

Scale Effect of Plastic Strain Rate

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Abstract: By analyzing some mechanical quantities and typical dynamic testing results for similar models, this paper studies the scale effect pertaining to similar models made of strain-rate dependent materials, and also describes the effect of plastic strain rate on the mechanical behavior of similar models under dynamic loading. It has been pointed out that the strain-rate sensitivity for dynamic behavior increases with the decrease of the characteristic dimension.

Key words: plastic strain rate; scale effect; similarity coefficient; dynamic constitutive equation; strain-rate sensitivity

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摘要: 通过对相似模型中若干力学量和典型动力学实验中部分结果和结论的探讨分析, 研究了动态载荷下率相关材料相似模型的尺度效应, 阐述了塑性应变率具有的尺度效应对相似模型在动态载荷下的影响, 指出了塑性应变率尺度效应的一个重要特征是, 随着结构特征尺度的减小, 其动力行为的应变率敏感效应增强。

关键词: 塑性应变率; 尺度效应; 相似系数; 动态本构方程; 应变率敏感性

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Structures will always respond to the action of external loading to some extent. With the gradual increase of the external loads, the structural deformation will be in the elastic range at beginning, and then elastic-plastic deformation will occur if the external loading exceeds the elastic limit loading capacity of the structure^[1]. Practical engineering structures are usually made up of many basic structural components such as beams, plates, arches, shells and so on. When these basic structural elements are subjected to intensive dynamic loading or impulsive velocity, large plastic deformation can occur, as a result excessive permanent deformation is produced, and even local or global failure could be caused in them^[2].

The distinctive difference between dynamic plastic and quasi-static plastic analyses is that inertia and strain-rate effects are no longer neglected for dynamic loading. The strain-rate effect is extremely important for the dynamic plastic analysis, and especially for strain-rate sensitive materials such as mild

steel, titanium alloy, OFHC copper and so forth. When these materials are subjected to external intensive loadings (such as high-velocity impact on aircraft structures from flying birds^[3]), their dynamic yield stresses are much larger than their static ones^[2~5]. Symonds^[4] *et al.* carried out studies on dynamic plastic response of cantilever beams under impact loading in 1965, and presented a dynamic constitutive equation which took both strain-rate sensitivity and strain hardening into account. In the meantime, Symonds noticed the scale effect in the dynamic plastic response of strain-rate sensitive materials, and pointed out that the smaller the structure, the stronger the effect of strain-rate sensitivity will be. In 1967, Jones^[5] studied the influence of both strain hardening and strain-rate sensitivity on permanent deflections of rigid-plastic beams under impulsive loading. His analysis showed that physically smaller beams are more sensitive to strain-rate sensitivity than larger ones. Recently, Zhao *et al.*^[6] studied the influence of strain-rate effect on the material

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intrinsic length scale in strain gradient plasticity and the conclusion was that for most metals, the material intrinsic length scale decreases with increasing strain rate. Classical plasticity theory cannot predict scale effect simply because no length scale is included in its various constitutive equations. This is also true for classical dynamic plasticity theory.

By analyzing some mechanical quantities and experimental results of similar models, this paper studies the influence of scale effect pertaining to plastic strain rate on the material and structural dynamic response.

1 Relationship between Scale Effect and Some Mechanical Quantities of Similar Models

Since it is very difficult to make theoretical or numerical analyses for complicated structural systems, a smaller model test is often adopted. The relationship will be discussed in this section between some mechanical quantities of similar models and variation of the model size.

1.1 Strain and stress

Engineering strain ε can be defined as the ratio of the length variation Δl to the initial length l

$$\mathbf{e} = \Delta l / l \quad (1)$$

This definition shows that engineering strain \mathbf{e} is a dimensionless quantity, and is independent of the geometric size of practical structures that are studied, *i.e.*, \mathbf{e} scales with length scale as

$$[\mathbf{e}] \sim L^0 \quad (2)$$

Thus in the model experiments, \mathbf{e} is independent of the scale effect.

According to Hooke's law, the engineering stress \mathbf{S} is related to engineering strain \mathbf{e} through

$$\mathbf{S} = E\mathbf{e} \quad (3)$$

Obviously, for the similar models made of same material, the stress will be the same for same strain, which indicates that the stress, \mathbf{S} , does not vary with the geometric size of the practical structures.

1.2 Stress wave speed and loading rate

If the transversal effect is ignored, the propagation velocity of the tensile or compressive distur-

bance along the axial direction in a one-dimensional linear elastic bar is

$$c_0 = \sqrt{E / \rho} \quad (4)$$

where E and ρ are the Young's modulus and the material density, respectively. Eq.(4) shows that c_0 only depends upon the material itself rather than the geometric size of the structure, namely c_0 is independent of the scale effect. Elementary stress wave theory indicates that stress \mathbf{S} is linearly proportional to particle velocity \mathbf{u} , *viz*

$$\mathbf{S} = \rho c_0 \mathbf{u} \quad (5)$$

Substituting Eq. (3) into Eq. (5), one can obtain the expression as follows

$$\mathbf{e} = \mathbf{u} / c_0 \quad (6)$$

When the disturbing velocity reaches the yielding velocity $\mathbf{u}_y = \mathbf{S}_y / (\rho c_0)$, *i.e.* $\mathbf{u} = \mathbf{u}_y$, the stress reaches the yield limit \mathbf{S}_y , *i.e.*, $\mathbf{S} = \mathbf{S}_y$, simultaneously $\mathbf{e} = \mathbf{e}_y$. When $\mathbf{u} > \mathbf{u}_y$, plastic deformation will occur and begin to propagate in the material.

Because the yield limit \mathbf{S}_y of material depends only upon material itself, \mathbf{u}_y is also determined by the material property. Due to Eq. (6), if the same physical characteristics of response are required between similar models made of the same material, then the loading rate must be identical, *i.e.*

$$\mathbf{u} \sim \mathbf{b}^0 \quad (7)$$

where β is the similarity coefficient of the similar models.

1.3 Strain rate

Strain rate $\dot{\mathbf{e}}$ is defined as^[1]

$$\dot{\mathbf{e}} = \frac{d\mathbf{e}}{dt} \quad (8)$$

as well as

$$\frac{d\mathbf{e}}{dt} = \frac{1}{l_0} \frac{d\Delta l}{dt} = \frac{\mathbf{u}}{l_0} \quad (9)$$

where l_0 is the initial length. From the above section, \mathbf{u} must be kept the same in the experiments of the similar models, consequently, when $\mathbf{u} \sim \mathbf{b}^0$, one can have

$$[\dot{\mathbf{e}}] \sim L^{-1} \quad (10)$$

It is clear from (10) that the strain rate possesses scale effect.

1.4 Coefficient of strain-rate sensitivity and rate sensitive parameter for strain hardening

As a characterization of parameter for material strain-rate sensitivity in plastic dynamics, the coefficient of strain-rate sensitivity m is defined as^[8,9]

$$m = \frac{\partial \sigma}{\partial (\ln \dot{\epsilon}^p)} \quad (11)$$

Since the plastic strain rate $\dot{\epsilon}^p$ is associated with the geometric scale of structures in similar model experiments, the above m varies with the length scale.

Klepaczko proposed a parameter I_h to determine the rate sensitivity of strain hardening for materials^[8]

$$I_h = \frac{\Delta t_h}{\log(\dot{g}_2 / \dot{g}_1) \big|_{T, g_i}} \quad (12)$$

where g is strain, \dot{g} is strain rate ($\dot{g}_2 > \dot{g}_1$), T is temperature, and t is stress. According to Eq. (10), if b is the similarity coefficient between the prototype and the model, $\dot{g}_2 / \dot{g}_1 = 1/b$, then

$$I_h \sim [\log(\dot{g}_2 / \dot{g}_1)]^{-1} \sim (-\log b)^{-1} \quad (13)$$

The above equation shows that I_h , the parameter of the rate sensitivity of strain hardening also varies with the model scale.

2 Several Typical Examples Related to Strain Rate

2.1 Expanding-ring experiment

An expanding ring experimental method was introduced by Johnson *et al.*^[8] in 1963. The strain rate of the expanding ring can be expressed by

$$\dot{\epsilon} = \frac{u}{r} \quad (14)$$

where u is the initial expanding velocity, and r is the initial radius of the expanding ring.

Considering two expanding rings with similarity coefficient b , *i.e.* the radii of prototype r' and the model r'' satisfy the relationship $r'' = b r'$. If the loading speed u is kept unchanged, according to expression Eq. (14), one can obtain the following relationship between the strain rate of model $\dot{\epsilon}''$ and that of prototype $\dot{\epsilon}'$,

$$\dot{\epsilon}'' = \frac{r'}{r''} \dot{\epsilon}' = \frac{\dot{\epsilon}'}{b} \quad (15)$$

The average length of the fragments of the expanding ring under impulsive loading is of the order^[8,10]

$$d \sim \left(\frac{K_{cr}}{r C_0 \dot{\epsilon}_0} \right)^{2/3} \sim \dot{\epsilon}_0^{-2/3} \quad (16)$$

where K_{cr} is the fracture toughness of material, r is the material density, C_0 is the velocity of elastic wave, and $\dot{\epsilon}_0$ is the bulk strain rate. From Eq. (15), one can obtain the ratio between the average fragmental length of model d'' and the one of prototype d' as follows

$$\frac{d''}{d'} \sim \left(\frac{\dot{\epsilon}_0''}{\dot{\epsilon}_0'} \right)^{-2/3} \sim b^{2/3} \quad (17)$$

This shows that the average length of fragments in the expanding ring experiments depends upon the scale of test specimens, *i.e.*, there is a scale effect in the expanding ring experiments.

2.2 Adiabatic shear bands and recrystallization of metal

Adiabatic shear bands are regions where plastic shear deformation in a material is highly concentrated. The formation of these regions is extremely important in dynamic deformation of materials because they often are precursors of the failure of materials. Bai, *et al.*^[11] obtained the approximate equation for d , the half-width of the adiabatic shear band, as follows

$$d \cong \left(\frac{IT}{t\dot{g}} \right)^{1/2} \sim \dot{g}^{-1/2} \quad (18)$$

where I is the thermal conductivity, and T , t , and \dot{g} are the temperature, shear stress, and shear rate in the shear band, respectively. Considering the scale effect of the shear rate \dot{g} , one can have the relationship for the characteristic width of the adiabatic shear bands between model d'' and prototype d'

$$\frac{d''}{d'} \sim \left(\frac{\dot{g}''}{\dot{g}'} \right)^{-1/2} \sim b^{1/2} \quad (19)$$

where b is the similarity coefficient. The above equation shows that there is a scale effect in the adiabatic shear deformation.

The adiabatic shear bands have been observed in a variety of alloys. Most quenched-and-tempered steels (martensitic) and a number of alloys under high-strain-rate deformation also exhibit the adiabatic shear bands. Actually, any metal material can exhibit adiabatic shear band formation if work hardening is appropriately depressed. In the adiabatic shear region, the size of recrystallizing grains decreases with the increase of strain rate. Sandstrom, *et al.* [8] predicted recrystallized grain sizes d_{ss} as

$$d_{ss} \propto \dot{g}^{-1/2} \quad (20)$$

where \dot{g} is the shear strain rate. In the same way, one can obtain the following expression

$$\frac{d_{ss}''}{d_{ss}'} \sim \left(\frac{\dot{g}''}{\dot{g}'} \right)^{-1/2} \sim \mathbf{b}^{1/2} \quad (21)$$

where d_{ss}'' and d_{ss}' are the recrystallized grain sizes of model and prototype, respectively, and \mathbf{b} is the similarity coefficient. It is obvious that d_{ss} depends on the scale.

2.3 Evolution of voids

Usually, any material contains some micro flaws to a certain extent. When these materials are subjected to certain external loads, the micro flaws can be activated and evolve into micro cracks and micro-voids, and then likely propagate to cause macro breakage of materials. Under intensive dynamic loading, the material strain-rate sensitivity has a major influence on both the size and the statistical distribution of micro voids [12]. The smaller the average radius of micro void is, the stronger the strain-rate sensitivity will be.

2.4 Clamped beam under impulsive loading

The dimensionless permanent mid-point deflection of a fully clamped rigid, ideally plastic beam of unit width loaded impulsively can be expressed in the form of

$$\frac{W_f}{H} = \frac{1}{2} \left[\left(1 + \frac{3R_n}{n} \right)^{1/2} - 1 \right] \quad (22)$$

where $2L$ is the length of the clamped beam, and V_0 is the impulsive velocity. W_f is the permanent deflection of the mid-point, and R_n is the dimensionless number (response number) suggested by

Zhao [13], which can be expressed as

$$R_n = \frac{\mathbf{r} V_0^2 L^2}{\mathbf{s}_0 H^2} \quad (23)$$

where \mathbf{r} is the material density, \mathbf{s}_0 is uniaxial yield stress, H is the height of cross section for beam, and n can be determined by the following expression [14]

$$n = \frac{\mathbf{s}_0'}{\mathbf{s}_0} = 1 + \left(\frac{V_0 W_f}{3\sqrt{2}DL^2} \right)^{1/q} \quad (24)$$

where D and q are material constants.

Considering the similar beam models made of identical material with the similarity coefficient \mathbf{b} ($\mathbf{b} = L''/L' = H''/H'$), if the response number R_n is kept unchanged, the impulsive velocity V_0 will be identical between beam models. If the parameter \mathbf{a} is assumed to be

$$\mathbf{a} = \left(\frac{2V_0 H}{DL^2} \right)^{1/q} \quad (25)$$

then there will be a relationship between the model and the prototype as

$$\left(\frac{\mathbf{a}''}{\mathbf{a}'} \right)^q = \frac{L'}{L''} = \frac{H'}{H''} = \frac{1}{\mathbf{b}} \quad (26)$$

Thus it can be seen that with the decrease of beam dimension (*i.e.*, $\mathbf{b} < 1$), $\mathbf{a}'' > \mathbf{a}'$ (*i.e.*, $n'' > n'$). Combining Eq. (22) yields

$$\left(\frac{W_f}{H} \right)'' < \left(\frac{W_f}{H} \right)' \quad (27)$$

The above inequality shows that W_f/H varies with the similarity coefficient, *i.e.*, W_f/H depends on the dimension of the beam.

From aforementioned several typical instances, one can find that whether the average length of expanding-ring fragment d , the characteristic width of adiabatic shear band \mathbf{d} , the recrystallized grain size d_{ss} , or the maximum dimensionless permanent lateral deflection of a clamped beam under the impulsive loading, are all related to the plastic strain rate of the material. Therefore, with the decrease of the dimension of similar models, there occurs correspondingly the scale effect due to the size effect of the plastic strain rate.

3 Discussions on Two Constitutive Equations Considering Strain Rate

3.1 Cowper-Symonds dynamic constitutive equation

The dynamic behavior of most metallic materials can be modeled by the Cowper-Symonds constitutive equation as follows

$$\mathbf{s}'_0 / \mathbf{s}_0 = 1 + (\dot{\mathbf{e}} / D)^{1/q} \quad (28)$$

where \mathbf{s}'_0 and \mathbf{s}_0 denote, respectively, the dynamic yield stress and static yield stress at a uniaxial plastic strain rate $\dot{\mathbf{e}}$; D and q are constants for a particular material. Some constants for several typical materials^[6] are listed in Table 1.

Table 1 Coefficients in Cowper-Symonds constitutive equation for some typical materials

Material	D/s^{-1}	q
Mild steel	40.4	5
Aluminum alloy	6500	4
a-Titanium(Ti 50A)	120	9
Stainless steel	100	10

With the decrease of the characteristic dimension of the similar model, its similarity coefficient \mathbf{b} ($\mathbf{b} < 1$) decreases proportionally, and then the strain rate in the smaller dimension model will become $\dot{\mathbf{e}}/\mathbf{b}$, its yield stress \mathbf{s}''_0 will be

$$\mathbf{s}''_0 / \mathbf{s}_0 = 1 + [\dot{\mathbf{e}} / (\mathbf{b}D)]^{1/q} \quad (29)$$

Comparing Eq. (28) to Eq. (29), leads to

$$\frac{\mathbf{s}'_0}{\mathbf{s}_0} = \frac{1 + [\dot{\mathbf{e}} / (\mathbf{b}D)]^{1/q}}{1 + (\dot{\mathbf{e}} / D)^{1/q}} \quad (30)$$

If $\dot{\mathbf{e}} = D$, Eq. (30) will be written as

$$\mathbf{s}''_0 / \mathbf{s}'_0 = [1 + (1/\mathbf{b})^{1/q}] / 2 \quad (31)$$

Taking the mild steel in Table 1 as an example, one can see from the expression (31) that

when $\mathbf{b} = 0.100$, $\mathbf{s}''_0 = 1.292\mathbf{s}'_0$;

when $\mathbf{b} = 0.010$, $\mathbf{s}''_0 = 1.756\mathbf{s}'_0$;

when $\mathbf{b} = 0.001$, $\mathbf{s}''_0 = 2.491\mathbf{s}'_0$.

It can be seen from the above example that with the decrease of the scale of similar model, the yield stress will increase apparently for the material satisfy-

ing the Cowper-Symonds dynamic constitutive equation, which obviously shows the scale effect of the plastic strain rate. Even for rate-insensitive materials such as aluminum, the dynamic yield stress will also increase with the decrease of the dimension of the similar model due to the scale effect of the plastic strain rate.

3.2 Johnson-Cook dynamic constitutive equation

Johnson and Cook presented a dynamic constitutive equation in 1983 as follows^[15],

$$\mathbf{s} = (A + B\mathbf{e}^n)(1 + C \ln \dot{\mathbf{e}}^*)(1 - T^{*m}) \quad (32)$$

where A, B, C, n , and m are yield strength, work hardening coefficient, work hardening exponent, strain rate sensitivity and thermal coefficient, \mathbf{e} is the equivalent plastic strain, $\dot{\mathbf{e}}^* = \dot{\mathbf{e}} / \dot{\mathbf{e}}_0$ is the dimensionless plastic strain rate taking $\dot{\mathbf{e}}_0 = 1s^{-1}$, and $T^* = (T - T_r) / (T_m - T_r)$ is the homologous temperature, with T being the immediate absolute temperature of the deformed specimen, T_r and T_m being the room temperature and the melting point, respectively.

Comparing with many other dynamic constitutive equations, Johnson-Cook equation considers not only the strain hardening and strain-rate effect of material, but also thermal softening effect of material.

If the dimensionless temperature T^* and the stress \mathbf{e} are kept invariable, Eq. (32) can be written as

$$\mathbf{s} \sim (1 + C \ln \dot{\mathbf{e}}^*) \quad (33)$$

When $\dot{\mathbf{e}}^* = 1$, and the similarity coefficient is \mathbf{b} ($\mathbf{b} < 1$), the relationship of the yield stresses between the larger model and the smaller model will be as follows (\mathbf{s}'' for larger model and \mathbf{s}' for smaller model)

$$\begin{aligned} \frac{\mathbf{s}''}{\mathbf{s}'} &= \frac{(1 + C \ln \dot{\mathbf{e}}^* - C \ln \mathbf{b})}{(1 + C \ln \dot{\mathbf{e}}^*)} \\ &= 1 - C \ln \mathbf{b}. \end{aligned} \quad (34)$$

Obviously, the yield stress for material satisfying the Johnson-Cook equation exhibits a size effect due to the scale effect of the plastic strain rate.

4 Example of Numerical Analysis

Consider a cylindrical mild steel bar, the prototype model, with initial length $L_0 = 32.4b$ mm and initial diameter $D_0 = 6.4b$ mm (here β is the similarity coefficient, β is taken as 1 for prototype model, and material parameters of mild steel refer to the Reference [16]) impacting a rigid target at a velocity V_0 of 227 m/s (illustrated in Fig. 1).

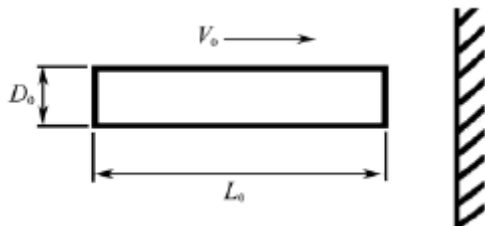


Fig. 1 A mild-steel bar impacting rigid target

The following Cowper-Symonds equation can be adopted

$$\mathbf{s} = (1 + \dot{\mathbf{e}} / D)^{1/q} \mathbf{s}_0(\mathbf{e}) \quad (35)$$

with $\mathbf{s}_0(\mathbf{e})$ being

$$\mathbf{s}_0(\mathbf{e}) = \mathbf{s}_0 + B E_p \mathbf{e}_{\text{eff}}^p \quad (36)$$

where \mathbf{s}_0 is the static yield stress, B is the hardening parameter, $\mathbf{e}_{\text{eff}}^p$ is the efficient plastic strain, and E_p is the plastic hardening modulus.

The finite element method is utilized, and models with β being 1, 0.1, 0.01, and 0.001 are calculated. The mesh of the bar has been shown in Fig. 2, and all cases have the same mesh configuration. Frictionless contact condition is assumed between the bar and the rigid target.

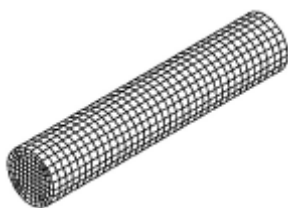


Fig. 2 Sketch of element mesh for numerical analysis

Fig. 3 shows the numerical results of the above similar bars with different similarity coefficient b . In this figure, the vertical coordinate denotes the ratio of instant length L to the initial length L_0 for the models under impact loading, *i.e.*, the dimensionless instant length of models L/L_0 , and the horizontal coordinate denotes the dimensionless time $V_0 t / L_0$.

According to the results of the numerical analysis, one can see that the relative deformation decreases steadily with the model dimension. It is the size effect of the plastic strain rate that results in the scale effect of similar models made of the same kind of rate-dependent material.

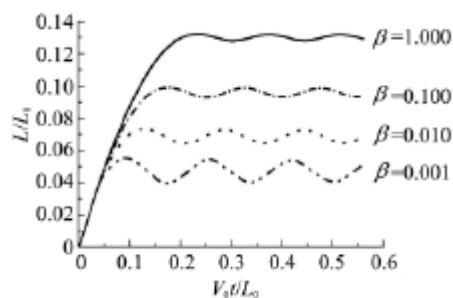


Fig. 3 Result curves of numerical analysis

5 Summary

In the first part of the present paper, an analysis is carried out on some mechanical quantities and some typical dynamic experiments results in similar models, then further study is made on the scale effect of the plastic strain rate for materials satisfying two well-known rate-dependent dynamic constitutive equations (*i.e.*, Cowper-Symonds and Johnson-Cook). It is demonstrated by numerical simulation that the size effect caused by the decrease of the dimension of rate-dependent material is due to the scale effect of the plastic strain rate. It should be emphasized that the scale effect cannot be well explained by constitutive equations of classical dynamic plastic theory, because such equations contain no length scales. With the decrease of length scale, the structures are more sensitive to the strain rate, thus there occurs a major difficulty: it is necessary to develop a dynamic constitutive equation containing internal material length scale in the micron or submicron domains (mainly in the field of micro-electronics and MEMS at present). Only by this way, the physical origin and mechanism of the scaling law and size effect can be clearly understood.

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