}{\partial x}, \tag{3}$$

it is easy to yield

$$\phi = G_0 + G_2r^2 + \frac{1}{2}g_2r^2\ln r + \frac{1}{4}g_1r^4, \tag{4}$$

where

$$G_0 = \frac{1}{4}(g_1\lambda^2 + g_2)\lambda^2, \quad G_2 = -\frac{1}{4}(2g_1 + g_2 + 2).$$

With the axisymmetrical axis being zero stream line, the stream function on the surface of the RBC is

$$\psi = F(r) = \begin{cases} -Q, & r = 1, \\ G_0 + G_2 r^2 + \frac{1}{2} g_2 r^2 \ln r + \frac{1}{4} g_1 r^2, & \lambda \leqslant r < 1, \\ 0, & 0 \leqslant r < \lambda. \end{cases} \tag{5}$$

It follows that the boundary conditions imposed on References [2—4] are all inconsistent with (5), and thus are somewhat unreasonable. It is the expression (5) that seems to be the only correct formulation for the boundary conditions in reality.

Now let us turn to the approximate equation for the stream function. Owing to the fact that the capillary has a diameter as slight as 3—15 μ, the speed of blood flow is as low as 0.1—2 mm/sec, it is found that the Reynolds number of the flow is as low as 10^{-2}—10^{-3}, so we may use the Stokes equation to describe the plasma flow in the Bolus Flow region by neglecting inertial force:

$$E^4\psi = 0, \tag{6}$$

where E is the differential operator:

$$E = \left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2}\right)^2 = \left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{L^2}\frac{\partial^2}{\partial \bar{x}^2}\right)^2.$$

According to the axisymmetry of the flow, the adherence of the plasma on the wall and Eq. (5), we have the following boundary conditions:

$$\begin{aligned} &r = 0, \quad \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial x}\right) = 0, \quad \psi = 0, \\ &r = 1, \quad \frac{\partial\psi}{\partial r} = -1, \quad \psi = -Q, \\ &\bar{x} = \pm 1, \quad \psi = F(r), \quad \frac{1}{r}\frac{\partial\psi}{\partial \bar{x}} = 0. \end{aligned} \tag{7}$$

Thus we finally lead to Eq. (6) with the boundary conditions (7) for the plasma flow in the Bolus Flow region.

III. The Galerkin Method and Numerical Smoothing

The Galerkin method is one of the approximate methods in common use in applied mathematics and mechanics. The main points for treating the above problem are as follows. We first assume

$$\psi = \psi_0 + \sum_{j=1}^{n} A_j\psi_j, \tag{8}$$

and then choose appropriate ψ_0 and $\psi_j (j = 1, 2, 3, \ldots, n)$, so that ψ_0 may satisfy the boundary conditions (7), and ψ_j the homogeneous boundary conditions (7′)

$$\begin{aligned}
&r = 0, \quad \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi_j}{\partial r}\right) = 0, \quad \psi_j = 0, \\
&r = 1, \quad \frac{\partial \psi_j}{\partial r} = 0, \quad \psi_j = 0, \quad (j = 1, 2, \cdots\cdots, n), \\
&\bar{x} = \pm 1, \quad \psi_j = 0, \quad \frac{1}{r}\frac{\partial \psi_j}{\partial \bar{x}} = 0.
\end{aligned} \tag{7'}$$

As a result, ψ itself strictly satisfies (7) as well. Then according to the Galerkin's general procedure, the requirements of integral

$$\iint \psi_j E\psi dr d\bar{x} = 0, \quad (j = 1, 2, \cdots\cdots n), \tag{9}$$

is needed. We thus lead to a linear algebra equation system of n-order for the coefficients A_j:

$$\sum_{j=1}^{n} K_{ij}A_j = -K_{i0}, \quad (i = 1, 2, \cdots, n), \tag{10}$$

where

$$K_{ij} = \int_0^1\int_0^1 \psi_i E^4 \psi_j dr d\bar{x}, \; K_{i0} = \int_0^1\int_0^1 \psi_i E^4 \psi_0 dr d\bar{x}.$$

As soon as A_j are found out in this way, ψ thus obtained is an approximate solution for the problem.

In this paper, we assume that $n = 12$. As to the concrete forms of ψ_j, they may be chosen as polynomials with even powers being no more than 10 which are symmetric with respect to both axial and radial axes. Hence, ψ can be written as

$$\begin{aligned}
\psi = \psi_0 &+ (1 - \bar{x}^2)^2 r^2 (1 - r^2)^2 [(A_1 + A_2 r^2 + A_3 r^4) \\
&+ (A_4 + A_5 r^2 + A_6 r^4)\bar{x}^2 + (A_7 + A_8 r^2 + A_9 r^4)\bar{x}^4 \\
&+ (A_{10} + A_{11} r^2 + A_{12} r^4)\bar{x}^6].
\end{aligned} \tag{11}$$

We notice the conclusion in the References [2] and [4] that the velocity profile of plasma is similar to that of Poiseuille flow in the region over 1.3 r_0 far from the surface of the RBC. We may, therefore, suppose ψ_0 as

$$\psi_0 = \psi_p(r)y(\bar{x}) + F(r)[1 - y(\bar{x})], \tag{12}$$

where

$$\psi_p(r) = \frac{1}{2}[(1 - 4Q)r^2 - (1 - 2Q)r^4] \tag{13}$$

is the stream function of Poiseuille flow with flux Q. The other notations would mean:

$$y(\bar{x}) = \begin{cases} 1.0, & \bar{x} \leqslant \bar{x}_0, \\ (1 - \xi^4)^2, & \bar{x}_0 < \bar{x} \leqslant 1, \end{cases} \tag{14}$$

$$\bar{x}_0 = \begin{cases} 0, & L \leqslant 1.3, \\ 1 - \dfrac{1.3}{L}, & L > 1.3, \end{cases} \quad \xi = \frac{\bar{x} - \bar{x}_0}{1 - \bar{x}_0}.$$

It is easy to verify that ψ_0 and $\psi_j (j = 1, 2, 3, \ldots, n)$ in the presentations (12) and (11) satisfy the boundary conditions (7) and (7′) respectively. What we have done enables us not only to get an analytic form for K_{ij}, K_{i0} in (10), but also to reach a higher accuracy with as few terms as possible.

Clearly, the approximation of a function does not mean the approximation of its derivatives. Most likely, the approximate function will oscillate nearby the original one. It follows that the differential procedure for predicting velocity, pressure, stress and other physical variables might always bring about the deterioration of the results even though the expression (11) itself is a fairly precise approximation. Hence we would differentiate it with piece-wise numerical smoothing technique to diminish errors instead of directly using analytic formulas available.

In this paper, we apply fitting programme of cubic curves to the smoothed and differentiated functions. Provided that the interval between points is h for the function itself and its derivative after smoothing, we have

$$\begin{aligned}
f_{-2} &= \frac{1}{70}(69y_{-2} + 4y_{-1} - 6y_0 + 4y_1 - y_2), \\
f_{-1} &= \frac{1}{35}(2y_{-2} + 27y_{-1} + 12y_0 - 8y_1 + 2y_2), \\
f_0 &= \frac{1}{35}(-3y_{-2} + 12y_{-1} + 17y_0 + 12y_1 - 3y_2), \\
f_1 &= \frac{1}{35}(2y_{-2} - 8y_{-1} + 12y_0 + 27y_1 + 2y_2), \\
f_2 &= \frac{1}{70}(-y_{-2} + 4y_{-1} - 6y_0 + 4y_1 + 69y_2),
\end{aligned} \tag{15}$$

$$\begin{aligned}
f'_{-2} &= \frac{1}{84h}(-125y_{-2} + 136y_{-1} + 48y_0 - 88y_1 + 29y_2), \\
f'_{-1} &= \frac{1}{42h}(-19y_{-2} - y_{-1} + 12y_0 + 13y_1 - 5y_2), \\
f'_0 &= \frac{1}{12h}(y_{-2} - 8y_{-1} + 8y_1 - y_2), \\
f'_1 &= \frac{1}{42h}(5y_{-2} - 13y_{-1} - 12y_0 + y_1 + 19y_2), \\
f'_2 &= \frac{1}{84h}(-29y_{-2} + 88y_{-1} - 48y_0 - 136y_1 + 125y_2).
\end{aligned} \tag{16}$$

The procedures are needed once either in r or in $\bar{x}$ directions. We may leave out them for boundary points because of the satisfaction of the boundary conditions. We had better, whenever possible, utilize the expressions of f_0 and f'_0 in (15) and (16) for internal points, except for those in the vicinity of the boundary. If $\lambda < 1$, we see that the second derivative of $F(r)$ is discontinuous at $r = \lambda$. A greater error might arise unless the procedures are carried out on both sides of $r = \lambda$, respectively.

The stresses τ_B, τ_S at the vessel and the surface of the RBC and the pressure gradient (dp_s/dr) are all connected with the first and the second derivatives of the velocity at the boundary, that is

$$\tau_B = \left(\frac{\partial u}{\partial r}\right)_{r=1}, \tag{17}$$

$$\tau_s = \frac{1}{L}\left(\frac{\partial v}{\partial \bar{x}}\right)_{\bar{x}=\pm 1}, \tag{18}$$

$$\frac{dP_s}{dr} = \frac{1}{L^2}\left(\frac{\partial^2 v}{\partial \bar{x}^2}\right)_{\bar{x}=\pm 1}. \tag{19}$$

In order to find out the above-stated physical variables by numerical derivation, we take five neighbouring points with interval $h(y_0, y_1, y_2, y_3, y_4)$ and put the endpoint at $r=1$ or $x=1$ with the value at y_0 fixed. We may also smooth and differentiate a function through the least square cubic polynomial in the same manner and produce

$$\left.\begin{aligned} f_0' &= -\frac{1}{h}(4.1324476622z_1 - 2.8985507225z_2 + 0.4714170689z_3, \\ f_0'' &= \frac{2}{h^2}(-2.89855077224z_1 + 2.1521739114z_2 - 0.3623188403z_3, \end{aligned}\right\} \tag{20}$$

(where $z_1 = y_1+2y_2+3y_3+4y_4-10y_0$, $z_2 = y_1+4y_2+9y_3+16y_4-30y_0$, $z_3 = y_1+8y_2+27y_3+64y_4-100y_0$.) Remark that Eq. (6) holds in the meaning of weighted average of (9); the peaks of the weighted functions are probably nearby $r=1$ or $x=1$. Accordingly we may expect to obtain better approximations of stress and pressure gradient by differential procedure after smoothing.

Integrating the pressure over the surface and considering the balance between the wall stress and the pressure drop, we immediately derive the pressure distribution on the surface of the RBC and average pressure $\langle P\rangle$ at the section $x = \text{Const}$.

$$\Delta P_s = P_s - (P_s)_{r=0} = \int_0^r \left(\frac{dP_s}{dr}\right)_{\bar{x}=\pm 1} dr, \tag{21}$$

$$\langle P\rangle - \langle P\rangle|_{\bar{x}=0} = 2L\int_0^{\bar{x}} \tau_B d\bar{x}. \tag{22}$$

In addition, another important physical variable — apparent viscosity μ_{app}, i.e. the ratio of the average axial pressure drop of the Bolus Flow to the corresponding one of Poiseuille flow without RBC (hence $Q=0$) is

$$\mu_{\text{app}} = \frac{\langle P\rangle|_{\bar{x}=0} - \langle P\rangle|_{\bar{x}=1}}{8L}, \tag{23}$$

which is a measurement of plasma drag in the capillary.

IV. Results and Discussion

The basic parameters for determining the Bolus Flow are the ratio L of the distance between two adjacent RBC's to their diameter; the ratio λ of the RBC diameter to that of the vessel and the leakback parameter Q.

The magnitude of L depends on the diameter of a capillary and haematocrit of the

RBC's (i.e. the volumetric percentage of the RBC's in capillary). If the RBC's are uniformly distributed, we have

$$H = \frac{V}{V + \frac{\pi}{4} D_T^{*3} L}, \tag{24}$$

where the volume of the RBC is assumed to be $94.1\,\mu^3$. By ranging from 0.5 to 5.0, the average L are listed in Table 1 including various cases of great interest. As the effects of L on Bolus Flow have been treated in most literatures available, we would rather restrict our attention to those of another two parameters λ and Q.

Several pairs of λ and Q chosen are shown in Table 2, in the first five rows of which λ and Q correspond to each other according to "the zero drag condition" (i.e. the resulting force of pressure and stress vanishes). With the ratio of velocity U^* for cylindrical disk-shaped RBC to average velocity V^* being $2/(1+\lambda^2)$ and the leakback satisfying $2\pi r_0^* Q^* = \pi r_0^{*2}(U^* - V^*)$, we get

$$Q = \frac{1}{4}(1 - \lambda^2), \tag{25}$$

according to its definition. Remark that the expression (25) is only applicable to the cylindrical disk model. Since for the real elastic RBC there seems to be no such simple relation, some other pairs of λ and Q except the first five rows have been calculated. Generally speaking, the thinner the capillary and the slower the velocity, the more λ and the less Q will become. The problem as to which λ and Q ought to be taken under certain physiological or pathological conditions is beyond our consideration.

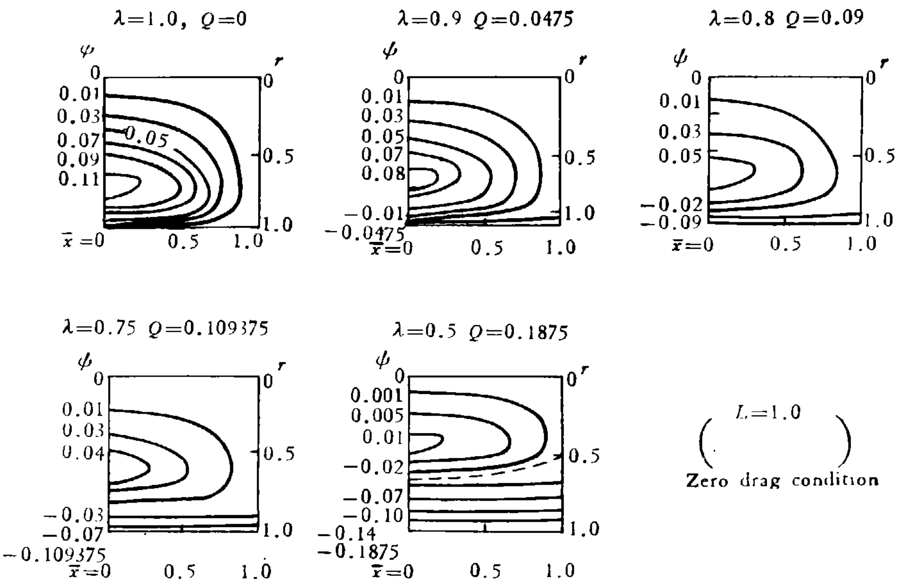

Fig. 2

In Fig. 2, we have sketched some of the typical Bolus Flow patterns with $L = 1$ and only a quarter of the flow field is drawn because of its symmetry in either $\bar{x}$ or r

direction. From this we may observe that the stream lines coming from the lubrication layer are nearly parallel to the wall of the vessel, and that there exists a high stress region there, whereas in the central part a closed circulation is formed. As λ is increasing and Q decreasing, the circulation grows weaker (i.e. the maximum of ψ drops) and closer to the axis. The changes of the pattern with L are essentially consistent with those given in Reference [4]. In Fig. 3, we have made comparison of velocity profiles between this paper and [2], which are fairly similar to each other with the exception that there ought to be a little difference at the exit and entrance of the lubrication layer due to different boundary conditions.

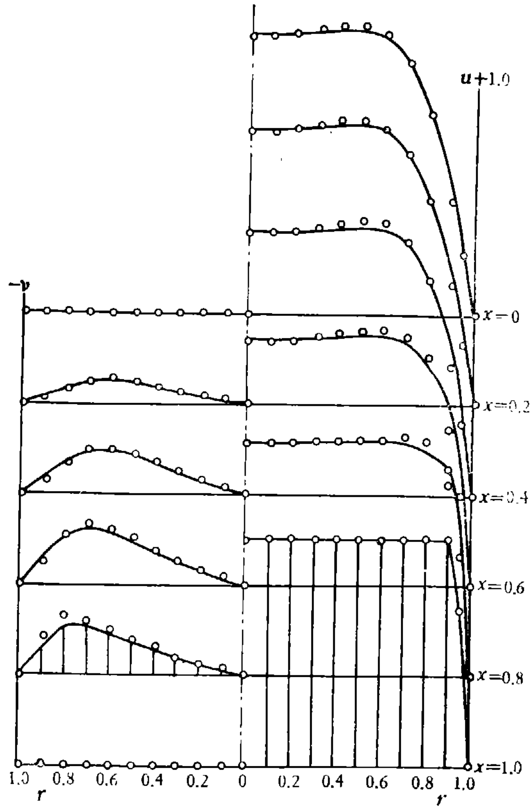

Fig. 3

Next, we attempt to analyse the pressure-flux relationship. The apparent viscosity μ_{app} (within the scope of 2—3 under general physiological conditions, Fig. 4) is rising as L approaches to 1, and so does it when λ is decreasing and the drag might even go below the value of Poiseuille Flow (Fig. 5). Whereas the lubrication theory will become worse with a smaller λ, we merely discuss the cases with greater λ in this paper. The leakback will considerably affect it with Q increasing and μ_{app} rapidly decreasing. Figs. 6 and 7 show that the stress τ_B on the wall approximates those of Poiseuille's at $\bar{x}=0$ and ascends violently at the exit or entrance of the lubrication layer ($\bar{x}=1$) to a certain value depending upon the velocity profile in this layer. The fact that as λ becomes smaller or Q greater, τ_B will fall a great deal perhaps may account for the tendency of μ_{app} in Figs. 4 and 5.

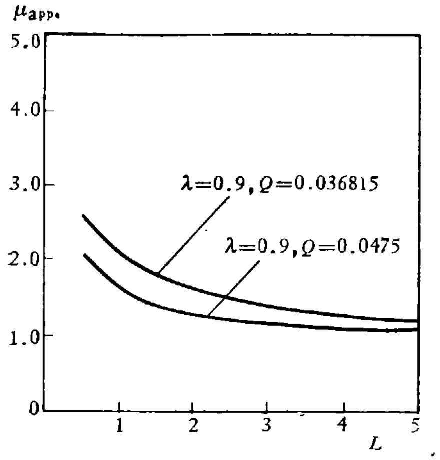

Fig. 4

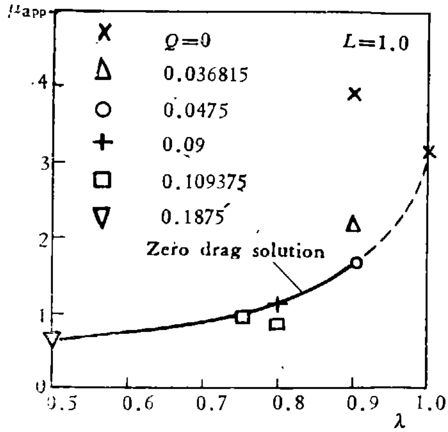

Fig. 5

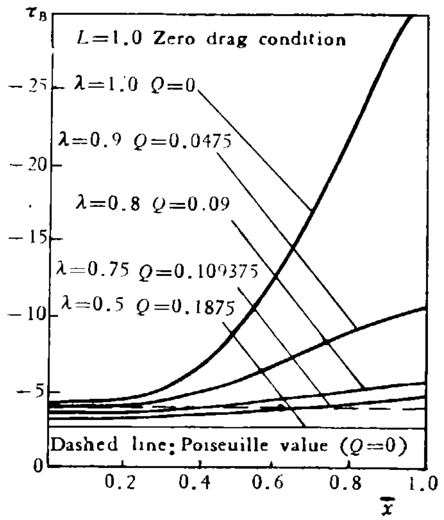

Fig. 6

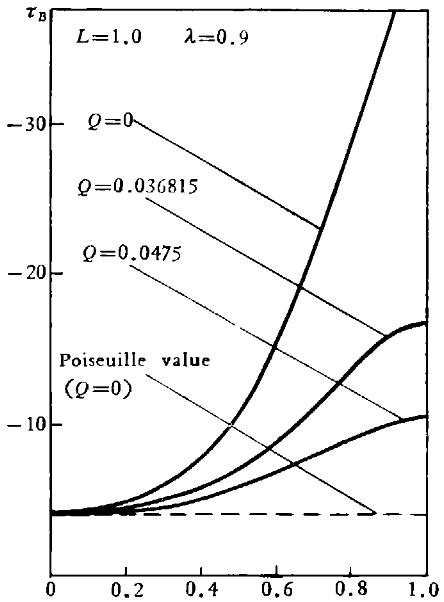

Fig. 7

Both the stress and the pressure distributions are essential for understanding haemolysis phenomenon and calculating the deformation of the RBC. In Figs. 8—11, we have shown the data of τ_S and ΔP_S at $\bar{x}=1$, that is equal to those at $\bar{x}=-1$ in absolute value with adverse sign. There we see that the stress and pressure gradients are smaller at the exit or entrance, while the pressure in the centre $r=0$ on the surface of the RBC is higher than that on the outer edge, which may account for the back recession of the RBC to form parachute shape after deformation. As we know, the pressure and stress are going up in a thinner capillary with greater λ or thinner lubrication layer as shown in Figs. 8 and 9. Consequently the pressure in the central part is much greater than that on the outer edge. Then we conclude that the recession of the RBC grows deeper in a thinner capillary, which is in complete agreement with

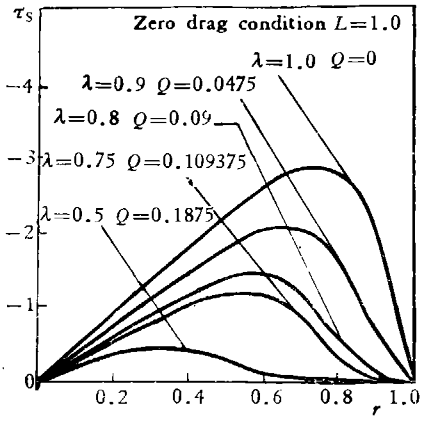

Fig. 8

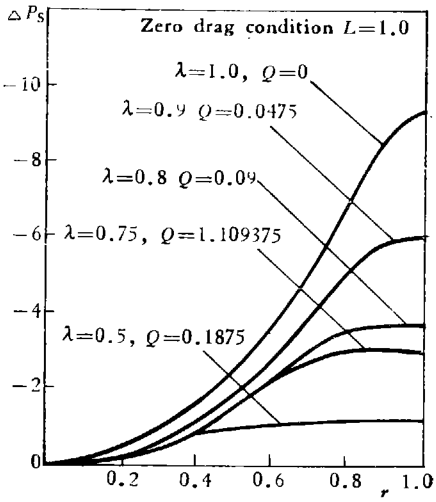

Fig. 9

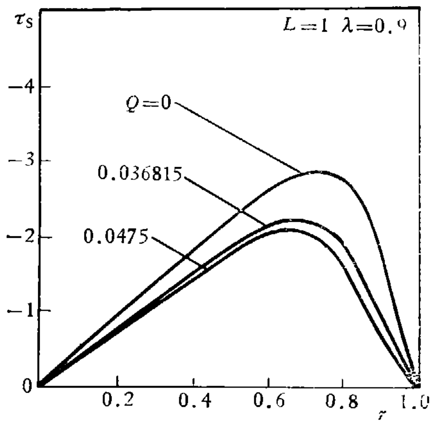

Fig. 10

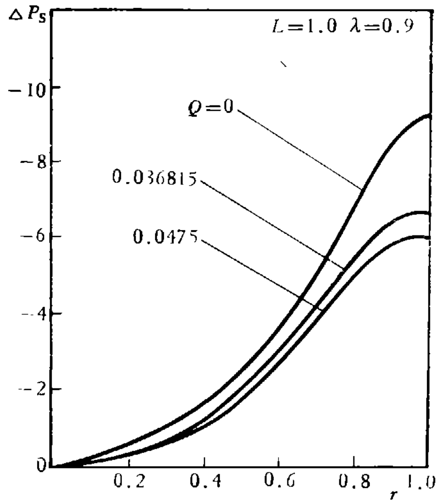

Fig. 11

modeling. Figs. 10, 11 illustrate that the leakback Q will slightly affect the stress and pressure distribution.

Finally, we intend to give a brief discussion on how the Bolus Flow will influence the mass transfer. Reference [4] has made a comparison between the durations needed for oxygen exchange through diffusion and convection. They believe that the circulation shows little influence on the gas transfer, nevertheless it appears to be important for high polymers such as protein. When the lubrication layer becomes thicker and λ smaller, the circulation will play a less important role in the process of mass transfer due to the slowing-down of the circulation and its greater distance from the wall. On the other hand, since more plasma saturated with oxygen from the lubrication layer (i.e. Q rises) keeps on supplying tissue nearby with it, the situation is advantageous to the exchange of gases.

V. Concluding Remarks

In this paper, something unreasonable in the mathematical model suggested in the references available is modified, and the correct boundary conditions at the entrance and exit of the lubrication layer are imposed.

We have paid more attention to the effects of the ratio λ of the diameter of the RBC to that of the capillary and those of the leakback parameter Q on the Bolus Flow. We have found that as the thickness of the lubrication layer and the leakback grow greater, the circulation will become weaker, the axial pressure drop lower and the period of circulation longer. With the thickness of the lubrication layer (or λ) fixed, the augment of the leakback will exert a remarkable influence on the stress at the wall of a capillary and on the axial pressure drop. It will also tend to strengthen gas exchange in the region nearby the wall. On the contrary, the leakback shows minor influence on both the stress and the pressure distributions on the surface of the RBC. The results are reasonably consistent with other theories.

The calculation in this paper has shown that the Galerkin method widely-used in solid mechanics is able to be applied effectively to fluid mechanics. The most prominent merit of the method lies in its minor numerical work. For example, it is enough to spend 6 minutes in operating CPU of a minicomputer with a speed of only 10,000 C/S in order to obtain 25 groups of data; whereas the numerical method for partial differential equations or double-series expansion method will involve some more calculation.

The assumption that the RBC is considered as a cylindrical disk is merely a rough approximation, which is justified on the ground that the flow at low Reynolds number is insensitive to local deformation of the RBC and the recess at the back of the RBC is shallow enough to be neglected.

To acquire a perfect understanding of the problem, we have to make close prediction of the RBC deformation and this is just the genuine task for future theory, as Fung once said. The author of [1] was a pioneer in this respect. The unsatisfactory qnantitative results are partly due to the rough model without taking into consideration the Bolus Flow on both sides of the RBC. It is expected that the stress and pressure on the surface of the RBC in the Bolus Flow in this paper and the physical parameters concerned in the lubrication layer by lubrication theory will comprise the entire approximate boundary conditions for the determination of the shape of the RBC. After putting forward a correct elastic model and considering the balance of the forces exerted on the RBC membrane, we are in a position to find an accurate shape of the RBC by iterating the procedure again and again. We have attempted to push a new step forward towards the task of elucidating the RBC deformation, which attaches the practical significance to this paper.

The author wants to take the present opportunity to express his deep indebtedness to Prof. H. S. Tan for his direction and help, and to Comrade Z. L. Tao for his comment on the first draft.

References

[1] Lighthill, M. J., *J. Fluid Mechanics*, **34**(1968), 113—143.
[2] Lew, H. S. & Fung, Y. C., The motion of the plasma between the red cells in the Bolus Flow, *Biorheology*, **6**(1969), 109—119.
[3] ——————, Plug effects of erythrocytes in capillary blood vessels, *Biophys. J.*, **10**(1970), 80—99.
[4] Bugliarello, G. & Hsiao, G. C., *Biorheology*, 7(1970), 5—36.
[5] Fitz-Gerald, J. M., *J. Fluid Mechanics*, **51**(1972), 463—476.
[6] 严宗毅，红细胞在毛细血管中的流动和变形，力学与实践，**3**(1981),2: 4—9.
[7] Fung, Y. C., *Microcirculation* (ed. Keley, G. & Altura, B. M.) 1(1977), 292—294.
[8] 冯康，数值计算方法，国防工业出版社，1978.