DRIVING MECHANISM FOR SEA FLOOR SPREADING

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ABSTRACT

In the present paper, the characteristic features of upwelling flow from beneath the mid-ocean ridges are analysed and from the basic equations of hydrodynamics a mathematical description of the upwelling flow is made; the velocity of the plate motion, the driving forces exerting on the plate and the energy transported by the upwelling flow have been calculated. The results are found to be in good agreement with geophysical observations. The following picture of what happens in the process of sea floor spreading may be suggestive. The upwelling flow of the deep mantle material through the central rift is believed to be the source of: (i) a continuous influx of material out of which the oceanic lithosphere is continually created, (ii) the force which enables the oceanic plate to overcome the resistance it encounters on its way and move forward with approximately constant velocity, and (iii) the energy released in an earthquake, volcanic eruption, etc. as a consequence of the build-up of strain energy due to plate motion.

I. INTRODUCTION

The hypothesis of sea floor spreading and plate tectonics serves to explain the tectonic and seismic activities of the upper layer of the earth. However, the important question where comes the force that causes sea floor spreading and the movement of the plates at almost constant velocity remains unsolved.

In the last decade, a number of geophysicists and specialists in fluid dynamics devoted themselves to the study of the driving mechanism of sea-floor spreading\(^1\). Some hold that the flows in the asthenosphere exert on the overlying lithosphere plate a viscous shear force that drags and moves the plate. The flows in the asthenosphere are considered to be a thermal convection resulting from the vertical temperature gradient. Since the horizontal scale of the convection cell in the asthenosphere is much smaller than that of the plate, as pointed out by Richter\(^1\) there must be several convection cells beneath each plate. As the flows in two neighbouring cells are opposite in direction, the force acting on the plate will certainly balance off each other. Therefore, the net shearing force acting on the plate will not be big enough to actually drag the plate along with it.

Recently, other authors have emphasized the action of the downgoing slab\(^1\). They suggested that the temperature of the downgoing lithospheric slab is lower than that in the surroundings, thereupon a downward force will act on the downgoing slab. This force is known as negative buoyancy and it could trigger the plate to move. It is to be noted that not all plates have downgoing slabs. Then how can such plates be driven?

We are of the opinion that for a complete dynamical model the interaction be-
tween the lithosphere and asthenosphere must be considered. In particular, we must examine in the mid-oceanic ridges the effect of the mass, momentum and energy imported to the lithospheric plate by upwelling lava in sea floor spreading. The upwelling of mantle material through the central rift may provide a continuous influx of matter by which the ocean-plate can be created, the force by the ocean-plate to overcome the resistance it encounters on its way and to move forward with an approximately constant velocity, and the energy to be released in an earthquake and volcanic eruption, etc. as a consequence of the build-up of strain energy due to plate motion.

From the basic equations of fluid dynamics, the partial differential equations governing upwelling flows and their detailed structure can be obtained on the basis of the features of the upwelling flow. The velocity of the movement of the plate, the force driving the plate and the energy provided by the upwelling material per unit time can be calculated from the parameters of flow in the upwelling channel. The three dynamical parameters of the plate as computed in the present paper all agree well with the geophysical observation data.

II. Basic Equations

The general equations of fluid mechanics govern the flows of mantle fluids. According to the features of mantle flows F. M. Richter and D. P. McKenzie have presented the following basic assumptions through the comparison between the orders of magnitudes: (1) The flows are two-dimensional and steady; (2) the Ekman number is very large, therefore we can neglect the Coriolis' force caused by the rotation of the earth; (3) Boussinesq approximation is used; (4) adiabatic lapse rate and the term of viscous dissipation is neglected; (5) a linear constitutive relation is assumed; (6) the Reynolds number is very small, about $10^{-17}$, so inertia terms can be neglected; (7) some parameters in earth's mantle, such as the specific heat at constant pressure $c_p$, the coefficients of expansion $a$ and thermal conductivity $K$ are all constants. In addition, we shall only examine the upward flow of the convection where the convection effect is important, so the internal heat generation due to radioactive decay can be neglected. Using the above assumptions, the hydrodynamic equations of the earth's mantle become

$$\frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \frac{\partial u}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} - \frac{\partial p}{\partial x} + \rho g \alpha (T - T_s) = 0,$$  \hspace{1cm} (2.1)

$$\frac{\partial}{\partial x} \left( \mu \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right) + \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} - \frac{\partial p}{\partial y} = 0,$$  \hspace{1cm} (2.2)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right),$$  \hspace{1cm} (2.3)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$  \hspace{1cm} (2.4)

where the $x$-axis is vertically upward, $y$ axis is horizontally rightward, $u$ and $v$ are velocity components in $x$- and $y$-directions respectively, $T$ is temperature of fluids and $T_s$ is the reference temperature, $\rho$ is the density, $g$ is the gravitational acceleration, $k = \frac{K}{\rho c_p}$.
is the thermometric conductivity, $p$ is the pressure caused by the motion.

For the reason that the Prandtl number and Rayleigh number are both very large in the earth's mantle, the central part of the convection cell will remain as a basically stationary nucleus in which the change of the temperature is small. On both sides are upward flows and downgoing flows, where the temperature and velocity change abruptly over a short distance. For a convection cell at a depth of 700 km, the horizontal distance over which dominant changes of temperature and velocity will occur ranges from a few kilometers to dozens of kilometers. Thus Eqs. (2.1)—(2.4) can be further simplified.

There is an upwelling flow channel beneath the mid-ocean ridges. This flow is the continuation of the upward flow in the asthenosphere (Fig. 1). Since the materials surrounding the upwelling flow in the asthenosphere are in fluid state, the upward flowing materials can spread out, or the surrounding materials can enter into the upward flow. As the upwelling materials pass into the lithosphere, they can only be pasted onto the walls of the channel and move sidewise along with the plate.

![Fig. 1. Sketch of the upwelling flows.](image)

As shown in Fig. 1, $y = 0$ is the symmetric axis (i.e. the boundary of the upward flows of two neighbouring cells). The zero point of the coordinate is the point where the axes of symmetry intersect with the ocean-base. Let us consider only the flows in the region $y > 0$, $x < 0$. The distance over which dominant changes in the upwelling velocity occur (i.e. from maximum at the axis of symmetry to zero at the boundary) is defined as the thickness of the upwelling flow, represented by $d$. The distance over which dominant changes in the temperature of the upwelling flow occur (i.e. from the highest temperature at the axis of symmetry to ambient temperature at the same
depth) is defined as the thermal-thickness of the upwelling flow, which is represented by \( \delta_r \). The parameters at \( y = 0 \) are represented by the subscript \( w \), while the parameters on both sides of the channel or the outer margins of the upward flow in the asthenosphere are represented by the subscript \( a \), \( \delta_a \) is the thickness of the lithosphere, \( l_1 \) is the depth at which convection starts. The typical values of \( u, v, p, \mu \) are represented by \( U, V, P, M \) respectively; the typical values of temperature difference in \( x \)- and \( y \)-directions are represented by \( \Delta u T \) and \( \Delta_v T \) respectively. The characteristic distance in \( x \)-direction is represented by \( l(l = l_1 \) for the lithosphere, \( l = l_2 - l_1 \) for the asthenosphere). From \( \delta_a \ll l \), it follows that

\[
\frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) \approx \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right),
\]

\[
\frac{\partial}{\partial x} \left( \mu \frac{\partial v}{\partial x} \right) \approx \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right).
\]

If

\[
\frac{\Delta_u T}{\Delta_v T} \approx \left( \frac{\delta_u}{l} \right),
\]

then

\[
\frac{\partial^2 T}{\partial x^2} \approx \frac{\partial^2 T}{\partial y^2}.
\]

From Eq. (2.4), we have

\[
\frac{V}{U} \approx \frac{\delta_u}{l}.
\]

So Eq. (2.3) yields

\[
P \approx \frac{MV}{\delta_u}.
\]

From Eq. (2.1) and by using Eqs. (2.8) and (2.9), it follows that

\[
\frac{P/l}{MU/\delta_u} \approx \left( \frac{\delta_u}{l} \right) \ll 1.
\]

Thereupon, in the set of equations (2.1)—(2.4), we can neglect the terms \( \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right), \frac{\partial}{\partial x} \left( \mu \frac{\partial v}{\partial x} \right), \frac{\partial^2 T}{\partial x^2}, \frac{\partial v}{\partial x} \) and so on. Eq. (2.2) does not couple with the rest of this set. Thus we obtain the following equations:

\[
\frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \rho g a (T - T_0) = 0,
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

\[
u \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2}.
\]
Eqs. (2.11)—(2.13) are the differential equations that govern the required parameters of the upwelling flow \( u, v \) and \( T \). Note that the set of Eqs. (2.11)—(2.13) governing the upwelling flow can be derived from the set of Eqs. (2.1)—(2.4) governing the mantle’s flow, provided \( \left( \frac{\partial z}{l} \right)^2 \ll 1 \) and \( \left( \frac{\partial x}{l} \right)^2 \ll \frac{\Delta z T}{\Delta x T} \). From the calculated results presented in this paper, we can see that these two conditions can be satisfied, except for the small starting region in the asthenosphere or near the cap of the upwelling channel.

III. Solution to the Differential Equations of the Upwelling Flow

In analogy to Karman-Pohlhansen’s single parameter approximation, we can solve the set of Eqs. (2.11)—(2.13) as follows:

Let

\[
\varphi(\eta) = \frac{u}{u_w} = a_0 + a_1 \eta + a_2 \eta^2 + a_3 \eta^3 + a_4 \eta^4, \tag{3.1}
\]

\[
\theta(\eta_T) = \frac{T - T_o}{T_w - T_o} = b_0 + b_1 \eta_T + b_2 \eta_T^2 + b_3 \eta_T^3 + b_4 \eta_T^4. \tag{3.2}
\]

For the flow in the channel within the lithosphere, \( T_o \) is the softening temperature of rock at some depth (the rock will flow at a temperature higher than this), and \( T_w \) is the ambient temperature \( T_w \) for the asthenosphere. We define

\[
\eta = \frac{y}{s_w}, \quad \eta_T = \frac{y}{s_T}, \quad \kappa = \frac{s_T}{s_w}, \tag{3.3}
\]

where \( \kappa \leq \kappa \). The boundary conditions of Eqs. (2.11)—(2.13) become

\[
\varphi = 0, \quad \text{when} \quad \eta = 1,
\]

\[
\varphi = 1, \quad \frac{\partial \varphi}{\partial \eta} = 0, \quad \frac{\partial^3 \varphi}{\partial \eta^3} = 0, \quad \text{when} \quad \eta = 0,
\]

\[
\theta = 0, \quad \text{when} \quad \eta_T = 1,
\]

\[
\theta = 1, \quad \frac{\partial \theta}{\partial \eta_T} = 0, \quad \frac{\partial^3 \theta}{\partial \eta_T^3} = 0, \quad \text{when} \quad \eta_T = 0. \tag{3.4}
\]

From these conditions, we obtain

\[
\varphi(\eta) = 1 + a_2 \eta^2 - (1 + a_2) \eta^4, \tag{3.5}
\]

\[
\theta(\eta_T) = 1 + b_2 \eta_T^2 - (1 + b_2) \eta_T^4. \tag{3.6}
\]

By substituting Eqs. (3.5) and (3.6) into Eqs. (2.11), comparing the coefficients and using the continuity condition of \( \frac{\partial u}{\partial y} \) at point \( y = \delta_r \), it follows that

\[
1 - 2 \left( 1 + \frac{1}{a_2} \right) \varepsilon^2 - \frac{\mu_w}{\mu_s} \left( \frac{4}{5} + \frac{2}{15} b_2 \right) = 0, \tag{3.7}
\]
$$\frac{1}{a_1} + 2 \varepsilon^2 \mu_w \int_0^{\eta \tau} \frac{b_2}{\mu / \mu_0} \eta^2 d\eta = - \left( \frac{8}{5} + \frac{4}{5} b_2 \right) (\varepsilon - \varepsilon^2) \frac{\mu_w}{\mu_a}, \quad (3.8)$$

$$12(1 + a_2) \varepsilon + 4(2 + b_2) [a_2 - 6(1 + a_2) \varepsilon^3] \frac{\partial \mu}{\partial T} \frac{T_w - T_a}{\varepsilon \mu_w}$$

$$+ 2 \left\{ - (6 + 5 b_2) \frac{\partial \mu}{\partial T} + 2(T_w - T_a)(2 + b_2) \frac{\partial^2 \mu}{\partial T^2} \right\} \frac{2 + a_2}{\varepsilon^2} (T_w - T_a)$$

$$+ \frac{\rho g \alpha (T_w - T_a) \delta_x^2 (2 + b_2)}{\varepsilon \mu_w} = 0, \quad (3.9)$$

if the effects on \( \mu \) of the pressure changes in \( y \)-direction are neglected. We can also obtain

$$\delta_x^2 = - \frac{4 a_2 b_2 \varepsilon^2 \mu_w}{\rho g \alpha \frac{dT_w}{dx}}, \quad (3.10)$$

$$u_w = - \frac{b_2 \rho g \alpha (T_w - T_a)^2}{a_2 \varepsilon^2 \mu_w \frac{dT_w}{dx}}. \quad (3.11)$$

Substituting Eqs. (3.5) and (3.6) into (2.13) and integrating for \( y \) over the region \([0, \delta_y]\), we have

$$\left[ \frac{2}{15} + \frac{2}{35} a_2 \varepsilon^2 + \frac{2}{63}(1 + a_2) \varepsilon^3 \right] \frac{db_2}{dx} + \left[ \left( \frac{4}{21} + \frac{2}{35} b_2 \right) \varepsilon^3 + \left( \frac{4}{45} + \frac{2}{63} b_2 \right) \varepsilon^4 \right] \frac{da_2}{dx}$$

$$+ \left[ 2 a_2 \left( \frac{4}{21} + \frac{2}{35} b_2 \right) \varepsilon + 3 \left( \frac{4}{45} + \frac{2}{63} b_2 \right)(1 + a_2) \varepsilon^3 \right] \frac{d\varepsilon}{dx}$$

$$+ \left[ \frac{2}{T_w - T_a} \frac{d(T_w - T_a)}{dx} + \frac{3}{4 b_2} \frac{db_2}{dx} - \frac{1}{4 \mu_w} \frac{d\mu_w}{dx} - \frac{1}{4 a_2} \frac{da_2}{dx} \right]$$

$$\frac{1}{2 s} \frac{d\varepsilon}{dx} - \frac{3}{4} \frac{dT_w}{dx} \left( \frac{dT_w}{dx} \right)^{-1} \times \left[ \frac{4}{5} + \frac{2}{15} b_2 + \left( \frac{4}{21} + \frac{2}{35} b_2 \right) a_2 \varepsilon^4 \right]$$

$$\times \left[ 1 + \frac{1}{3} a_2 \varepsilon^2 - \frac{1}{5}(1 + a_2) \varepsilon^3 \right] + \frac{2 + b_2}{b_2} \frac{dT_w}{dx} (T_w - T_a)^{-1} = 0. \quad (3.12)$$

The boundary conditions are

$$T_w - T_a = 0, \quad \text{at } x = 0 \text{ or } x = -l_z. \quad (3.13)$$

If the functions \( \frac{\partial \mu}{\partial T}, \frac{\partial^2 \mu}{\partial T^2} \) are known, \( u_a, b_2 \) and \( \varepsilon \) can be obtained from Eqs. (3.7)—(3.9), \( T_w \) from Eqs. (3.12) and (3.13), \( u_w, \delta_x \) and \( \delta_y \) from Eqs. (3.10), (3.11) and (3.3).
E. M. Parmentier\textsuperscript{151} has recently investigated the finite-amplitude convection in non-Newtonian fluid, and adopted a power-law constitutive relation with power $n$. He has found that the structure of the convection cell is very close to that of the fluids with a constant viscosity, when $n \leq 3$. Thereby, we shall study in detail the case of constant viscosity in the same way that other writers deal with the mantle's convection\textsuperscript{11,13,14} as a first step for the study of plate dynamics. In this case Eqs. (3.7)\textemdash(3.9) become

\begin{equation}
\frac{1}{5} - 2 \left( 1 + \frac{1}{a_2} \right) \varepsilon^2 - \frac{2}{15} b_2 = 0, \tag{3.14}
\end{equation}

\begin{equation}
\frac{1}{a_2} + 2 \varepsilon^2 \left( \frac{7}{15} + \frac{b_2}{20} \right) = -\left( \frac{8}{5} + \frac{4}{15} \right) \varepsilon (1 - \varepsilon), \tag{3.15}
\end{equation}

\begin{equation}
6 \left( 1 + \frac{1}{a_2} \right) - \frac{1}{\varepsilon^2} (2 + b_2) = 0. \tag{3.16}
\end{equation}

From which we obtain

\begin{equation}
a_2 = -\frac{6}{5}, \quad b_2 = -1, \quad \varepsilon = 1, \tag{3.17}
\end{equation}

so $\sigma_s = \sigma = \sigma$. Eqs. (3.12) and (3.13) become

\begin{equation}
\begin{cases}
ZZ'' - \frac{19}{204} Z'' - \frac{170}{204} \beta Z' + \frac{189}{204} \beta' - Z \beta' = 0, \\
Z = 0 \quad (\text{at} \quad x = 0 \quad \text{or} \quad x = -l_2),
\end{cases} \tag{3.18}
\end{equation}

where "'" and """" represent the first-order and second-order differentiations with respect to $x$, $Z = T_w = T_s$, $\beta = \frac{dT_s}{dx}$.

Eq. (3.18) is a boundary-value problem of the second-order non-linear ordinary differential equation, and should be solved by numerical method. For the asthenosphere, $\frac{dT_s}{dx} = \frac{dT_e}{dx} = \text{const.}$, but for the lithosphere, the law of $\frac{dT_s}{dx}$ has been little studied.

For the purpose of estimating the parameters of plate dynamics, using of the model put forward by the present authors, $\frac{dT_s}{dx} = \text{const.}$ is assumed. Thus, with $\beta'(x) = 0$, Eq. (3.18) becomes

\begin{equation}
\begin{cases}
ZZ'' - \frac{19}{204} Z'' - \frac{170}{204} \beta Z' + \frac{189}{204} \beta' = 0, \\
Z = 0 \quad (\text{at} \quad x = 0 \quad \text{or} \quad x = -l_2),
\end{cases} \tag{3.19}
\end{equation}

Eq. (3.19) has the following analytical solutions:

\begin{equation}
Z_i = \frac{x_i \left( \frac{189}{19} + \sigma \right)^{\frac{963}{98}}}{204} \left( 1 - \sigma \right)^{\frac{963}{98}} \frac{19}{19} \int_{\frac{189}{19}}^{\sigma} \left[ \left( \frac{189}{19} + \sigma \right)^{\frac{961}{98}} \right] d\sigma, \tag{3.20}
\end{equation}
and

\[
\begin{align*}
\ddot{z}_2 &= \frac{-\left(\ddot{x}_1 + \dot{z}_1\right) \left(\frac{189}{19} + \dot{\omega}\right) \left(1 - \dot{\omega}\right)^{0.61^{96.2}}}{204 \left[\left(\frac{189}{19} + \dot{\omega}\right) \left(1 - \dot{\omega}\right)^{0.61^{96.2}}\right]} d\dot{\omega},
\end{align*}
\]  

(3.21)

where \( \ddot{x} = x/l_1 \), \( \ddot{z}_1 = z_1/l_1 \beta_1 \), \( \ddot{z}_2 = z_2/l_2 \beta_2 \) and \( \dot{\omega} = \frac{d\ddot{x}}{dx} \). The parameters in the region \(-l_4 \leq x \leq 0\) are represented by the subscript "1", and those in the region \(-l_2 \leq x \leq -l_1\) by the subscript "2".

IV. COMPUTED RESULTS AND DISCUSSION

The parameters of the earth's mantle are substituted into the above formulas and the structure of the upwelling flow is computed to give the parameters of plate dynamics.

1. Selection of the Parameters of the Earth's Mantle

(1) Viscosity. The computed results in the present paper are given for Newtonian fluids with constant viscosity. The value of the viscosity was chosen as \(10^{11} - 10^{12}\) poises in earlier literatures, and some authors recently suggested a viscosity value of \(10^{10}\) poises for the asthenosphere beneath the continent\(^{[3]}\). Artushkov\(^{[3]}\) believed that the temperature in the asthenosphere beneath the ocean would be \(100^\circ - 200^\circ\) C higher than that in the asthenosphere beneath the continent. Consequently, the viscosity of the asthenosphere beneath the ocean is taken as \(10^{10} - 10^{11}\) poises in Ref. [8]. It is inferred that the temperature of the upwelling flow is even higher than that in the ambient asthenosphere, so the viscosity may still be lower. A calculation is made here for \(\mu = 10^{14}, 10^{15}, 10^{16}\) poises respectively.

(2) The thermometric conductivity \(k = \frac{K}{\rho c_p}\) of the mantle changes with temperature and pressure. \(k\) is assumed to be constant by most authors on the earth's mantle convection. McKenzie et al.\(^{[3]}\) take \(k = 1.5 \times 10^{-5}\) cm\(^2\)/sec. A depth-dependence investigation of \(k\) (Schatz et al.\(^{[3]}\)) showed \(k = 2 \times 10^{-3}\) cm\(^2\)/sec up to a depth of 400 km, and a still larger \(k\) for deeper mantle materials. \(k = 2 \times 10^{-2}\) cm\(^2\)/sec is taken in the present paper.

(3) The coefficient of expansion \(a\). Different authors adopted slightly different values of \(a\), varying from \(3 \times 10^{-5}/\)C to \(4 \times 10^{-5}/\)C. The value of \(3.5 \times 10^{-5}/\)C is taken in the present paper.

(4) With reference to the values of the density \(\rho\) and gravitational acceleration \(g\) adopted by most authors, we take \(\rho = 3.3\) g/cm\(^3\) for mantle materials and \(g = 10^7\) cm/sec\(^2\).

(5) The vertical gradient of temperature in the asthenosphere is generally accepted as \(1^\circ - 2^\circ\) C/km. In the present paper, \(\beta_1 = 1.5^\circ\) C/km is adopted. If the
gradient of softening temperature in the lithosphere is taken as a constant, its absolute value would be smaller than the gradient of the ambient temperature. \( \frac{dT}{dx} \) in the lithosphere is 12°—15°C/km. \( \beta_1 = 8°C/km \) is adopted for the first estimate in the present paper. The calculations show that the velocity of the plate and the energy imported to the plate are almost independent of \( \beta_n \), and the effect on the driving force can not cause a change by orders of magnitude.

2. The Upwelling Flow Structure

In order to compute the values of the functions in Eqs. (3.20) and (3.21), let

\[
f_1 = \frac{\left( \frac{189}{19} + \bar{\alpha} \right) \left( 1 - \bar{\alpha} \right)}{\left[ \left( \frac{189}{19} + \bar{\omega} \right) \left( 1 - \bar{\omega} \right) \right]_{\text{max}}}.
\]  

(4.1)

When

\( \bar{\omega} = 0, f_1 = 1, \)

where

\[
\left( \frac{189}{19} \right)_{\text{max}} = 5.416 \times 10^9,
\]

\[
f_2 = \frac{\int_{0}^{1} \left( \frac{189}{19} + \bar{\omega} \right) \left( 1 - \bar{\omega} \right) \frac{\delta \omega}{\gamma} d\bar{\omega}}{\int_{-\frac{189}{19}}^{1} \left( \frac{189}{19} + \bar{\omega} \right) \left( 1 - \bar{\omega} \right) \frac{\delta \omega}{\gamma} d\bar{\omega}},
\]

(4.2)

where

\[
\int_{-\frac{189}{19}}^{1} \left( \frac{189}{19} + \bar{\omega} \right) \left( 1 - \bar{\omega} \right) \frac{\delta \omega}{\gamma} d\bar{\omega} = 1.427 \times 10^9.
\]

Using Eqs. (4.1) and (4.2), Eqs. (3.20) and (3.21) become

\[
\bar{Z}_1 = -0.3535 \bar{z}_1 f_1 (1 - f_1)^{-1}, \quad \bar{Z}_2 = 0.3535 \left( \bar{z}_1 + \frac{l_1}{l_1} \right) f_1 f_2^{-1},
\]

(4.3)

(4.4)

Let \( f_3 = \frac{\delta u_y}{\delta u_x}, \) and by using Eqs. (3.10) and (3.11), it follows that

\[
f_3 = \frac{\delta u_y}{\delta u_x}_{\text{max}} = \frac{\left( \frac{189}{19} + \bar{\alpha} \right) \left( 1 - \bar{\alpha} \right) \frac{\delta \alpha}{\gamma}}{\left( \frac{2457}{3289} \right) \left( \frac{189}{19} \right) \left( \frac{832}{228} \right)}.
\]

(4.5)
When \( \bar{\omega} = \frac{7371}{9869} \), \( \delta_w \) reaches its maximum. \( f_1 \) is the ratio of mass flux to the maximum flux. When \( 1 > \bar{\omega} > \frac{7371}{9869} \), \( \delta_w \) increases with the decrease of \( \bar{\omega} \), such a case showing that the materials in the asthenosphere tend to enter into the upward flow. When \( -\frac{189}{19} < \bar{\omega} < \frac{7371}{9869} \), \( \delta_w \) decreases with the decrease of \( \bar{\omega} \), indicating that the uprising materials spread outward. The values of functions \( f_1(\bar{\omega}), f_2(\bar{\omega}) \) and \( f_3(\bar{\omega}) \) are given in Table 1.

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<th>( \bar{\omega} )</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( \bar{\omega} )</th>
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Fig. 2 shows the change of maximum horizontal temperature difference \( T_w-T_s \) with depth for the starting depths of 400 km, 500 km and 700 km, respectively. One can see from it that the maximum temperature difference is closely related to the starting depth. The deeper the starting depth is, the greater the temperature difference will be. But in the upwelling channel through the lithosphere, the change of maximum temperature difference with the starting depth suddenly narrows down to a very small range, with the three curves approaching each other. The distribution of the temperature difference is independent of viscosity.

As shown in Fig. 2 and the following figures, at the base of the lithosphere (-100 km), discontinuities appear in the derivatives of \( T_w-T_s, w_w, \delta \), as a result of the discontinuity of \( \frac{dT_w}{dx} = -\beta \) at this depth.

Fig. 3 shows the effect of various starting depths on the distribution of the maximum upwelling velocity \( w_w \), when the viscosity remains constant (\( \mu = 10^{19} \) poises). In the asthenosphere the maximum upwelling velocity \( w_w \) changes greatly with starting depths. The effect of the starting depth on the maximum upwelling velocity in the lithospheric channel is not very obvious due to the small effect of starting depth on the maximum temperature difference \( T_w-T_s \) in the lithospheric channel as can be seen from Eq. (3.11).
Fig. 2. Maximum horizontal temperature difference $T_w - T_s$ versus depth.

Fig. 3. Maximum upwelling velocity $u_w$ versus depth (for three starting depths).
Fig. 4 shows the effect of viscosity \(10^9, 10^{10}, 10^{12}\) poises respectively) on the maximum upwelling velocity \(u_w\) for the starting depth at 500 km. The lower the viscosity is, the larger the value of \(u_w\) will be. From Eq. (3.11), one can see that \(u_w \propto \frac{1}{\sqrt{\mu}}\).

![Graph showing the relationship between viscosity and upwelling velocity](image)

**Fig. 4.** Maximum upwelling velocity \(u_w\) versus depth (for three viscosities).

Fig. 5 shows the dependence of the thickness of the upwelling flow (it is the half width of the upwelling channel in the lithosphere) on depth. The width of the upwelling channel in the lithosphere is almost independent of the starting depth.

Fig. 6 shows the dependence of thickness of the upwelling flow on depth for different viscosities \(10^9, 10^{10}, 10^{12}\) poises and the same starting depth (500 km). It is evident from Fig. 6 that both the thickness of the upwelling flow in the asthenosphere and the half-width of the upwelling channel in the lithosphere are controlled by viscosity. The larger the viscosity is, the wider the upwelling channel will be. From Eq. (3.10), \(\sigma \propto \mu^4\).

From Fig. 6 one can see that the condition \((\sigma/l)^2 \ll 1\) is evidently satisfied by the upward flow in the asthenosphere except for the starting region. In the upwelling channel, the maximum value of \((\sigma/l)^2\) is about 0.02, so Eq. (2.10) is still satisfied.

From Fig. 2, the minimum value of \(\Delta x T\) is about 200°C in the upwelling channel (except for the flow near the cap), and from \(\Delta x T = 8^\circ C/\text{km} \times 100 \text{ km} = 800^\circ C\), we obtain \(\frac{\Delta x T}{\Delta x T} = 0.25\). But \((\sigma/l)^2 = 0.02\), and \(\frac{\Delta x T}{\Delta x T} \gg (\sigma/l)^2\) is still valid, so Eq. (2.7) is satisfied.
3. Calculations of the Parameters of the Plate Dynamics.

From the above results, the following important parameters of the plate dynamics can be computed, e.g. the velocity of the plate movement, the driving force, and the energy transported to the plate by the upwelling lava.

Since the mass flux entering into the upwelling channel equals the mass flux
transported by the plate, the velocity of the plate movement can be obtained

\[ V_p = 0.64 u_{w_1} \frac{\delta_1}{l_1}, \]  

where \( u_{w_1} \) and \( \delta_1 \) are the maximum upwellling velocity and half-width of the channel at the entrance of the channel respectively, \( l_1 \) is the thickness of the lithosphere, taking \( l_1 = 100 \text{ km} \).

The total buoyancy to which the upwelling matter is subject (half channel) is

\[ F_b = \left( \frac{144}{135} k \mu \right)^{\frac{1}{2}} (\rho g a \beta_1)^{\frac{1}{2}} l_1^2 \int_0^{\delta_1} \tilde{Z}(1 - \tilde{\alpha})^{-\frac{1}{2}} d\tilde{\alpha}. \]  

(4.7)

The buoyancy force exerted on unit length of the wall can be decomposed into a shear force along the wall and a horizontal pushing force. Thus the ratio of the pushing force to the buoyancy is \( \frac{d\delta}{dx} \). The total pushing force (half channel) is

\[ F_p = \left( \frac{2}{15} \rho g a \mu \beta_1 \right)^{\frac{1}{2}} l_1 \int_{\frac{19}{19}}^{\delta_1} \tilde{Z}(1 - \tilde{\alpha})^{-\frac{1}{2}} d\tilde{\alpha}. \]  

(4.8)

The inner energy contained in the lava entering into the lithosphere from the asthenosphere per unit time and per unit width can approximately be written as

\[ W_x = \int_0^{\delta_1} \rho c_p u_{w_1} T_i dy = 0.518 \rho c_p u_{w_1} \delta_1 (T_{w_1} - T_{a_1}) + 0.64 \rho c_p u_{w_1} \delta_1 T_{a_1}. \]  

(4.9)

The inner energy contained in the rock returning to the asthenosphere per unit time and per unit width of the plate can approximately be written as

\[ W_0 = \dot{m} c_p T_{\infty_1}, \]  

(4.10)

It is conservatively assumed that the temperature within the plate which returns to the asthenosphere, equals that at the base of the lithosphere \( T_{\infty_1} \) (the temperature of the returning slab of the lithosphere is actually smaller than \( T_{\infty} \)). \( \dot{m} \) in Eq. (4.10) represents the mass flux returning to the asthenosphere

\[ \dot{m} = \int_0^{\delta_1} \rho u_i dy, \]  

(4.11)

then

\[ W_0 = 0.64 \rho c_p u_{w_1} \delta_1 T_{\infty_1}. \]  

(4.12)

By noting \( T_{\infty_1} = T_{a_1} \), and from Eqs. (4.9) and (4.12), the energy transported to the plate by the upwelling materials per unit time can be obtained

\[ W = 0.518 \rho c_p u_{w_1} \delta_1 (T_{w_1} - T_{a_1}). \]  

(4.13)

If \( L \) represents the total length of the active mid-oceanic ridges \( (L = 6000 \text{ km}) \), the energy transported to the lithosphere of the whole earth by the upwelling materials per unit time will be

\[ W_{\text{total}} = 2WL = 1.036 \rho c_p u_{w_1} \delta_1 (T_{w_1} - T_{a_1}) L. \]  

(4.14)
$W_{\text{total}}$ will be exhausted in earthquakes, volcanic eruptions, surface heat flow and deformation in the marginal areas of the plate, and so on.

Table 2 gives the parameters of the plate dynamics for three viscosities ($\mu = 10^{14}$, $10^{19}$, $10^{20}$ poises) and three starting depths (400, 500, 700 km).

**Table 2**

<table>
<thead>
<tr>
<th>$l_4/l_1$</th>
<th>$V_s$ cm/yr</th>
<th>$F_s$ dynes/cm</th>
<th>$W_{\text{total}}$ cal./sec</th>
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<tr>
<td>$l_4/l_1 = 4$</td>
<td>$\mu = 10^{14}$ Poises</td>
<td>1.78</td>
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<td>0.56</td>
<td>$1.63 \times 10^{13}$</td>
</tr>
<tr>
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<td>1.33</td>
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<tr>
<td></td>
<td>$\mu = 10^{19}$ Poises</td>
<td>1.02</td>
<td>$0.500 \times 10^{13}$</td>
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<td>$\mu = 10^{20}$ Poises</td>
<td>0.58</td>
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<tr>
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<td>$1.52 \times 10^{13}$</td>
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From Table 2, we can see that the velocity of the plate movement computed agrees with that observed. If $W_{\text{total}} = 6.45 \times 10^{11}$ cal/sec (for $l_4 = 500$ km, $\mu = 10^{19}$ poises) is chosen as a typical value of the energy transported by the upwelling materials, it is found to be eighty times greater than the energy $0.8 \times 10^{10}$ cal/sec released by earthquakes of the whole earth, a value much larger than the lower limit of energy necessary for any driving mechanism, $4.6 \times 10^{10}$ cal/sec, as pointed out by McKenzie. The buoyancy is the driving force of the plate movement. An absence of observational data prevents a comparison of the pushing forces. Since the plate moves with a constant velocity, the total resistance must balance the total pushing force. The resistance is primarily the viscous resistance. The results of the calculations of resistance are not identical because there is no agreement on the models of the mantle's flows. F. M. Richter suggested a model for calculating the resistance, the result of which is used for our data. The obtained value, $1 \times 10^{13}$ dynes/cm, is of the same order of magnitude as the pushing force $0.5 \times 10^{13}$ dynes/cm listed in Table 2.

V. Concluding Remarks

Prof. Yin Zanxun (尹赞勋), an eminent geologist of China, and some well-known scientists abroad (J. T. Wilson and others) suggested that sea floor spreading may originate from the penetration of the upwelling lava into the cracks of the lithosphere. Now, the above view remains a conjecture, mainly because the upwelling flow can not be described quantitatively. Using the method of hydrodynamics we have given a mathematical description of the upwelling flow and obtained the size of the upwelling channel, the velocity of the plate movement, the pushing force and the energy transported to the lithosphere by the upwelling materials. The results obtained are found to be in good agreement with the observational data, indicating that the analysis from
fluid dynamics mentioned above appears to be valid. Hence the following point may be elaborated: the mass, momentum and energy imported to the oceanic plate by the upflow of the deep mantle materials into the lithospheric channel (crack) below the mid-oceanic ridges seems to constitute the most efficient driving mechanism for sea floor spreading.

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