

THERMAL MODEL OF CONTINENTAL LITHOSPHERE

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ABSTRACT

The aim of the present paper is to obtain the curves of continental lithospheric thickness and surface heat flow versus time, and the relation between surface heat flow and lithospheric thickness, using integral-relation method. The last relation can be written in the form $Q_s = a + bH_0$, which is just the empirical formula proposed by F. Birch^[1]. The coefficients a and b have been expressed as functions of the temperature of solidus T_m , the thermal conductivity k and the lithospheric thickness L . Results obtained in the present paper are found to be in good agreement with the observational data.

INTRODUCTION

The lithosphere constitutes the most important modern concept of global geology and geophysics. In the plate tectonics, the plate is defined as a block cutting lithosphere which moves laterally as a whole. So, some problems related to lithosphere have attracted much attention of earth scientists. How can the lithospheric thickness be determined is certainly an interesting subject.

Based on concept of thermodynamics, the assumption that the lithosphere-asthenosphere boundary is taken to be isothermal solidus of mantle materials is generally accepted. Thus, the determination of lithospheric thickness is considered as the discussion of thermal model of the lithosphere. The mathematical analysis of the thermal model of the oceanic lithosphere was suggested in the 1960's, but the study of continental lithosphere was initiated only recently^[1-3].

I. BASIC CONTENTS OF THERMAL MODEL OF CONTINENTAL LITHOSPHERE

According to the definition generally adopted, the base of the lithosphere is defined by the isothermal solidus T_m of the mantle material, and the materials of the lithosphere are just the solidified ones of the asthenosphere. If a vertical profile is considered only, we may take a two-dimensional frame, and put x -axis along horizontal direction and z -axis along vertical direction. The surface of the solid earth is put at $z = 0$, which is the isothermal at $T = 0^\circ\text{C}$. The base of the lithosphere is put at $z = L(t)$, where $T = T_m$. According to S. T. Crough and G. A. Thompson^[3] the heat flow transported to the lithospheric base from asthenosphere remains constant, and is denoted by

Q_b . This heat flow is supported by the thermal convection of asthenospheric materials, and transported to the earth surface by conduction, becoming a portion of surface heat flow. The heat generated by radioactive decay within the continental crust is another important component of the surface heat flow. The radioactive heat production within the upper layer of continental lithosphere adds a term of disturbed heat sources to the equation of heat transfer. As generally considered, the radioactive heat production rate decreases exponentially with depth index, and is given by

$$H = H_0 e^{-z/D},$$

where H_0 is the heat production generated by radioactive decay at $Z = 0$, D is the characteristic scale of the layer containing radioactive elements. The thermal model of continental lithosphere described above can be simulated mathematically by the following equation and boundary conditions:

$$\begin{cases} \rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + H_0 e^{-z/D}, & (1) \\ z = 0, \quad T = 0, \quad \frac{\partial T}{\partial t} = 0, & (2) \\ z = L(t), \quad T = T_m, \quad k \frac{\partial T}{\partial z} = Q_b & (3) \end{cases}$$

In the thermal model of continental lithosphere, we can assume that ρ and C_p are constants. To facilitate the following discussion, we assume k to be a constant, i.e. independent of temperature, and hence of depth. That is to say, an integral-average for k is to be taken for the subsequent integral-relation method,

$$k = \frac{k_c + k_m}{2},$$

where k_c is the conductivity of the crust and k_m is that of the upper mantle. It should be emphasized that the integral-relation method remains valid even when the function $k = k(T)$ is adopted.

In addition, the horizontal change in temperature is much smaller than the vertical one in the continental lithosphere, thus the term of $\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right)$ in Eq. (1) can be neglected. With these assumptions mentioned above, Eq. (1) becomes

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} + H_0 e^{-z/D}. \quad (4)$$

Eqs. (4), (2) and (3) are the basic equations in the thermal model of continental lithosphere.

II. INTEGRAL-RELATION METHOD

The integral-relation method was first adopted by fluid-dynamicsists in discussing the boundary layer problem. It is an efficient approximate method for solving partial

differential equation having an unknown boundary. Its basic approach is as follows: We first assume such a profile for the unknown function that only one parameter remains undefined after the boundary conditions are satisfied. This undefined parameter will then be determined by solving an ordinary differential equation obtained from the integration of the partial differential equation for a chosen variable.

According to the feature of Eq. (4), we can assume

$$T(z, t) = a_0(t) + a_1(t)z + a_2(t)z^2 + a_3(t)e^{-z/D}. \quad (5)$$

Satisfying the boundary conditions (2) and (3) by $T(z, t)$, we obtain

$$a_0 = \frac{D^2(2Q_b L + L^2 H_0 - 2kT_m)}{k(L^2 - 2D^2)}. \quad (6)$$

$$a_1 = \frac{2KLT_m - 2D^2Q_b - L^2Q_b - 2LD^2H_0}{k(L^2 - 2D^2)}, \quad (7)$$

$$a_2 = \frac{Q_b L + D^2H_0 - kT_m}{(L^2 - 2D^2)}, \quad (8)$$

$$a_3 = \frac{D^2(2kT_m - 2Q_b L - L^2H_0)}{k(L^2 - 2D^2)}. \quad (9)$$

The terms containing $e^{-L/D}$ are neglected in Eqs. (6)–(9) and (11), because $\text{Max}\{e^{-L/D}\} < 0.01 \ll 1$ for the continental lithosphere.

Substituting Eqs. (6)–(9) into Eq. (5), we get

$$\begin{aligned} T(z, t) &= \frac{D^2(2Q_b L + L^2 H_0 - 2kT_m)}{k(L^2 - 2D^2)} + \frac{2kLT_m - 2D^2Q_b - L^2Q_b - L^2Q_b - 2LD^2H_0}{k(L^2 - 2D^2)} z \\ &\quad + \frac{Q_b L + D^2H_0 - kT_m}{k(L^2 - 2D^2)} z^2 - \frac{D^2(2Q_b L + L^2 H_0 - 2kT_m)}{k(L^2 - 2D^2)} e^{-z/D}. \end{aligned} \quad (10)$$

By substituting Eq. (10) into Eq. (4), integrating for z over region $[0, L(t)]$ and assuming H_0 to be independent of t , the following relation can be obtained

$$\begin{aligned} &\frac{\rho C_p \dot{L}}{3k(L^2 - 2D^2)} [Q_b(4D^2L^3 + 6D^3L^2 - 12D^4L + 12D^5 - L^5) \\ &\quad + H_0LD^2(L^3 - 6D^2L + 12D^3) + 2kT_m(L^4 - 3L^2D^2 - 6LD^3 + 6D^4)] - \rho C_p T_m \dot{L} \\ &= \frac{(L - D)(2Q_b L + 2H_0D^2 - 2kT_m)}{L^2 - 2D^2}, \end{aligned} \quad (11)$$

where

$$\dot{L} = \frac{dL}{dt}.$$

The relation obtained from integrating the energy equation for z is just that of energy budget within the element volume with height $L(t)$ and width Δx . In [3], the relation of energy budget Eq. (10) cannot be obtained from the differential energy equation (4). We are of the opinion that their relation is essentially hypothetic. The

relation in the present paper is a result of direct integration. Of course, these two relations are different in form.

Eq. (11) can be rewritten as follows:

$$\left\{ \begin{aligned} \frac{dL}{dt} &= [3k(L^2 - 2D^2)(L - D)(2Q_bL + 2H_0D^2 - 2kT_m)] / \\ &\quad \{ \rho C_p [Q_b(4D^2L^3 + 6D^3L^2 - 12D^4L + 12D^5 - 5L^5) \\ &\quad + H_0LD^3(L^3 - 6D^2L + 12D^3) - kT_m(L^4 - 6L^2D^2 + 12LD^3)] \} \\ t &= 0, \quad L = L_0. \end{aligned} \right. \quad (12)$$

Eq. (12) is the basic relation of time-dependence of the lithospheric thickness. According to Eq. (12), the equilibrium value of continental lithospheric thickness can be derived from $Q_bL + D^2H_0 - kT_m = 0$. Then we have

$$L_\infty = \frac{kT_m - D^2H_0}{Q_b}. \quad (13)$$

The relation between surface heat flow and lithospheric thickness can be obtained from Eq. (10) as follows:

$$Q_s = \frac{2kT_m(L - D) - Q_b(L - D)^2 - Q_bD^2 + H_0DL(L - 2D)}{L^2 - 2D^2}. \quad (14)$$

By substituting the solution Eq. (12) $L = L(t)$ into Eq. (14), the relation $Q_s = Q_s(t)$ can be obtained.

Eq. (14) can also be written as

$$Q_s = a + bH_0, \quad (15)$$

where

$$a = \frac{2kT_m(L - D) - Q_b(L^2 - 2DL + 2D^2)}{L^2 - 2D^2}, \quad (16)$$

$$b = \frac{D(L^2 - 2LD)}{L^2 - 2D^2}. \quad (17)$$

The parameters on the right-hand side of Eq. (16) are related to thermal state in the deep. Apparently, a represents the component of surface heat flow due to deep thermal convection, b is a quantity having the dimension of length, bH_0 represents the component of surface heat flow due to radioactive decay. For a geological region L is about the same, and the parameters a and b are approximately constant, thus giving a linear relation between Q_s and H_0 .

In the process of mathematical simulation of the thermal model of continental lithosphere, the empirical relation $Q_s = a + bH_0$ has been established theoretically. The condition under which this linear relation remains valid is the small change in lithospheric thickness in a geological region. Hence the linear relationship cannot be assumed for active tectonic regions where the lateral change in lithospheric thickness is large.

III. COMPUTED RESULTS AND DISCUSSION

To get the time-dependence of lithospheric thickness, the solution of Eq. (12) can be written as

$$t = \int_{L_0}^{L(t)} \{ \rho C_p [Q_b(4D^3L^3 + 6D^3L^2 - 12D^4L + 12D^5 - 5L^5) + H_0LD^2(L^3 - 6D^2L + 12D^3) - kT_m(L^4 - 6L^2D^2 + 12LD^3)] \} / [3k(L^2 - 2D^2)(L - D)(2Q_bL + 2H_0D^2 - 2kT_m)] \cdot dL. \quad (18)$$

For convenience, the parameters in Eq. (18) should be made dimensionless. For this reason, the following dimensionless parameters are introduced

$$\begin{cases} \tilde{Q}_s = \frac{Q_s}{H_0D}, & \tilde{Q}_d = \frac{kT_m}{H_0D^2}, & \tilde{Q}_b = \frac{Q_b}{H_0D}, \\ \tilde{L} = \frac{L}{D}, & \tilde{t} = \frac{k}{\rho C_p D^2} t. \end{cases} \quad (19)$$

Substituting Eq. (19) into Eq. (18), we have

$$\tilde{t} = \int_{\tilde{L}_0}^{\tilde{L}(t)} [Q_b(-5\tilde{L}^5 + 4\tilde{L}^3 + 6\tilde{L}^2 - 12\tilde{L} + 12) + \tilde{L}(\tilde{L}^3 - 6\tilde{L} + 12) - \tilde{Q}_d(\tilde{L}^4 - 6\tilde{L}^3 + 12\tilde{L})] / [6(\tilde{L}^2 - 2)(\tilde{L} - 1)(\tilde{Q}_b\tilde{L} + 1 - \tilde{Q}_d)] \cdot d\tilde{L}. \quad (20)$$

The integral in Eq. (20) can be calculated by digital computer. In consideration of the parameters adopted in [3], we take the following values: $T_m = 1,200^\circ\text{C}$, $D = 10$ km, $Q_b = 0.56$ cal/cm³·sec, $k = \frac{k_c + k_m}{2} = 6 \times 10^{-3}$ cal/cm³·sec·°C, $\rho = 3.3$ g/cm³ and $C_p = 0.27$ cal/g·°C. To examine the effect of H_0 on L , we take $H_0 = 2 \times 10^{-13}$, 4×10^{-13} , 6×10^{-13} cal/cm³·sec, respectively. The value of L_0 is taken at some place where the lithospheric thickness is considered as that at $t = 0$. The value of L in the Basin and Range Province in North America is considered as L_0 in the following calculation and L_0 is obtained from Eq. (14) by using surface heat flux. The parameters used in calculation are listed in Table 1.

Table 1
Dimensionless Parameters Adopted in Calculation

H_0	\tilde{Q}_d	\tilde{Q}_b	L_0
2×10^{-13}	36	2.8	5.045
4×10^{-13}	18	1.4	5.360
6×10^{-13}	12	0.93	5.643

The cooling of earth surface resulted in a gradual thickening of lithosphere with the passing of time, as shown in Fig. 1. Beyond 500 m.y. the lithospheric thickness ceased to increase and reached an equilibrium value, which can also be obtained from Eq. (13). From Fig. 1, it can be seen that the effect of the radioactive heat produc-

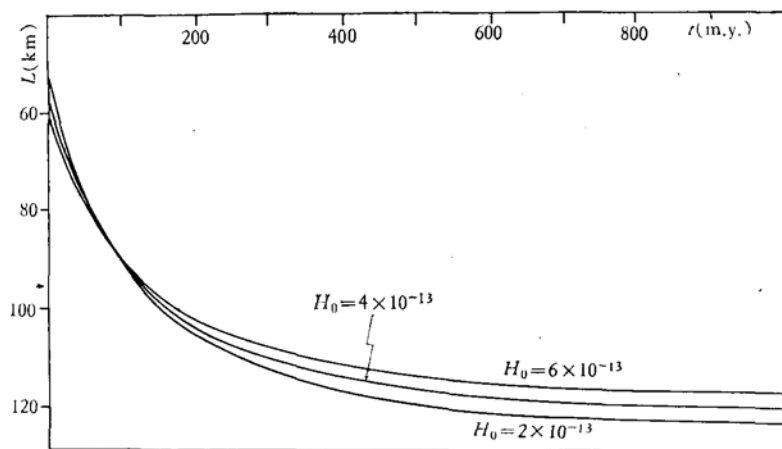


Fig. 1. Variation of continental lithospheric thickness with the time.

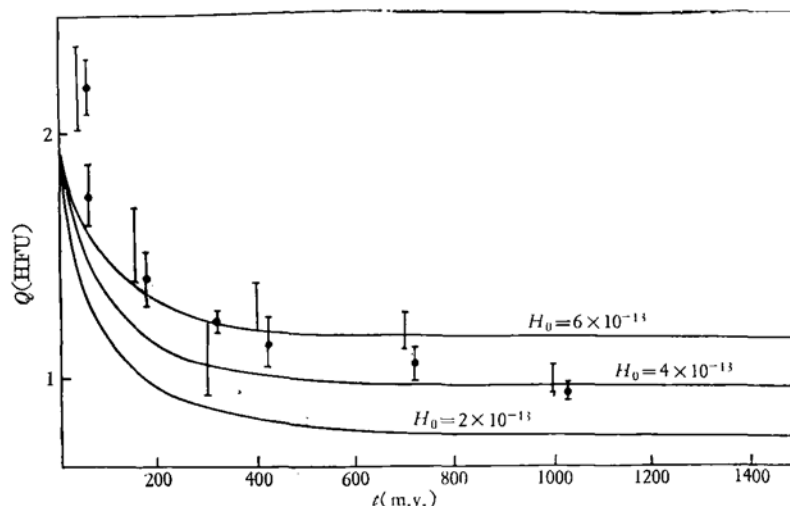


Fig. 2. Variation of surface heat flow with the time.

tion H_0 on the lithospheric thickness L is very small. Indeed, for the same age, increasing H_0 1 or 2 times results only in decreasing L_0 by 2 ~ 3%. The curves in Fig. 2 are given for these three values of H_0 (see Table 1).

The larger H_0 is, the higher the surface heat flow Q_s is, and for the same L increments in H_0 result in proportional increment in Q_s , which confirms again the linear relation between Q_s and H_0 . The observational data used in Fig. 2 are cited from [3], where the vertical bar indicates the error. The theoretical curve is certainly in good agreement with the observational data.

The observational data adopted in Fig. 3 are cited from [3], where the errors of the surface heat flow are indicated by the crossbars and those of the lithospheric thickness by the vertical bars. The three solid lines are obtained for three H_0 (see Table 1) according to the formula given in the present paper, and the broken curve is obtained for $H_0 = 2 \times 10^{-13}$ cal/cm²·sec according to Crough^[3] (the rest of the parameters are

the same with those adopted in the present paper). From Fig. 3 one can see that the present results are in very good agreement with the observational data, especially for $L \geq 50$ km.

The linear relationship between Q_s and H_0 is also evident in Fig. 3. Indeed, for the same L , increments in H_0 result in proportional increment in Q_s , which is certainly an intuitive expression of the relation $Q_s = a(L) + b(L)H_0$, i.e. Eq. (15).

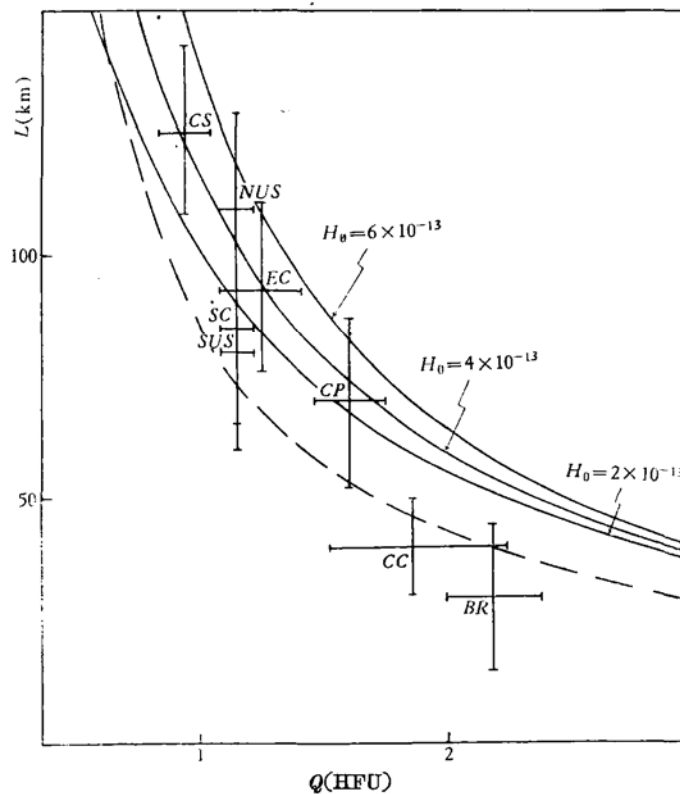


Fig. 3. The continental surface heat flow Q_s and lithospheric thickness L . CS, Canadian Shield; SC, Southern Canada; EC, Eastern Canada; CC, Canadian Cordillera; NUS, Northern United States; SUS, Southern United States; CP, Colorado Plateau; BR, Basin and Range.

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