Planet. Space Sci. Vol. 29, No. 6, pp. 695–702, 1981
Printed in Northern Ireland

0032-0633/81/060695-08$02.00/0
Pergamon Press Ltd.

THE MECHANISM OF MAGNETOSPHERIC SUBSTORM AND THE MHD WAVES OF THE SOLAR WIND

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(*Received* 13 *January* 1981)

Abstract—A mechanism of the Earth's magnetospheric substorm is proposed. It is suggested that the MHD waves may propagate across the magnetopause from the magnetosheath into the magnetotail and will be dissipated in the plasma sheet, heating the plasma and accelerating the particles. When the solar wind parameters change, the Poynting flux of the waves transferred from the magnetosheath into the tail, may be greater than 10^{18} erg s^{-1}. The heated plasma and accelerated particles in the plasma sheet will be injected into the inner magnetosphere, and this may explain the process of the ring current formation and auroral substorm.

The Alfvén wave can only propagate along the magnetic force line into the magnetosphere in the open magnetosphere, but the magnetosonic wave can propagate in both the open and closed magnetosphere. When the IMF turns southward, the configuration of the magnetosphere will change from a nearly closed model into some kind of open one. The energy flux of Alfvén waves is generally larger than that of the magnetosonic wave. This implies that it is easy to produce substorms when the interplanetary magnetic field (IMF) has a large southward component, but the substorm can also be produced even if the IMF is directed northward.

1. INTRODUCTION

The magnetospheric substorm is one of the important problems in the magnetospheric physics (Akasofu, 1977). It seems also that the processes associated with solar flares are similar to those of the Earth's magnetospheric substorm. We have proposed a model of the solar flare, in which the fast magnetosonic wave excited in the convective region of the Sun will develop into fast magnetosonic wave and twist the magnetic field in the active region and store the energy in the form of magnetic energy. The twisted instability will result when the field is screwed tight enough, and then the configuration of the magnetic field deforms. As a result, the plasma is driven to move and form shock waves and the magnetic energy is converted into the kinetic energy of the shock waves. The shock sequence heats the gas and results in a thermal flare. The collision between the wave fronts can be considered to be similar to the collision between the magnetic mirrors, which accelerates the particles. As the Alfvén velocity decreases outward in the corona, the shock wave propagates easily along the magnetic arch in the corona. Only a few shocks can propagate along the magnetic arch downward into the chromosphere, where a new shock sequence is produced. As the magnetic field is much stronger in the chromosphere than in the corona, such a few events release a larger amount of energy and produce a double shock sequence moving back and forth, resulting in the features of a two-ribbon flare. The shock fronts produced in the chromosphere propagate upward, which supply the mass source of type II radio burst, and the collision between the wave fronts again accelerates the particles. This model explains the basic features of solar flares (Hu, 1979a, 1981a,b).

Perreault and Akasofu (1978) and Akasofu (1979) proposed a magnetospheric substorm mechanism based on the idea of solar wind–magnetosphere dynamo. We proposed the substorm model by considering the energy flux of MHD waves, which flows from the magnetosheath into the magnetosphere (Hu, 1979b, 1981c). In this paper, we consider our idea further.

2. THE FLUX ENERGY PROPAGATED INTO THE MAGNETOSPHERE

The magnetopause is regarded as the tangential discontinuity in the closed model of the magnetosphere. Keeping the total pressure $(p+B^2/8\pi)$ constant across the magnetopause, we obtain

$$p_s = 2k_1 nmv^2 \cos^2\psi = \frac{B_\tau^2}{8\pi} \qquad (2.1)$$

where ψ is the flaring angle of the magnetopause, and k_1 is a constant. In equation (2.1), we assume that the magnetic pressure is much smaller than the pressure in the magnetosheath and much larger than that in the magnetotail. Let us ignore small quantities and divide the quantity into two parts:

$$n = n_0 + n_1, \quad \mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1, \quad \mathbf{B}_\tau = \mathbf{B}_{\tau 0} + \mathbf{B}_{\tau 1} \tag{2.2}$$

where the subscripts 0 and 1 correspond, respectively, with the basic and perturbed states. Then the relation of the perturbed state is

$$2k_1 m(n_1 v_0^2 + 2n_0 \mathbf{v}_0 \cdot \mathbf{v}_1)\cos^2\psi \simeq \frac{\mathbf{B}_{\tau 0} \cdot \mathbf{B}_{\tau 1}}{4\pi}. \tag{2.3}$$

If we take the solar wind velocity $v_0 \simeq 400\ \mathrm{km\ s^{-1}}$, the perpendicular velocity of fluctuation $v_{1n} \simeq 10\ \mathrm{km\ s^{-1}}$, $n_0 = 10\ \mathrm{cm^{-3}}$, the diameter of the magnetotail $d \simeq 20R_E$, the section length of the magnetotail $L \simeq 50R_E$ and $k\cos^2\psi = 0.2$, then the energy flux perpendicular to the magnetopause is given as follows:

$$E_c = 2mn_0 v_0 v_{1n}^2 \cdot \pi d \cdot L \cdot (k_1 \cos^2\psi) \simeq 1 \times 10^{17}\ \mathrm{erg\ s^{-1}}. \tag{2.4}$$

The energy estimated in (2.4) is less than the dissipated energy of the substorm. Sometimes, the velocity and the density of the solar wind can be increased to values much larger than what is shown above, and the v_{1n} can also be increased several times. In this case, the energy flux can be increased to 10^{18}–$10^{19}\ \mathrm{erg\ s^{-1}}$ or even larger, which is enough to supply the energy of the magnetospheric substorm.

In the case of the open model of the magnetosphere, the Alfvén wave can propagate along the open magnetic force line from the magnetosheath into the magnetosphere. The perturbed electric field can be written as:

$$\mathbf{E}_1 = -\frac{1}{c}(\mathbf{v}_1 \times \mathbf{B}_0 + \mathbf{v}_0 \times \mathbf{B}_1). \tag{2.5}$$

If the density $\rho = \rho_0$, the relation of Alfvén fluctuation is

$$\mathbf{v}_1 = \pm \frac{\mathbf{B}_0}{\sqrt{4\pi\rho_0}}. \tag{2.6}$$

Then, the energy flux is

$$\mathbf{E}_A = -\rho_0 \mathbf{v}_1 \times \left(\mathbf{v}_0 + \frac{\mathbf{B}_0}{\sqrt{4\pi\rho}}\right) \times \frac{\mathbf{B}_0}{\sqrt{4\pi\rho}} \cdot \pi d \cdot L \tag{2.7}$$

where the relation $|\mathbf{v}_0| \gg |\mathbf{B}_0|/\sqrt{4\pi\rho}$ is used. Here, $\mathbf{B}_0$ is the magnetic field in the solar wind. If we take $n_0 = 10\ \mathrm{cm^{-3}}$, $v_0 = 400\ \mathrm{km\ s^{-1}}$, $B_0 = 5\gamma$, $v_1 = 0.1v_0$, then the order of the average Poynting flux of the Alfvén wave is

$$E_A = \pi d \cdot L \cdot \langle E_A \rangle \simeq 4 \times 10^{17}\ \mathrm{erg\ s^{-1}}, \tag{2.8}$$

which is larger than E_c. In some cases, the solar wind velocity v_0, IMF B_0, and the perturbed velocity v_1 can be much larger than the value shown above, and the E_A may be larger than 10^{18}–$10^{19}\ \mathrm{erg\ s^{-1}}$, which is enough to supply a magnetospheric substorm and even magnetic storm, too.

It shows that the MHD wave in the solar wind may be the energy source of the substorm. The dissipation process of the MHD waves will dominate the process of the magnetospheric substorm.

3. THE SUBSTORM MECHANISM FOR THE CLOSED MAGNETOSPHERE

(a) *The propagating features of the fast magnetosonic waves in non-uniform medium*

The equations of magnetohydrodynamics may be written in Cartesian system as follows:

$$\begin{gathered}\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\mathbf{v}) = 0, \\ \frac{\rho\,\mathrm{d}v}{\mathrm{d}t} = -a^2\nabla\rho + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}, \\ \frac{\partial\mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \\ \nabla \cdot \mathbf{B} = 0.\end{gathered} \tag{3.1}$$

The propagation features of the fast magnetosonic wave in the magnetotail can be approximated as a one-dimensional problem. The basic state is chosen as

$$\mathbf{B}_0 = [B_0(Z), 0, 0], \quad \mathbf{v}_0 = 0, \quad \rho_0 = \rho_0(Z), \quad a_0 = \text{const.} \tag{3.2}$$

where the B_0 and ρ_0 are not uniform. The equilibrium relation of the basic state is

$$\rho_0 a_0^2 + \frac{B_0^2}{8\pi} = P_T\ (\text{const.}) \tag{3.3}$$

The perturbed equation for the z-component of velocity is

$$\frac{\partial^2 w}{\partial t^2} = a_0^2 \frac{P_T + B_0^2/8\pi}{P_T - B_0^2/8\pi}\frac{\partial^2 w}{\partial z^2} + \frac{a_0^2}{P_T - B_0^2/8\pi}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{B_0^2}{8\pi}\right)\frac{\partial w}{\partial z}. \tag{3.4}$$

Equation (3.4) describes the fast magnetosonic wave propagating perpendicular to $\mathbf{B}_0$. Substituting the wave form $w(t,z)=\hat{w}(t)\exp(i\omega t)$ into (3.4), we obtain

$$\left(P_T+\frac{B_0^2}{8\pi}\right)\frac{d^2\hat{w}}{dz^2}+\frac{d}{dz}\left(\frac{B_0^2}{8\pi}\right)\frac{d\hat{w}}{dz}+\left(\frac{\omega}{a_0}\right)^2\left(P_T-\frac{B_0^2}{8\pi}\right)\hat{w}=0. \quad (3.5)$$

Using the transformation between z and ζ,

$$\frac{d\zeta}{d\zeta}=P_T+\frac{B_0^2(z)}{8\pi}>0. \quad (3.6)$$

Then, equation (3.5) can be reduced to the Stürm equation:

$$\frac{d^2\hat{w}}{d\zeta^2}+\left(\frac{\omega}{a_0}\right)^2\left[P_T-\left(\frac{B_0^2}{8\pi}\right)^2\right]\hat{w}=0 \quad (3.7)$$

It shows that equation (3.7) is certainly a wave equation for non-uniform medium, because we have $P_T>B_0^2/8\pi$.

The dispersion relation can be given by the WKB method for short waves if the basic state varies only slowly. Substituting the relation $\hat{w}\propto\exp(i\int k\,dz)$ into (3.4), we obtain

$$\left(P_T+\frac{B_0^2}{8\pi}\right)k^2-i\frac{d}{dz}\left(\frac{B_0^2}{8\pi}\right)k-\left(\frac{\omega}{a_0}\right)^2\left(P_T-\frac{B_0^2}{8\pi}\right)=0. \quad (3.8)$$

Then the dispersion relation is

$$k=\frac{i\dfrac{d}{dz}\left(\dfrac{B_0^2}{8\pi}\right)\pm\sqrt{4\left(\dfrac{\omega}{a_0}\right)^2\left[P_T^2-\left(\dfrac{B_0^2}{8\pi}\right)^2\right]-\left[\dfrac{d}{dz}\left(\dfrac{B_0^2}{8\pi}\right)\right]^2}}{2\left(P_T+\dfrac{B_0^2}{8\pi}\right)}. \quad (3.9)$$

It shows that the wave will amplify if it propagates into the region with a weaker magnetic field and a lower density, and *vice versa*. Equation (3.2) gives the relation of basic state in the tail; the strength of magnetic field is increasing as the density decreases, and *vice versa*. The quantitative feature of the fast waves can be given only by a computational method. The two independent solutions of (3.5) are calculated for the initial values:

$$\text{(i)}\ \hat{w}(o)=0,\quad \frac{d\hat{w}(o)}{dz}=1;$$

$$\text{(ii)}\ \hat{w}(o)=1,\quad \frac{d\hat{w}(o)}{dz}=0 \quad (3.10)$$

Two typical basic states are chosen as follows:

$$\text{(i)}\ \frac{B_0^2}{8\pi}=P_T-P_1(1+\alpha e^{-z/z_0}),\quad \frac{P_T}{P_1}=2.2,\quad \alpha=1; \quad (3.11)$$

$$\text{(ii)}\ \frac{B_0^2}{8\pi}=\beta\left(\frac{z}{z_0}\right)^{\delta},\quad \beta=1,\quad \delta=2,1,0.5. \quad (3.12)$$

The calculated results are given in Figs. 1 and 2. It shows that the amplitude is nearly constant.

We apply the above results to explain the wave propagation in the magnetotail. The fast magnetosonic wave excited by the turbulence in the solar wind propagates across the magnetopause toward the plasma sheet. The amplitude of the wave is nearly a constant in the magnetomantle and plasma sheet, so that it will amplify in the magnetotail.

(b) *The dissipation of the wave energy*

The velocity of fast magnetosonic waves is $\sqrt{a_0^2+v_{A0}^2}$ in the direction perpendicular to B_0 while $v_{A0}=B_0/\sqrt{4\pi\rho_0}$ is the Alfvén velocity in the basic state. The velocity at the wave peak and the valley is u and $-u$ respectively, so the relative velocity between them is $2u$. The fast wave will develop into a fast MHD shock wave. The time scale for producing a shock wave is determined by the overlapping of the wave between the peak and the valley. The order of the time is

$$T=0\left(\frac{\lambda/2}{2u}\right)=0\ \frac{\pi}{2}\frac{\sqrt{a_0+v_{A0}}}{\omega u}\Bigg) \quad (3.13)$$

where λ is the wave length.

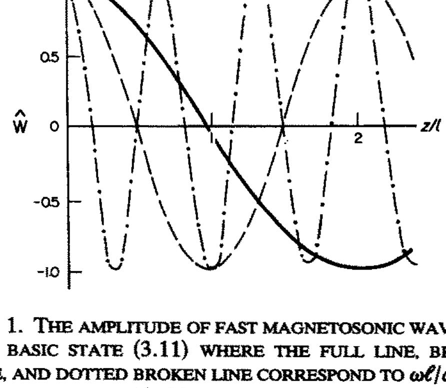

FIG. 1. THE AMPLITUDE OF FAST MAGNETOSONIC WAVE FOR THE BASIC STATE (3.11) WHERE THE FULL LINE, BROKEN LINE, AND DOTTED BROKEN LINE CORRESPOND TO $\omega\ell/a_0=5$, 10, 30, RESPECTIVELY.

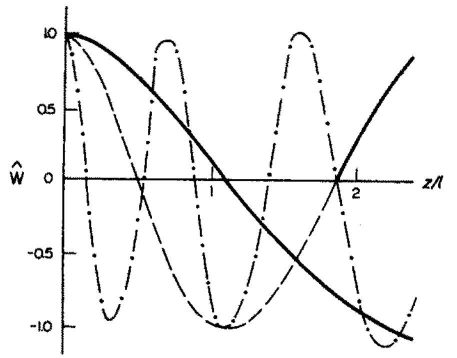

FIG. 2. THE SAME AS FIG. 1 EXCEPT FOR THE BASIC STATE (3.12).

In the magnetotail lobe, we have $a \ll v_{Ao}$. Taking $v_{Ao} > 10^2\ \text{km s}^{-1}$, $u = 10\ \text{km s}^{-1}$, $\omega = 10^2\ \text{s}^{-1}$, we shall have $T \simeq 10$ s, and the corresponding distance for producing the shock is $v_{Ao}T > 10^5$ km, which is larger than the distance between the magnetopause and the plasma sheet. The higher the wave frequency is, the shorter the distance is. In the region of the plasma sheet, we have $v_{Ao} \leqslant a$. Taking $a_0 \simeq 20\ \text{km s}^{-1}$, $u \geqslant 10\ \text{km s}^{-1}$, it gives $T < 10^2$ for the same frequency. This corresponds to a propagating distance of about $a_0T < 4 \times 10\ \text{km} < 1R_E$, which is smaller than the thickness of the plasma sheet. However, the measured results by satellites show the frequency of maximum power spectrum is near $10^2\ \text{s}^{-1}$ (Fairfield *et al.*, 1970).

The average decay length of the shock wave is

$$L_s = 0(4t_0\sqrt{a_0^2 + v_{Ao}^2}/\eta) \simeq 5000\ \text{km} \quad (3.14)$$

where the typical time $t_0 \simeq 1/8\omega$, and the strength of the shock wave is taken as 0.2 in the plasma sheet.

For a weak shock, the propagating equation (Osterbrock, 1961) is

$$\frac{d}{dh}(\sqrt{a_0^2 + v_{Ao}^2}\ \pi F_+) = -\frac{1}{4}(a_0^2 + v_{Ao}^2)^{\alpha - 1/2}\eta\frac{\pi F_+}{t_0} \quad (3.15)$$

where h is the depth along the propagating direction. The energy flux is

$$\pi F_+ = \frac{\rho_0\eta^2}{8}\sqrt{a_0^2 + v_{Ao}^2} \times \left\{a_0^2 + \frac{2v_{Ao}^2}{3[1 - v_{Ao}^2/3(a_0^2 + v_{Ao}^2)]^2}\right\} \quad (3.16)$$

and the coefficient

$$\alpha = \begin{cases} 0 & \text{if} \quad a_0^2 + v_{Ao}^2 < (a_0^2 + v_{Ao}^2)_i, \\ 1 & \text{if} \quad a_0^2 + v_{Ao}^2 > (a_0^2 + v_{Ao}^2)_i, \end{cases} \quad (3.17)$$

with the subscript i denoting the initial value. In the plasma sheet, we omit v_{Ao}, which is smaller than a_0. Then equation (3.18) reduces to

$$\frac{d}{dh}[\ell n(\rho_0 a_0^{2\alpha+3}\eta^2)] + \frac{1}{4a_0t_0}\eta = 0 \quad (3.18)$$

The above equation can be changed into a linear equation of $1/\eta$, and its solution will be

$$\frac{1}{\eta} = \frac{\sqrt{\rho_0 a_0^{2\alpha+3}}}{(\rho_0 a_0^{2\alpha+3})_i}\left\{\frac{1}{\eta_i} + \int_0^h \frac{1}{8a_0t_0}\left[\frac{(\rho_0 a_0^{2\alpha+3})_i}{\rho_0 a_0^{2\alpha+3}}\right]^{1/2} dh\right\}. \quad (3.19)$$

As both ρ_0 and a_0 increase, η, as well as energy flux πF_+, will decrease when h is increasing.

For the shock wave with a finite amplitude, the evolutionary relation of energy and amplitude can be written as:

$$\frac{\hat{w}}{w_i} = \frac{1}{1 + \frac{2\alpha_1 t}{\lambda}\hat{w}_i}, \quad \frac{\pi F_+}{(\pi F_+)_i} = \frac{1}{\left(1 + \frac{2\alpha_1 t}{\lambda}\hat{w}_i\right)^2} \quad (3.20)$$

for waves with a saw shape profile, where $\alpha_1 = (\gamma + 1)/2$ for perfect gas. It is easy to give

$$\frac{\pi F_+}{(\pi F_+)_i} = \frac{1}{\left(1 + \frac{\gamma + 1}{2\pi}\frac{\omega t\hat{w}_i}{\sqrt{a_0^2 + v_{Ao}^2}}\right)^2}. \quad (3.21)$$

Then, the typical length of decay for the shock energy is

$$L_{1/2} = \frac{0.8\pi}{\gamma + 1}\frac{\sqrt{a_0^2 + v_{Ao}^2}}{\omega} \simeq \frac{1}{\omega}\sqrt{a_0^2 + v_{Ao}^2}. \quad (3.22)$$

The result of (3.22) agrees with the result from (3.14). The particles will be heated and accelerated by the dissipative energy of the shock wave.

(c) *The acceleration of the particle in the plasma sheet*

The observations show that the parameters in the plasma sheet have rapid and finite fluctuations, which can be explained as the result of the heating by intermittent shocks. In the plasma sheet, the energy dissipated from the shock waves will excite the secondary shock, the MHD waves, the plasma waves, and so on. The wave fronts collide with each other, and the relative motion of wave fronts along the direction of the magnetic field can be understood as the random Fermi acceleration process, and the accelerated particles will be scattered after every collision. Of course, only the particles with energy larger than the threshold energy can be accelerated step by step.

According to these ideas, a hot plasma sheet is the inevitable result of the wave energy flux. The thermal particles are produced by the energy dissipation of the shock waves, and the energetic particles are produced by the Fermi accelerating process. When the Poynting flux is increased, the plasma is much heated to produce more and more energetic particles, which are injected into the nearby Earth space or the polar region to form the ring current and to produce the aurora. This mechanism of the magnetospheric substorm can be applied to the open model of the magnetosphere.

4. THE SUBSTORM MECHANISM FOR THE OPEN MAGNETOSPHERE

(a) *The propagation of Alfvén wave in a shear flow field*

In the magnetosheath and the magnetomantle, the velocity is large and non-uniform. The propagation of the Alfvén wave is considered in this case. For simplicity, the basic state in the Cartesian coordinate is as follows:

$$\mathbf{v}_0=(v_{xo}(Z),0,V_{zo}),\quad \mathbf{B}_0=(B_{xo}(Z),0,B_{zo}). \tag{4.1}$$

where v_{zo} and B_{zo} are constants, and are smaller than v_{xo} and B_{xo}, respectively. From equations (4.1), the equilibrium relation of the basic state is

$$\rho_0 v_{zo}\frac{\mathrm{d}v_{xo}}{\mathrm{d}z}=\frac{B_{zo}}{4\pi}\frac{\mathrm{d}B_{xo}}{\mathrm{d}z},\quad \frac{\mathrm{d}P_0}{\mathrm{d}z}=-\frac{B_{xo}}{4\pi}\frac{\mathrm{d}B_{xo}}{\mathrm{d}z}, \tag{4.2}$$

which can be integrated to give

$$\rho_0 v_{zo}v_{xo}(Z)-\frac{B_{zo}B_{xo}(Z)}{4\pi}=c_1, \tag{4.3}$$

$$P_0(Z)+\frac{1}{8\pi}B_{xo}{}^2(Z)=c_2 \tag{4.4}$$

where c_1 and c_2 are integration constants.

Considering the perturbed B_{y1} and v_{y1}, the equations for the Alfveń wave are derived by using the MHD equations (3.1), that is

$$\frac{\mathrm{D}B_{y1}}{\mathrm{D}t}\equiv\frac{\partial B_{y1}}{\partial t}+(\mathbf{v}_0\cdot\nabla)B_{y1}=B_{xo}\frac{\partial v_{y1}}{\partial x}+B_{zo}\frac{\partial v_{y1}}{\partial Z}, \tag{4.5}$$

$$\frac{\mathrm{D}v_{y1}}{\mathrm{D}t}\equiv\frac{\partial v_{y1}}{\partial t}+(\mathbf{v}_0\cdot\nabla)v_{y1}=\frac{1}{4\pi\rho_0}\left(B_{xo}\frac{\partial B_{y1}}{\partial x}+B_{zo}\frac{\partial B_{y1}}{\partial z}\right). \tag{4.6}$$

The term in the left hand side is the time differentiation by following the fluid particles, and that in the right hand side is the gradient along the magnetic force line. Combining equations (4.5) and (4.6) gives

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{\mathrm{D}B_{y1}}{\mathrm{D}t}\right)=\frac{1}{4\pi\rho_0}(\mathbf{B}_0\cdot\nabla)[(\mathbf{B}_0\cdot\nabla)B_{y1}]. \tag{4.7}$$

Expanding the $B_{y1}(t,z)$ in the spectrum form, we can denote

$$B_{y1}(t,z)=\sum_n b_n\exp[i(\omega_n t-\mathbf{k}_n\cdot\mathbf{r})]. \tag{4.8}$$

Substituting (4.8) into (4.7), we obtain the component relation as

$$(\omega_n-\mathbf{k}\cdot\mathbf{v}_0)^2=\frac{(\mathbf{B}_0\cdot\mathbf{k})^2}{4\pi\rho_0}+ik_x\left[v_{zo}\frac{\mathrm{d}v_{xo}}{\mathrm{d}z}-\frac{B_{zo}}{4\pi\rho_0}\frac{\mathrm{d}B_{xo}}{\mathrm{d}z}\right]. \tag{4.9}$$

By using the first relation of (4.2), the equation (4.9) becomes

$$(\omega_n-\mathbf{k}\cdot\mathbf{v}_0)^2=\frac{(\mathbf{B}_0\cdot\mathbf{k})^2}{4\pi\rho_0}. \tag{4.10}$$

Then the dispersion relation of the Alfvén wave in a shearing flow is

$$\frac{\omega_n}{k_n}=\mathbf{k}_n{}^0\cdot\mathbf{v}_0\pm\frac{B_{on}}{\sqrt{4\pi\rho_0}} \tag{4.11}$$

where the subscript n denotes the normal component, and $\mathbf{k}_n{}^0$ is the unit vector of wave number.

The above results may be applied to the open model of magnetosphere. Solar wind velocity in the magnetosheath and the plasma velocity in the magnetomantle are described by $v_{xo}(z)$, which is decreased to a very small value in the magnetotail, and the flow is a shear flow. On the other side, the strength of the magnetic field is increasing from the magnetosheath to the magnetomantle and the magnetotail. The Alfvén wave can propagate from the magnetosheath across the magnetomantle into the magnetotail. Then the energy flux of the Alfvén wave in the solar wind is transferred into the magnetotail, which can supply the energy of the substorm in the open model of the magnetosphere.

(b) *The dissipation of the Alfvén wave*

In the magnetotail, the magnetic viscosity is much smaller than the kinetic viscosity. As the Alfvén wave propagates along the magnetic field the typical length of decay can be written as (see Osterbrock, 1961)

$$L_A = \frac{v_A{}^3}{4\pi^2\nu^2\left(\frac{c^2}{4\pi\sigma} \pm \frac{\mu}{\rho}\right)} \simeq \frac{v_A{}^3}{4\pi^2\nu^2(\mu/\rho)}. \tag{4.12}$$

The order of the kinetic viscosity is the product of the mean free path of particles λ and the sonic velocity a, that is

$$\frac{\mu}{\rho} \simeq \lambda \cdot a = \frac{\sqrt{\gamma RT}}{\sqrt{2}\pi\sigma_1{}^2 n_0}. \tag{4.13}$$

where σ_1 is the radius of the atom, R the gaseous constant. In the plasma sheet, $n_0 \simeq 0.01\text{–}1\ \text{cm}^{-3}$, $T \simeq 10^7$ K; and in the magnetotail, $n_0 \simeq 10\ \text{cm}^{-3}$, $T \simeq 10^5$ K. Taking $\sigma_1 = 2.3 \times 10^{-8}$ cm, the order of the viscosity is

$$\left(\frac{\mu}{\rho}\right)_{\text{tail}} \simeq 4 \times 10^{15}\ \text{cm}^2\,\text{s}^{-1}, \quad \left(\frac{\mu}{\rho}\right)_{\text{sheet}} = 4 \times 10^{14}\ \text{cm}^2\,\text{s}^{-1}. \tag{4.14}$$

If the magnetic field is 10γ in the lobe of magnetotail, and 0.5γ in the plasma sheet, then the Alfvén velocity is

$$(v_A)_{\text{tail}} \simeq 10^8\ \text{cm}\,\text{s}^{-1}, \quad (v_A)_{\text{sheet}} \simeq 10^6\ \text{cm}\,\text{s}^{-1}. \tag{4.15}$$

Substituting (4.14) and (4.15) into (4.12), we obtain

$$(L_A)_{\text{tail}} \simeq 10^{12}\ \text{cm}, \quad (L_A)_{\text{sheet}} = 10^7\ \text{cm}, \tag{4.16}$$

where the frequency ν is taken as 1/300 s. The results of (4.16) show that the Alfvén wave carries energy flux nearly without dissipation as it propagates through the lobe of the magnetotail into the plasma sheet, where the waves dissipate rapidly.

The geomagnetic field above the polar region is connected with the IMF for the open model of the magnetosphere. So the Alfvén waves propagate also from the magnetosheath into the polar region. The effect of the collision between ions and neutral atoms dissipates the Alfvén wave, heating the gas in the polar region.

(c) *Acceleration process*

The heating and accelerating process by dissipation of the Alfvén wave in the plasma sheet for the open magnetosphere may be similar to that suggested for the closed magnetosphere. Then, the energetic particles and the high speed plasma are injected into the inner magnetosphere to form the ring current and produce the aurora.

Another possibility is a local acceleration process. There are fluctuations in the closed geomagnetic field region which is surrounded by the open magnetic field. Alfvén waves propagate along the open magnetic force line and will disturb the state of the closed geomagnetic field region. Then, there will be an effective electric field

$$\mathbf{E}_{\text{eff}} = \langle \mathbf{v}_1 \times \mathbf{B}_1 \rangle \tag{4.17}$$

where the bracket $\langle\ \rangle$ is the average of the value. According to the turbulent dynamo theory, we can calculate the effective electric field in some cases. Such effective electric field may accelerate the particle in the inner magnetosphere. When the Alfvén fluctuation intensifies, the effective electric field in the closed geomagnetic field region will also intensify, as well as the ring current.

The process given above can be used to explain the features of substorm for the open magnetosphere. In this case, the magnetosonic wave can also operate, but its energy flux is smaller than that of Alfvén wave.

5. THE SUBSTORM PROCESS AND THE INTERPLANETARY MAGNETIC FIELD

Substorm processes are dominated by the parameters of solar wind which determine the energy flux from the magnetosheath into the magnetosphere. The configuration of the magnetosphere is influenced by the IMF. When IMF points northward, the Earth's magnetosphere keeps its configuration nearly to the closed model. As the IMF turns southward, the configuration of the Earth's magnetosphere is transformed from the closed model to an open one. Hence, the energy flux of the Alfvén wave flows into the magnetosphere, and is converted into the internal or kinetic energy of particles. As the energy flux of the Alfvén wave is often larger than that of the magnetosonic wave, it is easier to produce the substorm in the case of the open magnetosphere than in the case of the closed one. In other words, when the IMF turns southward, the configuration of magnetosphere is transformed into the open one, and the substorm appears often. But the relation between the southward turning of the IMF and the appearance of a substorm do not coincide in every case. The substorm may appear even in the case without the southward component of the IMF.

By using the wave mechanism to explain the substorm process, the energy flux of waves is supplied continuously from the solar wind to the magnetosphere. In the lobe of the magnetosphere, the Alfvén velocity is about $10\,\mathrm{km\,s^{-1}}$. The typical propagation time of waves from the magnetopause to the plasma sheet has the order of

$$T_1 \simeq \frac{L}{v_{\mathrm{Ao}}} \simeq \frac{15R_E}{10^2\,\mathrm{km\,s^{-1}}} \simeq 10^3\,\mathrm{s}. \qquad (5.1)$$

As the IMF turns southward, there appears a relax time for the magnetosphere to change its configuration from a nearby closed model into an open one. We expect that the relax time has the same order of the time estimated in (5.1). Once the parameters of the solar wind are changed, it may give an energy flux of wave larger than $10^{18}\,\mathrm{erg\,s^{-1}}$; such an amount of energy flux can be transferred to the plasma sheet after a duration about $10^3\,\mathrm{s^{-1}}$. Then, the energy particles will be injected toward the inner magnetosphere from the plasma sheet. For a distance of $50R_E$ away from Earth, and a typical velocity of $300\,\mathrm{km\,s^{-1}}$, the typical flow time is

$$T_2 \simeq \frac{50R_E}{300\,\mathrm{km\,s^{-1}}} \simeq 10^3\,\mathrm{s}. \qquad (5.2)$$

This shows that the auroral substorm and the ring current will appear after $\frac{1}{2}$ an hour when the energy flux of the solar wind is intensified. In this process, it is not necessary to have a storage process to store the magnetic energy beforehand. The storage process and the triggering process will combine together. The substorm will be produced if a large energy influx of MHD waves is maintained long enough. The criterion for the appearance of a substorm can be written as

$$a_e \int_0^T \mathbf{E}_A \cdot \mathbf{e}_z\,dt > 10^{21}\text{–}10^{22}\,\mathrm{ergs}. \qquad (5.3)$$

where a_e is a coefficient smaller than 1, and T the duration of the expansive phase of substorm. From the point of view on the energy, the substorm cannot appear if the energy is not large enough although the energy flux of waves may be larger for a short duration.

Akasofu (1979) has discussed the solar-magnetosphere dynamo process. By using the Poynting flux function

$$\varepsilon = v_0 B_0^2 f(\theta) \ell_0^2 \qquad (5.4)$$

where $\ell_0 \simeq 7\ R_E$ is a constant, and $\theta = \tan^{-1}(B_y/B_z)$. He obtained a good correlation between ε and the index AE. For simplicity, the function $f(\theta)$ is chosen as

$$f(\theta) = \sin^4\left(\frac{\theta}{2}\right). \qquad (5.5)$$

Comparing the energy flux given here with that given in (5.4), we imagine that the perturbed velocity v_1 is approximately proportional to the Alfvén wave velocity, that is

$$v_{1n} = k_2 \frac{B_0}{\sqrt{4\pi\rho_0}} \qquad (5.6)$$

where k_1 is a coefficient, which may be dependent on the angle θ and ϕ. Substituting (5.6) into (2.4), we obtain the Poynting flux of magnetosonic wave as follows:

$$E_c = B_0^2 v_0 [2k_2^2 \pi\, dL(k_1 \cos^2\psi)] = B_0^2 v_0 f_1(\theta, \phi) \ell_0^2 \qquad (5.7)$$

where

$$f_1(\theta, \phi) = k_2^2(\theta, \phi) \frac{2\pi\, dL(k_1 \cos^2\psi)}{\ell_0^2}. \qquad (5.8)$$

In other words, if we assume that the perturbed magnetic field is also approximately proportional to the magnetic field of the basic state:

$$B_1 = k_3 B_0. \qquad (5.9)$$

Then, the equation (2.7) gives

$$E_A = v_0 B_0^2 f_2(\theta, \phi) \ell_0^2 \qquad (5.10)$$

where

$$f_2(\theta, \phi) = |k_2(\mathbf{v}_1^0 \times \mathbf{v}_0^0) \times \mathbf{B}_0^0 + k_3(\mathbf{v}_0^0 \times \mathbf{B}_1^0) \times \mathbf{B}_0^0| \frac{\pi\, dL}{\ell_0^2}. \qquad (5.11)$$

Qualitatively, the $f_2(\theta, \phi)$ is larger when the IMF has a larger southward component, and $f_2(\theta, \phi)$ is larger than $f_1(\theta, \phi)$ in general. So, our results have the similar function form with Akasofu's, and both results are proportional to the $v_0 B_0^2$. However, the dependence of angles is quantitatively different, but the general tendency has some similar features qualitatively.

During the expansive phase of a substorm, the magnetopause is compressed and the plasma sheet changes its thickness. There occurs a Poynting flux

TABLE 1. THE TIME SEQUENCE OF MAGNETOSPHERIC SUBSTORM

Time (min)	Configuration features	Theoretical explanation
−30∼−10	1. The IMF turn to south or the fluctuation of solar wind intensification 2. The magnetopause is concentrated and the radius of the magnetotail decreases 3. The fluctuation of plasma parameters in magnetotail lobe is increased	1. The energy flux of MHD waves which propagates from magnetosheath into magnetotail increases 2. Solar wind or interplanetary shock compress the magnetosphere 3. MHD wave and fluctuation propagate to the plasma sheet
−10∼0	1. The plasma sheet is thinning 2. The fluctuations of parameters are increasing. in the plasma sheet 3. The *H*-component of geomagnetic field starts to decrease in polar region	1. The plasma sheet responds to the concentration of the magnetopause 2. The heating and accelerating process is intensive in the plasma sheet 3. Energy particles escape and precipitate into the polar region
0∼20	1. The ring current intensifies 2. Auroral substorms appear 3. The parameters in the magnetosphere vary rapidly 4. High speed plasma flows towards the Earth and then backward in the plasma sheet	1. Energy particles inject from plasma sheet 2. The precipitation of energy particles is intensified 3. The secondary response for injected energy particles and for MHD waves 4. The source of energetic particles and heated plasma shift backward
20∼100	1. The magnetopause and plasma sheet recover, they expand farther and then to the quiet state 2. The parameters in the magnetotail vary slowly and the parameters in the magnetosphere recover quiet	1. The energy flux of MHD waves in magnetosheath is decreasing, the Poynting flux transfer from magnetotail into magnetosheath, then recover 2. The solar wind parameters recover quiet, the energy flux of waves recovers quiet

flow towards the plasma sheet. If the radius of the magnetotail decreases $2R_E$ in 30 min, the energy flux is given by

$$E_e = \frac{2}{4\pi} v_n B_0^2 \pi \, dL \simeq 5 \times 10^{17} \text{ erg s}^{-1}. \tag{5.12}$$

This is nearly half of the dissipative energy flux for a small substorm. On the other hand, the Poynting flux will flow from the magnetotail into the magnetosheath in the recovering phase of the substorm. However, the flux is less than (5.12), because the duration of the recovery is longer than that of the expansive phase. The process of the compression or expansion of the magnetopause may be explained by the interplanetary compressive and expansive waves. The interplanetary shock wave is a compressive wave, which compresses the magnetosphere and the magnetotail. After the shock wave passes, there come the expansive waves. As a result, pressure in the magnetosheath decreases and then the magnetotail is recovered. The kinetic energy of the solar wind is transferred into the magnetosphere in the former case, and the energy is extracted from magnetosphere in the latter case.

According to the above discussion, we can explain the main features of substorms by the energy flux of the MHD waves in the solar wind, which is summarized in Table 1. Of course, the substorm is a complex natural phenomenon; there are many characteristic features which may be explained by different mechanisms.

Acknowledgements—Many thanks are due to Professor S.-I. Akasofu for his helpful discussions.

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