FLOW LASERS

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ABSTRACT

The general stable oscillation condition for high speed flow lasers is derived directly from the radiation differential equation and the boundary conditions. It has been pointed out that the stable oscillation condition postulated in [1] is approximately satisfied under certain condition. The analytical formula of GDL output power is derived from our general oscillation condition and the saturated gain formula given in [2]. The relation between the GDL output power and various parameters is also shown. Analytical expression of the optical resonator modes for high speed flow lasers is derived from geometrical optical approximation. The mode characters in the Fabry-Perot resonator which contains flowing active medium are manifested and the typical case of Gerry's experiment[3] is calculated.

I. ORIGIN OF THE PROBLEM

With the appearance of high-power high-energy lasers, the high speed flow techniques have been widely adopted for removing unavailable energy and freezing the upper energy level of laser medium. The radiation field distribution and the output power characters in the resonators of high speed flow lasers have ever attracted universal attention. But as the interaction between the radiation field and the active medium is nonlinear, the problems are rather complex. Early in 1969, Cool first calculated the radiation field distribution and the output power in the parallel-plane resonator which contains flowing active medium under the assumption that the light propagates along the straight line perpendicular to the mirror surfaces. He obtained the stable oscillation condition of the resonator in high speed flow lasers: $g = -\frac{1}{2L}\ln(R_1 R_2)$, where R_1, R_2 are the reflectivities of the mirrors M_1, M_2 respectively, L is the distance between the two mirror surfaces (see Fig. 1), and g is the average saturated gain along the light axis in the resonator (refer to [4], formula (14)). This condition is similar to the stable oscillation condition in non-flow laser. From this condition, as the flow active medium enters the inlet of the resonator, its gain is saturated to the value $-\frac{1}{2L}\ln(R_1 R_2)$ assigned by the mirror surface losses. Therefore on the mirror surface of the flow inlet, the radiation field reaches the high peak value. Explicitly, this phenomenon is not in accordance with the experimental observations[5,6].

In 1974, Lee also pointed out that this condition was unreasonable and suggested another stable oscillation condition: $\frac{1}{s}\int_0^s\left(g-\frac{1}{2L}\ln\frac{1}{R_1R_2}\right)dx=0$, where s is the mirror length along the flow direction. He did not prove the correctness of this condition, but he supposed that this condition is better than Cool's as it seems to have included the diffraction effect. He calculated the output power with this stable oscillation condition under the assumption that the radiation field distribution is uniform. In this paper, we derive a general stable oscillation condition for the flow lasers and point out that as long as $g(x)$ does not vary violently, Lee's condition is approximately available. But his assumption that the radiation field distribution is uniform, is not well founded in certain cases.

Recently, there have been issued some results of the unstable resonators in high speed flow lasers[2,7,8], obtained by solving the gain dynamic equations and the radiation field equations simultaneously. With these calculations, we may obtain the three-dimensional distribution of the radiation field and calculate the effects of shocks, mirror distortions and mirror tilts on the radiation field distribution and on the output power. But for the parallel-plane resonators, we fail to get a steady convergent solution with these numerical iterations. As will be seen later in this paper the oscillation in these resonators is of multi-mode generally.

In this paper, we discuss the mode structure and the output power in the parallel-plane resonators which contain flowing active medium. First, we derive the general stable oscillation condition from the radiation differential equation and the boundary conditions. From this condition, we obtain the GDL output power formula. With the geometrical optical approximation, we get the transverse mode expression. The typical case of Gerry's experiments is also calculated.

II. The Basic Equation of Radiation Field and Boundary Conditions

Suppose that the active medium flows along the x direction (see Fig. 1). Let two plane reflective mirrors M_1, M_2 be placed along the x direction. The light axis is in parallel with the z axis and perpendicular to the flow direction. The excited region is upstream above the origin point. As the gas reaches the upstream boundary of the resonator $x=0$, the initial vibrational population inversion of the gas has been generated. In the region of the optical resonator where $x\geqslant 0$, the energy transfer generated from the molecular collisions and the stimulated radiation in the laser resonator makes the vibrational population inversion decay gradually. Suppose that all physical quantities are independent of y, i.e., the problem may be treated as a two-dimensional problem. Suppose also that within one wavelength scope, the change of the complex dielectric constant ε is small (because the change of the saturated gain is small within one wavelength scope). Then the radiation equation in the resonator becomes

$$\nabla^2E+\frac{\omega^2}{c^2}\varepsilon E=0, \tag{1}$$

where E is the component of the electrical vector, ω is the angular frequency, c is the light speed in vacuum; $\varepsilon = \varepsilon_0 - \frac{c}{\omega} gi$, where ε_0 is the real part of the complex dielectric constant and is taken as 1 here, g is the saturated gain. In our cases, g can be taken as the function of x only. In fact, as we took the numerical calculations with Rensch's method[7], we found that the change of g with z is small, and g is considered as the average value in L length along z direction.

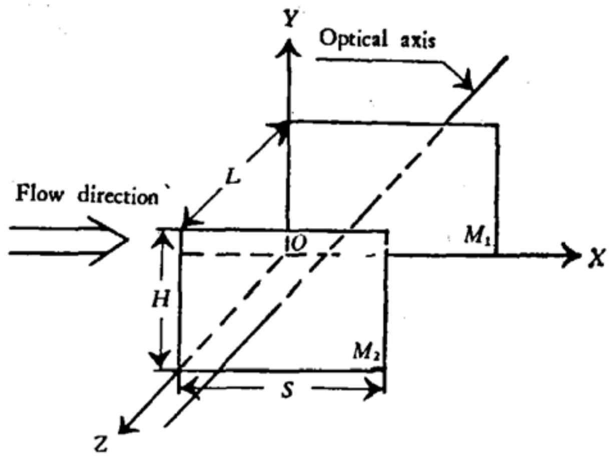

Fig. 1. The sketch of the resonator in high speed flow lasers.

Suppose that the boundary conditions in the inlet and the outlet of the resonator are

$$E(0, z) = E(s, z) = 0 \tag{2}$$

respectively. In fact, since the Fresnel number of the resonator in high speed flow laser is high in general, from the results of the passive parallel plane resonators, we know that the conditions (2) are satisfied approximately for high Fresnel number. This means that for high Fresnel number, the diffraction losses may be neglected. Therefore, in the inlet of the resonator, we may suppose that the value of $g(0)$ is equal to the small signal gain g_0 in the inlet.

Eq. (1) is to be solved under the boundary conditions (2). We may assume that the solution of the equation has the form:

$$E(x, z) = X(x)Z(z). \tag{3}$$

Substituting (3) into (1) and separating the variables, we obtain

$$X'' + \left(\frac{\omega^2}{c^2} - k_z^2 - i\frac{\omega}{c} g\right) X = 0, \tag{4}$$

$$Z(z) = C_1 e^{ik_z z} + C_2 e^{-ik_z z}, \tag{5}$$

where k_z is the separate constant, and C_1, C_2 are integrating constants. They can be determined by the boundary conditions in the two mirror surfaces.

Let the reflectivities of the two plane reflective mirrors M_1, M_2 be R_1, R_2 respectively. For mirror M_1, from (5) the incident radiation field (it means the electric vector here) may be assumed as

$$\boldsymbol{E}_i(x, z)|_{z=0} = X(x)e^{-ik_z z}\boldsymbol{e}_1|_{z=0},$$

where $\boldsymbol{e}_1$ is the unit vector in the polarization direction of the incident wave, and k_z is a complex number in general. Then the radiation field reflected from mirror M_1 is

$$\boldsymbol{E}_r(x, z)|_{z=0} = -\sqrt{R_1}\, X(x)e^{ik_z z}\boldsymbol{e}_1|_{z=0}.$$

So the radiation field on the M_1 mirror surface may be written as

$$E(x, z)|_{z=0} = (X(x)e^{-ik_z z} - \sqrt{R_1}\, X(x)e^{ik_z z})_{z=0}.$$

From this we obtain the boundary condition on the M_1 mirror surface as

$$k_z \frac{E(x, 0)}{\dfrac{\partial E(x, 0)}{\partial z}} = i\,\frac{1-\sqrt{R_1}}{1+\sqrt{R_1}}. \tag{6}$$

For the same reason, the boundary condition on the M_2 mirror surface is

$$k_z \frac{E(x, L)}{\dfrac{\partial E(x, L)}{\partial z}} = -i\,\frac{1-\sqrt{R_2}}{1+\sqrt{R_2}}. \tag{7}$$

Substituting (3), (5) into (6), (7), we obtain

$$E(x, z) = c_2(-\sqrt{R_1}\, e^{ik_z z} + e^{-ik_z z}) \cdot X(x), \tag{8}$$

where

$$k_z = \frac{m\pi}{L} - i\,\frac{1}{4L}\ln\frac{1}{R_1R_2}. \tag{9}$$

Now substitute (9) into (4) and let $k_x^2 = \dfrac{\omega^2}{c^2} - \left(\dfrac{m\pi}{L}\right)^2$, where m denotes the longitudinal mode order $\left(m \sim \dfrac{2L}{\lambda}\right)$ and is a large integer $\left(\dfrac{m\pi}{L}\right)\Big/\left(\dfrac{\omega}{c}\right) \approx 1$. For high speed flow lasers, there are generally $\dfrac{1}{16L^2}\left(\ln\dfrac{1}{R_1R_2}\right)^2 \approx 0$. In so doing, (4) and the boundary conditions (2) become

$$X'' + \left[k_x^2 - i\,\frac{\omega}{c}\left(g - \frac{1}{2L}\ln\frac{1}{R_1R_2}\right)\right]X = 0, \tag{10}$$

$$X(0) = 0, \tag{11}$$

$$X(s) = 0. \tag{12}$$

We know that the radiation-flux density is that

$$I(x, z) = \frac{c}{8\pi} E(x, z) \cdot E^*(x, z), \tag{13}$$

where $*$ denotes the conjugate complex number taken. Let $I(x)$ denote the average value of $I(x, z)$ between 0 and L along the Z direction. $g(x)$ in (10) is generally attributed to the function which varies with $I(x)$. So substituting (8) into (13), taking the average between 0 and L along the Z direction and introducing the suitable constant in $X(x)$, we obtain $I(x) = X(x) \times X^*(x)$, where $X(x)$ satisfies (10)—(12). The problem is attributed to solving Eq. (10) under the boundary conditions (11) and (12).

III. Stable Oscillation Condition in High Speed Flow Lasers and Output Power Expression of GDL

In (10), let $X(x) = f(x) \cdot e^{i\varphi(x)}$, where $f(x)$, $\varphi(x)$ are the real functions of x. Substituting it into Eq. (10) and letting the real part and the imaginary part equal zero respectively, we obtain

$$f'' + (k_x^2 - \varphi^{2'})f = 0, \tag{14}$$

$$f\varphi'' + 2f'\varphi' - k\left(g - \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right) f = 0, \tag{15}$$

where $k = \frac{\omega}{c}$. The boundary conditions (11) and (12) become

$$f(0) = 0, \tag{16}$$

$$f(s) = 0. \tag{17}$$

In (15) according to the first-order linear ordinary differential equation to solve $\varphi'(x)$, we obtain

$$\varphi'(x) = \frac{1}{f^2}\left[C + \int_0^x k\left(g - \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right) f^2 dx'\right], \tag{18}$$

where C is the integrating constant. From the boundary condition (16), we see that in order to make $\varphi'(0)$ finite, it is necessary to have $C = 0$. At the same time, from the boundary condition (17), in order to make $\varphi'(s)$ finite, we must have

$$\frac{\int_0^s g f^2 dx}{\int_0^s f^2 dx} = \frac{1}{2L} \ln \frac{1}{R_1 R_2}. \tag{19}$$

This is the general oscillation condition in high speed flow laser. It is the condition that is necessary for flowing active medium to create the radiation field from the inlet of the resonator to the outlet of it.

In (19), the left-hand side of the equation is weighted average where the weighted function is $f^2(x)$. If $I(x) = f^2(x)$ approaches the uniform distribution (at $x = 0$ and

$x = s$, $f^2(x)$ might have large gradients), then the left-hand side of (19) approaches the average value of $g(x)$ along the flow direction more correctly, and the condition (19) becomes the stable oscillation condition in [1]. In general cases, as will be pointed out later, $f^2(x)$ has the distributed shape of Fig. 2. The position of the peak value is just approximately at x_{max} which satisfies Eq. $g(x_{max}) = \frac{1}{2L}\ln\frac{1}{R_1R_2}$. However, on both sides of x_{max}, $f^2(x)$ decreases gradually with the increase of the distance from x_{max}. Hence, if $g(x)$ does not change violently (corresponding to the radiation intensity not too large), (19) becomes approximately,

$$\frac{1}{s}\int_0^s g(x)dx \doteq \frac{1}{2L}\ln\frac{1}{R_1R_2}. \tag{20}$$

This explains that Lee's stable oscillation condition can be available approximately in general cases.

For the case of CO_2 GDL, we adopt Siegmans' expression of the flowing saturated gain (refer to the formulas (23)—(25) in [2]):

$$g(x) = \left[\frac{g_0(x)}{1 + W(x)}\right]\exp\left[-\frac{\chi_{CO_2}\beta}{\chi_{N_2}V}\int_0^x \frac{W(x')}{1 + W(x')}dx'\right], \tag{21}$$

where

$$g_0(x) = g_0\exp(-\chi_{CO_2}\alpha x/\chi_{N_2}V), \tag{22}$$

$$W(x) = \sigma I(x)/h\nu\beta, \tag{23}$$

g_0 is the small signal gain in the inlet, α, β are the collisional exchange rates between the vibrational energy level (001) and the ground state and those between the vibrational energy level (100) and the ground state respectively, V is the flow velocity, χ_{CO} χ_{N_2} are the mole fractions of CO_2 and N_2 respectively, ν is the photon frequency, h is the Planck constant, and σ is the optical cross-section of the laser transition. As Siegman et al.[2] pointed out, Formula (21) was derived under certain assumptions. Although these assumptions are not satisfied permanently, particularly in the front edge of the light beam (i.e. approaching the inlet of the resonator), where the change of $I(x)$ may be large, yet even in these cases, Formula (21) can also offer a correct description of the saturated tendency of the GDL gain.

Let

$$\frac{\chi_{CO_2}}{\chi_{N_2}}\cdot\frac{\alpha}{V} = a, \quad \frac{\chi_{CO_2}}{\chi_{N_2}}\cdot\frac{\beta}{V} = B, \quad \frac{h\nu\beta}{\sigma} = A_0,$$

then from (21)—(23), we obtain

$$g(x) = \frac{g_0\cdot e^{-ax}}{1 + f^2/A_0}\cdot\exp\left[-B\int_0^x \frac{f^2/A_0}{1 + f^2/A_0}dx'\right]. \tag{24}$$

Substituting (24) into (19), from the boundary condition (17), and with the integration by parts, and letting $\bar{g} = \frac{1}{s}\int_0^s g(x)dx$ be the average saturated gain along the flow direction, we obtain

$$\int_0^s f^2 dx = \frac{2LA_0}{(a+B)\ln\frac{1}{R_1R_2}} \cdot [g_0 - g(s) - as\bar{g}].$$

Let the transmittance of the mirror M_2 be t. The output power P is

$$P = tH\int_0^s f^2 dx = \frac{2tHLA_0}{(a+B)\ln\frac{1}{R_1R_2}} \cdot [g_0 - g(s) - as\bar{g}]. \tag{25}$$

This is the output power expression derived from the stable oscillation condition (19) and the saturated gain formula (21).

Substituting the approximate stable oscillation condition (20) into (25), we obtain the approximate expression of the output power,

$$P \doteq \frac{2tHLA_0}{(a+B)\ln\frac{1}{R_1R_2}} \cdot \left[g_0 - g(s) - \frac{as}{2L}\ln\frac{1}{R_1R_2}\right]. \tag{26}$$

Substituting the expressions of A_0, a, B into (26) and from [12], $\sigma \doteq \frac{718}{NT}$ (where the value of the numerator changes in fact slightly with the temperature T, N is the population density in the resonator, and T is the translation temperature in the resonator), we obtain

$$P \doteq \frac{2tHLh\nu\beta\chi_{N_2}VNT}{718\chi_{CO_2}(\alpha+\beta)\ln\frac{1}{R_1R_2}} \cdot \left[g_0 - g(s) - \frac{\chi_{CO_2}\cdot\alpha\cdot s}{2\chi_{N_2}\cdot V\cdot L}\ln\frac{1}{R_1R_2}\right].$$

From this formula, we may see in general the relation between the output power and the various parameters.

Since $g(s) \ll g_0$ is required for the design of GDL, therefore, from (26), the maximum output power which does not take into account the energy of the flowing loss, is

$$P_{max} = \frac{2tHLA_0}{(a+B)\ln\frac{1}{R_1R_2}} \cdot \left[g_0 - \frac{as}{2L}\ln\frac{1}{R_1R_2}\right].$$

With respect to the typical experiment of GDL as was done by Gerry[3], we have calculated the output power with (26). The calculated results are as follows:

Gerry's primary data:

$$\chi_{CO_2} = 0.08, \quad \chi_{N_2} = 0.91, \quad \chi_{H_2O} = 0.01,$$

the temperature of the combustion chamber = 1300°K, the pressure of the combustion chamber = 17atm, the exit Mach number = 4, the area ratio = 14, the loss of the mirror surface = 0.02, $t = 0.02$, $H = 3$cm, $s = 20$cm, $L = 30$cm.

We adopt the small signal gain $g_0 = 4\times10^{-3}\,\text{cm}^{-1}$, as has been done in [7] and [13] for Gerry's 55 kilowatts installation. The calculated results are in the following:

Let γ be taken as 1.35 (γ is the specific heat ratio), then we have

$$T = 342°\text{K},\quad p = 0.0987\,\text{atm},\quad V = 1.45\times 10^5\,\text{cm/sec},$$
$$N = 2.145\times 10^{18}\,\text{cm}^{-3},\quad \sigma = 0.913\times 10^{-18}\,\text{cm}^2,$$

from the data of the relaxation time in [14], we obtain

$$\frac{1}{\alpha} = 25.4\times 10^{-6}\text{sec},\quad \frac{1}{\beta} = 3.3\times 10^{-6}\,\text{sec},\quad a = 0.024\,\text{cm}^{-1},$$

$$B = 0.184\,\text{cm}^{-1},\quad A_0 = 6716\,\text{Watts/cm}^2,\quad \frac{1}{2L}\ln\frac{1}{R_1R_2} \doteq 1\times 10^{-3}\text{cm}^{-1}.$$

In the expression (21) of $g(x)$, let $x = s$, then we obtain the expression $g(s)$. Suppose that $I(x)$ is uniform approximately, then $I(x) \doteq \frac{P}{tHs}$. We find the value P with the expression $g(s)$ and the output power formula (26) by iteration. After three iterations, the output power and the saturated gain of the outlet reach the steady values: $P =$ 6280 watts, $g(s) = 0.278\times 10^{-3}\,\text{cm}^{-1}$.

With (26) to calculate the optimum output coupling, we obtain $t_{op} \doteq 0.09$, but if we adopt Rigrod's formula to calculate it, we obtain $t_{op} \doteq 0.06$.

IV. The Approximate Distribution of the Transverse Mode in the Resonator of Flow Laser

We treat the mirror surface losses as the absorbtion distributed in the medium, so the radiation field equation in the resonator becomes

$$\nabla^2 E + \left[k^2 - ik\left(g - \frac{1}{2L}\ln\frac{1}{R_1R_2}\right)\right]E = 0. \tag{27}$$

From (21), we know that $g(x)$ is the function of $I(x)$, and therefore, is the function of EE^*. Eq. (27) is nonlinear. But as $g(x)$ influences the amplitude primarily, so Eq. (27) can also be solved by geometrical optical approximation. With respect to the light wave, k is large. Let E be the form

$$E(x, z) = f(x, z)\cdot e^{iks(x,z)}. \tag{28}$$

Substituting (28) into (27), dividing the both sides of the equation by k^2, neglecting the terms containing $1/k^2$, and making equal the coefficients of the equal power terms of $1/k$ in both sides of the equation, we obtain that:

from the coefficients of $\left(\frac{1}{k}\right)^0$,

$$\nabla s\cdot\nabla s = 1\quad\text{(eiconal equation)}; \tag{29}$$

from the coefficients of $\left(\frac{1}{k}\right)^1$,

$$2\nabla s \cdot \nabla f + f \cdot \left[\nabla^2 s - \left(g - \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right)\right] = 0 \text{ (transport equation).} \tag{30}$$

With the characteristic method to solve (29)[16], we obtain

$$\frac{d^2 \boldsymbol{r}}{d\sigma^2} = 0, \tag{31}$$

$$S = S_0 + \sigma, \tag{32}$$

where σ is the arc length of the ray, S_0 is the integrating constant, $\boldsymbol{r}$ is the radius vector of the point on the ray. From (31), we know that under the geometrical optical approximation, the ray is the straight line. Substituting (32) into (30), and in consideration of $\frac{dx}{d\sigma} = \pm \frac{k_x}{k}$ along the ray, then (30) becomes

$$\pm \frac{2k_x}{k} \cdot \frac{df}{dx} - f \cdot \left(g - \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right) = 0.$$

To solve f, we obtain

$$f = f_0 \exp\left[\pm \frac{k}{2k_x} \int_0^x \left(g - \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right) dx'\right], \tag{33}$$

where f_0 is the integrating constant. Formula (33) denotes that the amplitude changes with x. From (28), we see that the phase factors which change with x is $e^{\pm i k_x x}$. Therefore, we obtain two approximate solutions of Eq. (10):

$$X_1(x) = C_1 \exp\left[\frac{k}{2k_x} \int_0^x \left(g - \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right) dx' + i k_x x\right],$$

and

$$X_2(x) = C_2 \exp\left[-\frac{k}{2k_x} \int_0^x \left(g - \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right) dx' - i k_x x\right],$$

where C_1, C_2 are the integrating constants. Obviously, $X_1(x)$ denotes the wave propagating to the positive X direction, and $X_2(x)$ denotes the wave propagating to the negative X direction. The real wave is the superposition of these two waves. In consideration of the boundary condition (11), the approximate solution of (10) is

$$X(x) = C_1 \left\{\exp\left[\frac{k}{2k_x} \int_0^x \left(g - \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right) dx' + i k_x x\right] - \exp\left[-\frac{k}{2k_x} \int_0^x \left(g - \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right) dx' - i k_x x\right]\right\},$$

From this expression, to find $|X(x)|^2$, we have

$$|X(x)|^2 = X_0^2 \left\{\sin^2 k_x x + \sinh^2 \left[\frac{k}{2k_x} \int_0^x \left(g - \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right) dx'\right]\right\}, \tag{34}$$

where X_0^2 is a constant. In consideration of the approximate stable oscillation condition

(20) and the boundary condition (12), and from (34), we obtain

$$k_x = \frac{(n+1)\pi}{s} \quad (n = 0, 1, 2, \cdots). \tag{35}$$

If k_x satisfies (35), the solution (34) approximately satisfies the boundary condition (12). Eqs. (34) and (35) are the analytical expressions of the transverse modes we want to find.

From (34), we see that the larger k_x is, the flatter the curve of $|X(x)|^2$ is, and then the larger the mode volume is (see Fig. 2). As has been pointed out in [17], the experimental results indicate that the radiation intensity distributions are flat and the mode structure is of high order. In fact, since the energy of the vibrational energy level $v = 1$ of N_2 is liberated through the near resonant exchange with the vibrational energy level (001) of CO_2, the liberation of the energy requires a certain time, i.e., requires a certain length along the flow direction. The high-order modes which have the larger mode volume can obtain more energy from the medium than the lower-order modes, so from the result of the mode competitions, the higher-order modes will be predominant. But with the increase of the order of the transverse modes, the diffraction loss will increase too. From [18], we know that for the Fabry-Perot resonators which contain the uniform active medium, and for high Fresnel number, the diffraction loss primarily corresponds to the walk-off loss, which is then,

$$-\frac{\cos\theta_0}{L}\ln\left(1 - \frac{2L}{s}\sin\,\theta_0\right),$$

where $\sin\theta_0 = \frac{k_x}{k}$. In order to have it predominant in the competitions, we require that the mode volume is large, but it is also necessary to make the diffraction loss far less than the mirror surface losses, i.e.,

$$-\frac{\cos\theta_0}{L}\ln\left(1 - \frac{2L}{s}\sin\,\theta_0\right) \ll \frac{1}{2L}\ln\frac{1}{R_1R_2}.$$

Since $k_x \ll k$ and $\sin\theta_0 = \frac{k_x}{k}$, we get

$$\frac{2k_x}{sk} \ll \frac{1}{2L}\ln\frac{1}{R_1R_2}. \tag{36}$$

Of course, the small signal gain must be not so large (if g_0 is very large, the modes which correspond to the larger diffraction losses will oscillate too). Although we require that k_x has to correspond to the higher-order modes and is restricted by (36), k_x can still take many values. This indicates that the oscillation is of multi-mode in general.

From (34), we see that the first term in the brackets corresponds to the modes in the passive resonators and the second term indicates the effect of the flow gain deflection. Except that k_x is large or g_0 is very close to $\frac{1}{2L}\ln\frac{1}{R_1R_2}$, the shapes of the modes are determined primarily by the second term in the brackets of (34). We may discuss this case briefly. From the second term in the brackets of (34), We see that $|X(x)|^2$ is zero at $x=0$, and $|X(x)|^2$ increases gradually with the increase of x (in the neighborhoods of $x=0$ and $x=s$, owing to the effect of $\sin^2 k_x X$, $|X(x)|^2$ has some small waves) until reaching the position of the peak value x_{max}, which can be determined approximately by

$$g(x_{max}) = \frac{1}{2L}\ln\frac{1}{R_1R_2}, \tag{37}$$

and if x goes over x_{max}, $|X(x)|^2$ will decrease with the increase of x, until $|X(s)|^2=0$ (See Fig. 2). From (37), we know that in spite of the differences of the modes, the position of the peak value is basically identical and is determined by (37).

In order to express x_{max} with analytical expression, we substitute the average value along the flow direction,

$$\bar{f}^2 = \frac{1}{s}\int_0^s f^2 dx = \frac{p}{tHs},$$

for f^2 in (24). (Of course, this is allowed only in case that $f^2(x)$ varies not so large.) Substituting $g(x)$ in (24) into (37) and solving x_{max}, we obtain

$$x_{max} \doteq -\frac{A_0+\bar{f}^2}{aA_0+(a+B)\bar{f}^2}\ln\left[\frac{1+\bar{f}^2/A_0}{2g_0L}\ln\frac{1}{R_1R_2}\right]. \tag{38}$$

Eq. (38) gives the relation between the position of the peak value and the various parameters. For example, from (38), we see that if the output coupling increases, the position of the peak value will generally move toward the origin point, and since a, B are proportional to χ_{CO_2}/χ_{N_2}, therefore, if the CO_2 component increases, the posi-

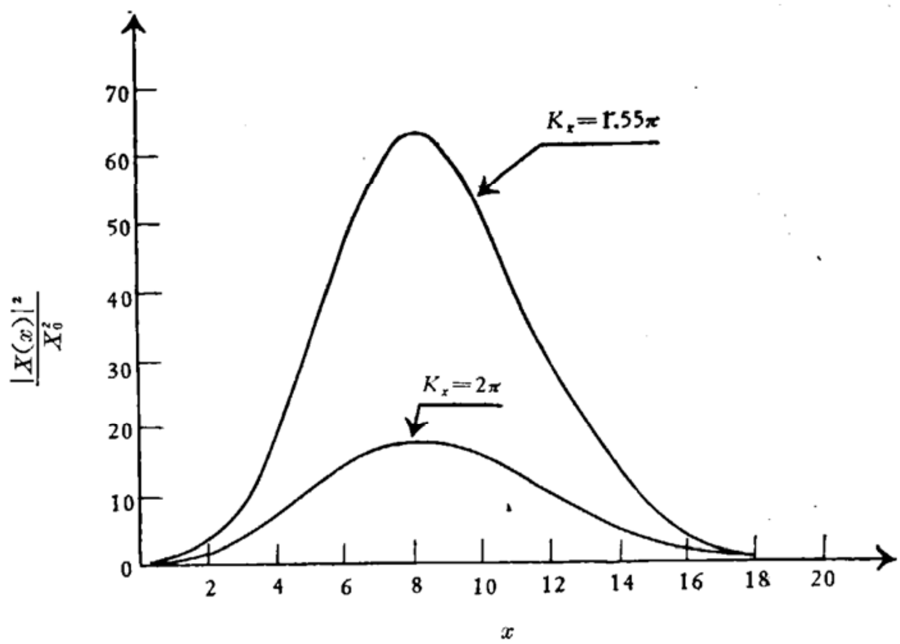

Fig. 2. For Gerry's typical experiments[3], the transverse mode distributions calculated by (34) (k_x is taken as 1.55π and 2π respectively).

tion of the peak value will move toward the origin point too.

For Gerry's typical experiments[3], we have calculated the patterns of the transverse modes (See Fig. 2).

Since $\frac{1}{2L}\ln\frac{1}{R_1R_2} \doteq 1\times 10^{-3}$, $s = 20$ cm and in consideration of condition (36) we can take $k_x = 1.55\pi$ and $k_x = 2\pi$ respectively. By substituting these k_x value into (34) and in to (24), substituting approximately $f^2 = 5000$ watt/cm^2 for f^2, and then substituting (24) into (34), we obtain the distribution curves of the transverse modes in Fig. 2. From (38), we obtain the position of the peak value $x_{max} \doteq 8.1$ cm. It can be seen from Fig. 2 that the value of k_x influences the shape of the curve sensitively. The curve corresponding to the larger k_x value is similar to the experimental burning results of the many-hole coupling in [6]. Therefore, the transverse mode in reality is of high order.

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