A FLUID-DYNAMIC MODEL OF THE ASCENDING FLOW IN THE MANTLE PLUME

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ABSTRACT

It is of significance, both for the exploitation of geothermal resources and for an understanding of the causes of volcanic and seismic activities and the continental drift, to make a thorough study of the ascending flows in the mantle plumes that lie beneath the hot spots of the earth. In this paper, a fluid-dynamic model of this ascending flow is presented. The extent of the mantle plume, the maximum temperature difference and the maximum velocity of the ascending flow in the mantle plume, and the heat transported by the mantle plume to the lithosphere have all been estimated. The results are found to be in good accord with the available data from geophysical observations.

I. INTRODUCTION

The presence of hot spots on the earth's surface has now attracted increasing attention of most earth scientists[1-3]. Discrete, small areas of volcanic activity referred to as "hot spots" on the earth's surface are so far known to exceed 100 in number. In such regions there are, in general, high heat flow together with volcanism. The hot spots have the following features: (i) Hot spots are not necessarily restricted to the boundaries of the lithospheric plates, not a few occurring in their interior. (ii) The volcanic lava at the hot spots are found to be mainly alkalic basalts indicating a deep mantle origin. (iii) The hot spots are believed to be "anchored" at the earth's depths; they do not move together with the plate, often leaving the traces of extinct volcanoes on the earth's surface.

According to these features, some have suggested that these hot spots may be the surface expressions of the mantle plumes rising as cylindrical flow of hypothermal material, probably from a depth of hundreds of kilometers beneath the lithosphere[4]. Most geophysicists believe that the natural convection of the mantle materials is responsible for the upwelling mechanism: as the materials reach the lower surface of the lithosphere by buoyancy, they spread out laterally and cool down, and finally return to the earth's interior. Calder[5] pointed out that if the solid rocks could move as a very slow and viscous flow, then it is more difficult, if not impossible, to estimate its movement. It appears that a plausible dynamic explanation for the ascending flow of the mantle plume has not yet been given, though the idea of the mantle plume was formulated many years ago.

An attempt has been made to estimate the flow in [2], with two problems left untouched: (i) The ascending movement of a single hot sphere was analysed only; the flow in the mantle plume was not considered in detail. (ii) A very important effect in natural convection — effect of heat transfer on the movement — was not taken into account. Although the cylindrical plume was examined in [6], the effect of heat transfer on the movement was again not considered. Therefore, the results of calculation in [2] and [6] cannot be expected to be in good agreement with practical observation.

II. Basic Equations of the Ascending Flow in the Mantle Plume

The mantle flow satisfies general fluid-dynamic equations. In terms of tensor symbol, the equations can be written as

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + 2\varepsilon_{ijk}\Omega_j u_k = -\frac{1}{\rho}\frac{\partial \pi}{\partial x_i} + \frac{\partial \Phi}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j}, \tag{2.1}$$

$$\rho c_p \left\{\frac{\partial T}{\partial t} + u_i \left[\frac{\partial T}{\partial x_i} - \left(\frac{\partial T}{\partial x_i}\right)_s\right]\right\} = \frac{\partial}{\partial x_i}\left(\kappa \frac{\partial T}{\partial x_i}\right) + H + \tau_{ij}\frac{\partial u_i}{\partial x_j} \tag{2.2}$$

$$\frac{\partial(\rho u_i)}{\partial x_i} = 0. \tag{2.3}$$

The state equation can be written as

$$\rho = \rho_0[1 - \alpha(T - T_0)], \tag{2.4}$$

where ρ is the density, u_i the ith component of velocity, T the temperature, c_p the specific heat at a constant pressure, k the thermal conductivity, H the inner heat rate generated by radioactive decay, τ_{ij} the shear tensor element, Ω_j the jth component of the earth's angular velocity, π the pressure, α the thermal expansion coefficient, $\left(\frac{\partial T}{\partial x_i}\right)_s$ the adiabatic temperature gradient, and Φ the gravitational potential in rotating reference frame:

$$\Phi = G + \frac{1}{2}|\boldsymbol{\Omega} \times \boldsymbol{r}|^2, \tag{2.5}$$

where G is the gravitational potential, Ω the vector of the earth's angular velocity, and $\boldsymbol{r}$ the position vector.

Accepting the assumptions and analyses similar to those made in sections 2 and 3 in [7], and by changing from two-dimensional into axisymmetric flow, we can rewrite equations (2.1)—(2.3) as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r \frac{\partial u}{\partial r}\right) + \rho g \alpha (T - T_\infty) = 0, \tag{2.6}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{2.7}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial x} = 0, \tag{2.8}$$

where u and v are velocity components in x- and r-directions, respectively; (x, r) coordinate are shown in Fig. 1; μ is the viscosity, g the gravitational acceleration, T_∞ the stationary surrounding temperature, and $\kappa/\rho c_p$ the thermometric conductivity.

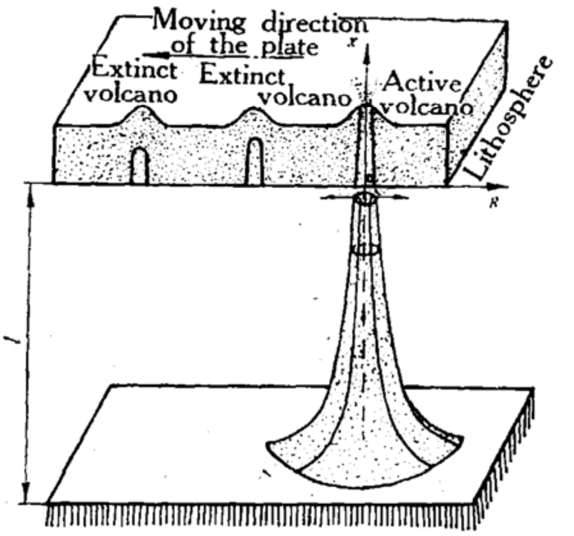

Fig. 1. Ascending flow in the mantle plume.

If we assume that the viscosity is constant, Eq. (2.6) can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{\rho g \alpha}{\mu}(T - T_\infty) = 0. \tag{2.9}$$

Eqs. (2.7), (2.8) and (2.9) are the set of differential equations governing the parameters of the ascending flow u, v and T respectively. Starting from the set of differential equations, we shall solve in the next section the problem of the ascending flow in the mantle plume.

III. Solution of the Differential Equations of the Ascending Flow

In analogy to Karman-Pohlhausen single parameter approximate method, we let

$$\varphi = \frac{u}{u_w} = a_0 + a_1\eta_u + a_2\eta_u^2 + a_3\eta_u^3 + a_4\eta_u^4, \tag{3.1}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} = b_0 + b_1\eta_T + b_2\eta_T^2 + b_3\eta_T^3 + b_4\eta_T^4, \tag{3.2}$$

where the subscript w represents the parameter at the axis; ∞ is the surrounding parameter; a_i, $b_i (i = 0, 1, 2, 3, 4)$ are constant, and

$$\eta_u = \frac{r}{\delta_u}, \tag{3.3}$$

$$\eta_T = \frac{r}{\delta_T}, \tag{3.4}$$

$$\varepsilon = \frac{\delta_T}{\delta_u}, \tag{3.5}$$

where δ_u and δ_T are the velocity radius and temperature radius of the ascending flow, respectively. They depend only on x. Since the driving force of the ascending flow is buoyancy, we have $\delta_u \geqslant \delta_T$, i.e. $\varepsilon \leqslant 1$.

From the boundary conditions on velocity, we obtain

$$\varphi = 0, \quad \text{when } \eta_u = 1\,, \tag{3.6}$$

and

$$\varphi = 1, \quad \frac{\partial \varphi}{\partial \eta_u} = 0, \quad \text{when } \eta_u = 0\,. \tag{3.7}$$

From the boundary conditions on temperature, we obtain

$$\theta = 0, \quad \text{when } \eta_T = 1\,, \tag{3.8}$$

and

$$\theta = 1\,, \quad \frac{\partial \theta}{\partial \eta_T} = 0\,, \quad \text{when } \eta_T = 0\,. \tag{3.9}$$

From Eqs. (3.1) and (3.7) we obtain

$$\varphi = \frac{u}{u_w} = 1 + a_2\eta_u^2 + a_3\eta_u^3 + a_4\eta_u^4\,. \tag{3.10}$$

By substituting Eq. (3.10) into Eq. (2.9), it follows that

$$\frac{u_w}{\delta_u^2}(4a_2 + 9a_3\eta_u + 16a_4\eta_u^2) + \frac{\rho g \alpha}{\mu}(T - T_\infty) = 0\,. \tag{3.11}$$

Differentiating Eq. (3.11) with respect to η_u, we obtain

$$\frac{u_w}{\delta_u^2}(9a_3 + 32a_4\eta_u) + \frac{\rho g \alpha}{\mu}(T_w - T_\infty)\frac{1}{\varepsilon}\frac{\partial \theta}{\partial \eta_T} = 0\,. \tag{3.12}$$

Taking the values at the point $\eta_u = \eta_T = 0$, Eq. (3.12) gives $a_3 = 0$. In terms of Eq. (3.6), Eq. (3.10) can be reduced to

$$\varphi = \frac{u}{u_w} = 1 + a_2\eta_u^2 - (1 + a_2)\eta_u^4. \tag{3.13}$$

Similarly, we obtain

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} = 1 + b_2\eta_T^2 - (1 + b_2)\eta_T^4. \tag{3.14}$$

By substituting Eqs. (3.5), (3.14) and $a_3 = 0$, $a_4 = -(1 + a_2)$ into Eq. (3.11), it follows that

$$4a_2 = -\frac{\rho g\alpha(T_w - T_\infty)}{\mu u_w}\delta_u^2, \tag{3.15}$$

$$b_2 + 1 = 0, \tag{3.16}$$

$$-4(1 + a_2)\varepsilon^2 - a_2 b_2 = 0. \tag{3.17}$$

When $\delta_T \leqslant r \leqslant \delta_u$, $(T - T_\infty) = 0$. From Eq. (2.9) we obtain

$$[-4(1 + a_2)\varepsilon^4 + 2a_2\varepsilon^2]\ln\varepsilon = 1 + a_2\varepsilon^2 - (1 + a_2)\varepsilon^4. \tag{3.19}$$

From Eqs. (3.16), (3.17) and (3.19), we obtain the following solutions:

$$a_2 = -\frac{4}{3}, \quad b_2 = -1, \quad \varepsilon = 1.$$

Thus $\delta_u = \delta_T = \delta$, $\eta_u = \eta_T = \eta$. Therefore, Eqs. (3.13) and (3.14) become

$$\varphi = \frac{u}{u_w} = 1 - \frac{4}{3}\eta^2 + \frac{1}{3}\eta^4, \tag{3.20}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} = 1 - \eta^2. \tag{3.21}$$

Eq. (3.15) becomes

$$\delta^2 = \frac{16}{3}\frac{\mu u_w}{\rho g\alpha(T_w - T_\infty)}. \tag{3.22}$$

Substituting Eq. (3.21) into Eq. (2.7), and taking the value at point $r = 0$, we obtain

$$\delta^2 = \frac{4k(T_w - T_\infty)}{u_w \dfrac{dT_w}{dx}}. \tag{3.23}$$

From Eqs. (3.22) and (3.23), we have

$$\delta^4 = \frac{64k\mu}{3\rho g\alpha\left(-\dfrac{dT_w}{dx}\right)}, \tag{3.24}$$

$$u_w^2 = \frac{3\rho g\alpha k(T_w - T_\infty)^2}{4\mu\left(-\dfrac{dT_w}{dx}\right)}. \tag{3.25}$$

Using Eqs. (2.7), (2.8) and the definitions of φ and θ, we easily obtain

$$\frac{\partial}{\partial x}(\varphi\theta)+\left[\frac{1}{T_w-T_\infty}\frac{d(T_w-T_\infty)}{dx}+\frac{1}{u_w}\frac{du_w}{dx}\right]\theta\varphi+\frac{dT_\infty/dx}{T_w-T_\infty}\varphi$$
$$+\frac{\partial}{\partial r}\left(\frac{u}{u_w}\theta\right)+\frac{1}{u_w}\frac{v}{r}\theta=\frac{k}{u_w(T_w-T_\infty)}\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial(T-T_\infty)}{\partial r}\right]. \quad (3.26)$$

Substituting Eqs. (3.20) and (3.21) into Eq. (3.26), and integrating r over the region $[0, \delta]$, we obtain

$$\frac{160}{315}\frac{1}{\delta}\frac{d\delta}{dx}+\frac{160}{315}\left[\frac{1}{T_w-T_\infty}\frac{d(T_w-T_\infty)}{dx}+\frac{1}{u_w}\frac{du_w}{dx}\right]+\frac{196}{315}\frac{dT_w/dx}{T_w-T_\infty}$$
$$-\frac{24}{315}\frac{1}{\delta}\frac{d\delta}{dx}-\frac{92}{315}\frac{1}{u_w}\frac{du_w}{dx}=-\frac{4k}{u_w\delta^2}.$$

By using Eqs. (3.24) and (3.25), the above equation can be further written as

$$\frac{68}{315}\frac{d^2T_w/dx^2}{dT_w/dx}+\frac{87}{315}\frac{1}{T_w-T_\infty}\frac{d(T_w-T_\infty)}{dx}+\frac{119}{315}\frac{1}{T_w-T_\infty}\frac{d(T_w-T_\infty)}{dx}=0. \quad (3.27)$$

Letting $Z=T_w-T_\infty, \dfrac{dT_w}{dx}=-\beta=$ constant, we can rewrite the above equation as

$$68ZZ''+87Z'^2-206\beta Z'+119\beta^2=0, \quad (3.28)$$

where primes represent the order of differentiation with respect to x. The boundary conditions of Eq. (3.28) are

$$Z=0, \text{ when } x=0 \text{ or } x=-l, \quad (3.29)$$

Letting $x=l\tilde{x}$, $Z=\beta l\tilde{x}$, we have $Z'=\beta\tilde{Z}'$, and Eqs. (3.28) and (3.29) become

$$\left.\begin{array}{l}68\tilde{Z}\tilde{Z}''+87\tilde{Z}'^2-206\tilde{Z}'+119=0,\\ \tilde{Z}=0 \text{ when } \tilde{x}=0 \text{ or } \tilde{x}=-1.\end{array}\right\} \quad (3.30)$$

Eq. (3.30) is a boundary value problem of the second-order nonlinear ordinary differential equation. And the following analytic solutions may be obtained:

$$\tilde{Z}=\frac{(1-\tilde{\omega})^{\frac{17}{8}}\left(\frac{119}{87}-\tilde{\omega}\right)^{-\frac{2023}{696}}}{\frac{68}{87}\int_{-\infty}^{1}(1-\tilde{\omega})^{\frac{783}{696}}\left(\frac{119}{87}-\tilde{\omega}\right)^{-\frac{2719}{696}}d\tilde{\omega}} \quad (-\infty<\tilde{\omega}\leqslant 1), \quad (3.31)$$

$$\tilde{x}=\frac{-\int_{-\infty}^{\tilde{\omega}}(1-\tilde{\omega})^{\frac{783}{696}}\left(\frac{119}{87}-\tilde{\omega}\right)^{-\frac{2719}{696}}d\tilde{\omega}}{\int_{-\infty}^{1}(1-\tilde{\omega})^{\frac{783}{696}}\left(\frac{119}{87}-\tilde{\omega}\right)^{-\frac{2719}{696}}d\tilde{\omega}} \quad (-\infty<\tilde{\omega}\leqslant 1), \quad (3.32)$$

where $\tilde{\omega}=d\tilde{Z}/d\tilde{x}$. As cited in [7], the above solutions are valid in the ascending flow region, except for the small regions in the neighborhood of $\tilde{x}=0$ and $\tilde{x}=-1$.

IV. Results of Calculation and Discussion

Using Eqs. (3.31) and (3.32), we can obtain the dependence of $\tilde{Z}$ and $\tilde{x}$ on $\tilde{\omega}$. Their typical values are given in Table 1. Eqs. (3.24) and (3.25) can further be written in the following dimensionless form:

Table 1

The Dependence of $\tilde{Z}$ and $\tilde{x}$ on $\tilde{\omega}$ $(-\infty < \tilde{\omega} \leqslant 1)$

$\tilde{\omega}$	$\tilde{Z}$	$\tilde{x}$	$\tilde{\omega}$	$\tilde{Z}$	$\tilde{x}$
1	0	−1	0.1000	0.5332	−0.1690
0.9500	0.02889	−0.9970	0.0500	0.5345	−0.1515
0.9000	0.09073	−0.9902	0	0.5349	−0.1356
0.8500	0.1599	−0.8234	−0.0500	0.5345	−0.1210
0.8000	0.2253	−0.7441	−0.1000	0.5335	−0.1075
0.7500	0.2833	−0.6694	−0.1500	0.5320	−0.09513
0.7000	0.3329	−0.6011	−0.2000	0.5300	−0.08370
0.6500	0.3744	−0.5397	−0.2500	0.5276	−0.07312
0.6000	0.4089	−0.4847	−0.3000	0.5250	−0.06331
0.5500	0.4372	−0.4356	−0.3500	0.5220	−0.05420
0.5000	0.4602	−0.3917	−0.4000	0.5188	−0.04573
0.4500	0.4789	−0.3526	−0.4500	0.5154	−0.03783
0.4000	0.4938	−0.3175	−0.5000	0.5119	−0.03045
0.3500	0.5056	−0.2861	−0.5500	0.5083	−0.02355
0.3000	0.5149	−0.2577	−0.6000	0.5046	−0.01710
0.2500	0.5219	−0.2322	−0.6500	0.5008	−0.01104
0.2000	0.5271	−0.2091	−0.7000	0.4969	−0.005349
0.1500	0.5308	−0.1881	−∞	0	0

$$\tilde{\delta} = \delta \Big/ \sqrt[4]{\frac{64k\mu}{3\rho g\alpha\beta}} = \frac{1}{\sqrt[4]{1-\tilde{\omega}}} \quad (-\infty < \tilde{\omega} \leqslant 1), \tag{4.1}$$

$$\tilde{u}_w = u_w \Big/ \sqrt{\frac{3\rho g\alpha k\beta l^2}{4\mu}} = \frac{\tilde{Z}}{\sqrt{1-\tilde{\omega}}} \quad (-\infty < \tilde{\omega} \leqslant 1). \tag{4.2}$$

Using the values in Table 1, we readily find the dependence of $\tilde{Z}$, $\tilde{\delta}$ and $\tilde{u}_w$ on $\tilde{x}$.

The $\tilde{Z}$ versus $\tilde{x}$ curve is shown in Fig. 2. The negative values of $\tilde{x}$ indicate that the ascending flow happens beneath the lithosphere (Fig. 1). When $\tilde{x}=-1$, $\tilde{Z}=0$, and the lava starts to ascend, therefore l is called the starting depth. $\tilde{Z}$ increases with the increase of $\tilde{x}$ (its absolute value decreases). When $\tilde{x}$ equals -0.1356 (which corresponds to $\tilde{\omega}=0$), $\tilde{Z}$ takes the maximum value (0.5349). When $\tilde{x}$ increases further, $\tilde{Z}$ slowly decreases. But $\tilde{Z}$ decreases suddenly near the point $\tilde{x}=0$. From Eq. (3.32) one can see that when $\tilde{x}=0$, $\tilde{\omega}\to-\infty$; there is a singular point. Therefore, when discussing the parameters at point $\tilde{x}=0$ below, we shall approximately use the parameters at point $\tilde{x}=-0.0100$ instead. Since the temperature change near point $\tilde{x}=0$ is sharp, enormous heat may be transported by the mantle plume to the lithosphere. The

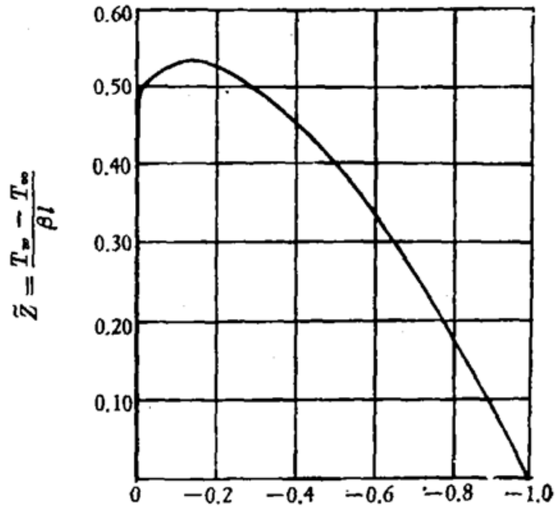

Fig. 2. The dimensionless temperature difference $\tilde{Z}$ versus the dimensionless depth $\tilde{x}$.

mantle plume acts in a way similar to a "hot drill" steadily working its way into the lithosphere. On penetrating through the lithospheric plate, the lava overflows in large quantities to produce a volcano. The formation of the volcano will tend to change the boundary conditions of Eq. (3.30) so that the heat transported to the plate will decrease and the volcano will decline gradually to extinction. A continuation of such a process will eventually lead to a volcanic chain now seen on the earth's surface (see Fig. 1) as the plate slides part over it with time.

Fig. 3 shows the $\tilde{\delta}$ versus $\tilde{x}$ curve. When $\tilde{x}=-1$, $\delta \to \infty$. As $\tilde{x}$ increases (its absolute value decreases), $\tilde{\delta}$ continuously decreases. When $\tilde{x}=-0.0100$, $\tilde{\delta}=0.8811$.

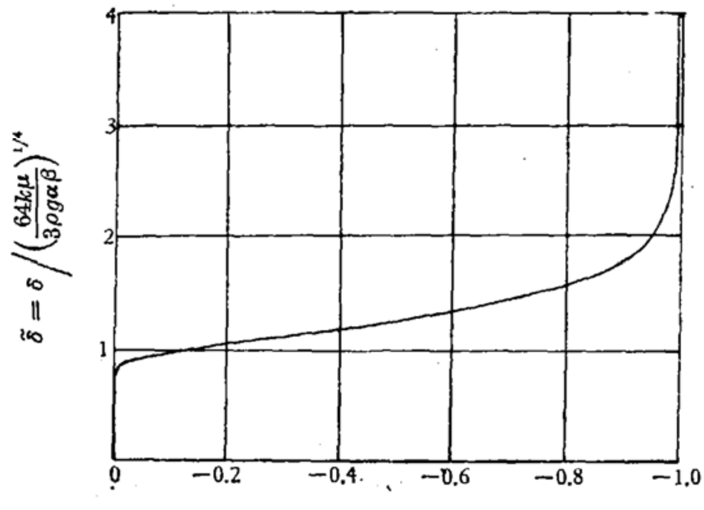

Fig. 3. The dimensionless radius $\tilde{\delta}$ versus the dimensionless depth $\tilde{x}$.

Fig. 4 shows the $\tilde{u}_w$ versus $\tilde{x}$ curve. At $\tilde{x}=-1$, $\tilde{u}_w=0$. As $\tilde{x}$ increases (its absolute value decreases), $\tilde{u}_w$ increases until it reaches its maximum. Beyond the maximum value, $\tilde{u}_w$ decreases with the increase of $\tilde{x}$. When $\tilde{x}=-0.0100$, $\tilde{u}_w=0.3882$.

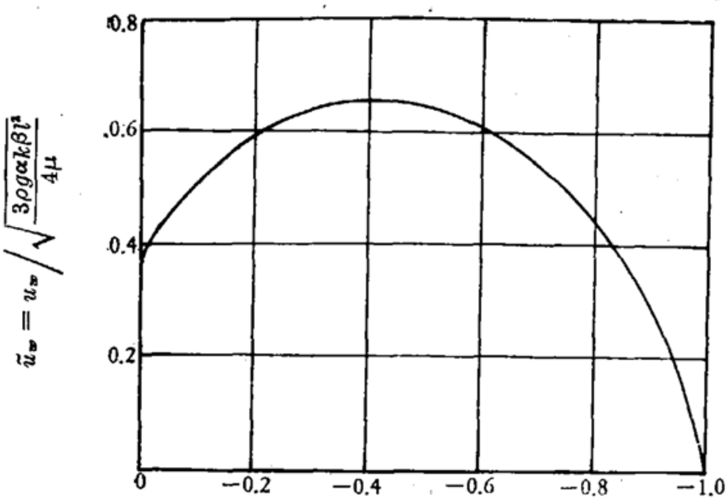

Fig. 4. The dimensionless velocity $\tilde{u}_w$ versus the dimensionless depth $\tilde{x}$.

If the physical properties, ρ, α, k, μ and β of the lava are known, we can obtain the dependences of $(T_w - T_\infty)$, δ and u_w on x, using the dimensionless charts (i.e. Figs. 2—4). According to the discussion cited in [7], we now take $\rho = 3.3\,\text{g/cm}^3$, $\alpha = 3.5 \times 10^{-5}$/deg, $k = 2 \times 10^{-2}\,\text{cm}^2$/sec, and $\mu = 10^{18}$, 10^{19} and 10^{20} g/cm sec, and $\beta = 1.0$ and 1.5 deg/km, respectively.

Fig. 5 shows the dependences of $(T_w - T_\infty)$ on the starting depth l. The value at $\tilde{x} = -0.1356 (\tilde{\omega} = 0)$ corresponds to the maximum value of $(T_w - T_\infty)$ and the value at $\tilde{x} = -0.0100$ $(\tilde{\omega} = 0.6601)$ corresponds to that of $(T_w - T_\infty)$ near the lower surface of the lithosphere. When $\tilde{x}$ is constant, $(T_w - T_\infty)$ is directly proportional to βl.

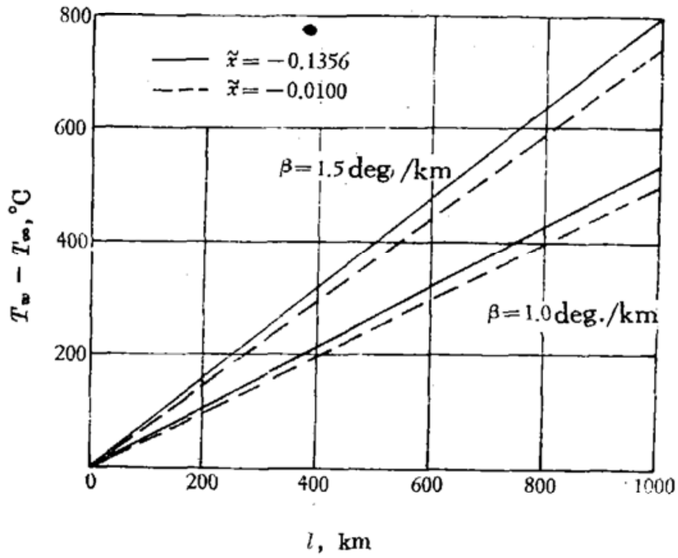

Fig. 5. The temperature difference $(T_w - T_\infty)$ versus the starting depth l.

Fig. 6 shows the δ versus μ curves in logarithmic coordinates. In this figure four curves are given: The $\tilde{x}$-values are -0.1356 and -0.0100, and β-values are 1.0 and 1.5 deg/km, respectively; they are all found to be straight lines. From this figure one can see that δ varies over a small region. when μ varies from 10^{18} g/cm sec to 10^{20} g/cm sec, δ varies from several kilometers to twenty kilometers or so. From Eq. (4.1) we find that when $\bar{x}$ is constant, δ is directly proportional to $(k\mu)^{\frac{1}{4}}$, and inversely proportional to $(\rho g\alpha\beta)^{\frac{1}{4}}$

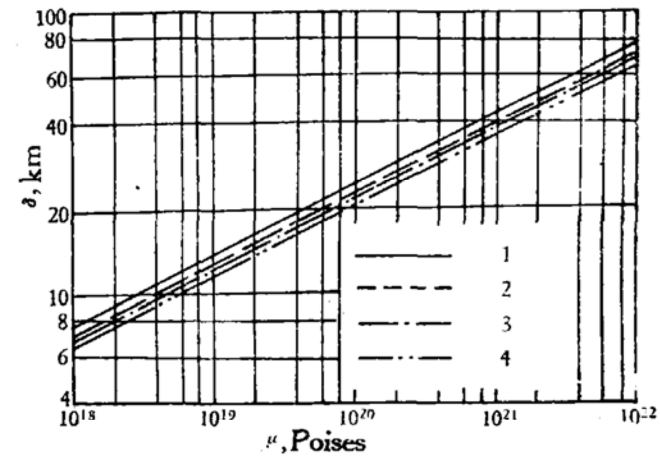

Fig. 6. The radius δ versus the viscosity μ.

1—$\beta = 1.0$ deg/km, $\bar{x} = -0.1356$; 2—$\beta = 1.0$ deg/km, $\bar{x} = -0.0100$;

3—$\beta = 1.5$ deg/km, $\bar{x} = -0.1356$; 4—$\beta = 1.5$ deg/km, $\bar{x} = -0.0100$.

Fig. 7 shows u_w as a function of the starting depth l. The $\bar{x}$-values of the curves in the figure are -0.1356 and -0.0100, their β-values are 1.0 and 1.5 deg/km, and their μ-values are 10^{18}, 10^{19} and 10^{20} g/cm sec, respectively. From Eq. (4.2) one can see that when $\bar{x}$ is constant, u_w is directly proportional to l and to the square root of $(\rho g\alpha k\beta)$, and inversely proportional to the square root of μ.

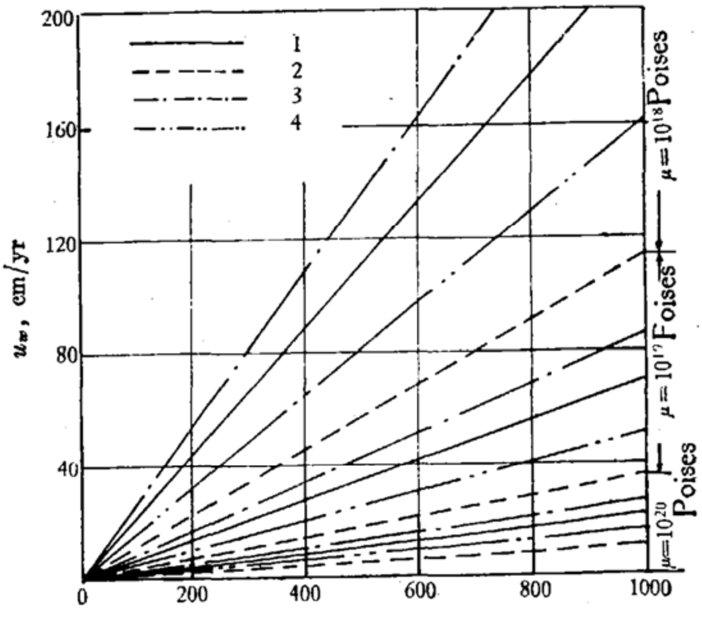

Fig. 7. The velocity u_w versus the starting depth l.

1—$\beta = 1.0$ deg/km, $\bar{x} = -0.1356$; 2—$\beta = 1.0$ deg/km, $\bar{x} = -0.0100$;

3—$\beta = 1.5$ deg/km, $\bar{x} = -0.1356$; 4—$\beta = 1.0$ deg/km, $\bar{x} = -0.0100$.

The heat $\dot{Q}$ transported by the mantle plume to the lower surface of the lithosphere is the most interesting parameter for us. Since the point $\tilde{x}=0$ (i.e. the lower surface of the lithosphere) is the singular point of the solution of the equations governing the ascending flow in the mantle plume, $\dot{Q}$ can not be obtained from the temperature gradient at this point. Thus, we can, to an approximation, substitute the section plane at $\tilde{x}=-0.0100$ for the lower surface of the lithosphere. Using the relation of energy balance, and expressing $\dot{Q}$ in terms of the difference of the convective heat transfer, we obtain

$$\dot{Q}=\int_0^{\delta} 2\pi r \rho u c_p (T - T_\infty) dr = \frac{11}{9}\pi \frac{\tilde{Z}^2}{1-\tilde{\omega}} \rho c_p k \beta l^2. \tag{4.3}$$

Here we have neglected the conductive heat due to the temperature gradient both at section plane $\tilde{x}=-0.0100$ and at the side surface of the mantle plume, and assumed that the temperature of the lateral flow of the mantle plume approximately equals the surrounding temperature at section plane $\tilde{x}=-0.0100$.

Fig. 8 shows $\dot{Q}$ as a function of the starting depth l; the solid curve for $\beta=1.0$ deg/km, and the broken curve for $\beta=1.5$ deg/km. From this figure it can be clearly seen that $\dot{Q}$ is of the order of 10^8 cal/sec. For example, at a starting depth of 600 km, $\dot{Q}$ is approximately 3.7×10^8 cal/sec, assuming $\beta=1.0$ deg/km.

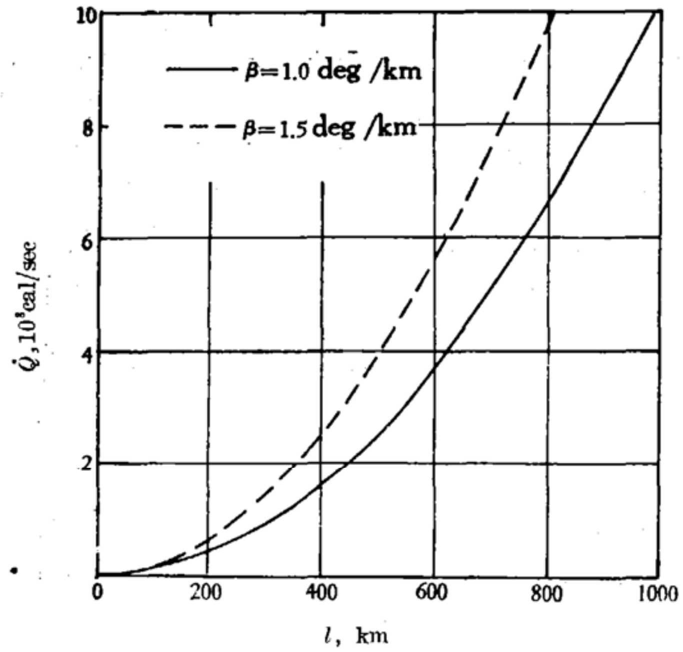

Fig. 8. The heat flow $\dot{Q}$ versus the starting depth l.

It was pointed out in [8] that the energy released by the volcanoes of the entire earth is about 2×10^{10} cal/sec. Up to the present, no exact estimate of the ratio of the heat released by the hot spot volcanoes to the above figure is available. The authors believe, however, that it will not probably exceed one half. Thus, the heat released by the volcanoes of all hot spots of the earth may approximately be considered as 1×10^{10} cal/sec. It was pointed out in [4] that there are 122 hot spots on the earth. It follows that the heat released by volcanism per hot spot is about 1×10^8 cal/sec. In addition, there is a high heat flow where hot spots occur. Practical measurements show that the heat flow in these regions is about 8×10^{-6} cal/cm^2 sec[9]. If a hot spot on the earth's surface is of the same size as the mantle plume on the lower surface of the lithosphere, the radius of the high heat flow region on the earth's surface will be from several kilometers to 20 kilometers. If this radius is considered to be 20 km, the heat flow per hot spot will be about 1×10^8 cal/sec. The sum of the two will be about 2×10^8 cal/sec. It agrees quite well with the results of calculation shown in Fig. 8 in their order of magnitude.

The heat transported by a single mantle plume to the earth's surface also was given in [2], its value being about 1.8×10^{11} cal/sec. If the number of hot spots of the whole earth is 122[4], the total heat supplied by all mantles of the whole earth is 2.2×10^{13} cal/sec. This is three times as large as the total heat flow on the earth's surface. Clearly this is unreasonable. This discrepancy may come from an overestimate of the radius of the mantle plume (about 75 km) and the ascending velocity of the hot material (about 200 cm/yr) in [2].

V. Conclusion

In the present paper, the extent of the mantle plume beneath a hot spot, the maximum ascending velocity and the maximum temperature difference in the mantle plume, and the heat transported by the mantle plume to the lithosphere have all been estimated. The results agree well with the available data from geophysical observations. This shows that the calculation presented in the present paper is valid and the mantle plume hypothesis proposed by W. J. Morgan[1-2] seems plausible. However, an overestimate by him of the extent of the mantle plume and the ascending velocity of hot material has resulted in an exaggeration of the efficiency of the mantle plume.

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