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Harmonic growth of spherical Rayleigh-Taylor instability in weakly nonlinear regime

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Harmonic growth in classical Rayleigh-Taylor instability (RTI) on a spherical interface is analytically investigated using the method of the parameter expansion up to the third order. Our results show that the amplitudes of the first four harmonics will recover those in planar RTI as the interface radius tends to infinity compared against the initial perturbation wavelength. The initial radius dramatically influences the harmonic development. The appearance of the second-order feedback to the initial unperturbed interface (i.e., the zeroth harmonic) makes the interface move towards the spherical center. For these four harmonics, the smaller the initial radius is, the faster they grow.
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I. INTRODUCTION

Rayleigh-Taylor instability (RTI)[1,2] occurs at the interface between horizontal layers of fluid, and also in cylindrical or spherical geometry. Relevant investigations have been undertaken by several authors in many applications;[3–8] specifically, an extra instability due to the curvature of the interface (i.e., the Bell-Plesset (BP) effects), the nonlinear evolution of the interface, and numerical solutions including magnetic effects have been widely discussed. RTI also plays a significant role in astrophysics[9,10] and inertial confinement fusion (ICF).[11–23]

RTI happens on an interface separating two different fluids when a light fluid supports a heavy fluid in a gravity field or accelerates it.[1,2] Assuming that the heavy fluid is superposed over the light one in a gravitational field $-g\boldsymbol{e_y}$, where g is gravitational acceleration, an initial interface perturbation between the two fluids of densities ρ_h and ρ_l is in the form of $\eta(x, t=0) = \varepsilon\cos(kx)$ with perturbation wave number $k = 2\pi/\lambda$ and perturbation amplitude $\varepsilon \ll \lambda$, where λ is perturbation wavelength. According to the classical linear theory,[1,2] the initial cosine modulation grows exponentially in time t, $\eta_L = \varepsilon e^{\gamma t}$, where

$$\gamma = \sqrt{Akg} \tag{1}$$

is the linear growth rate with $A = (\rho_h - \rho_l)/(\rho_h + \rho_l)$ being the Atwood number. When the typical perturbation amplitude is close to its wavelength, the second and third harmonics are generated successively, and then the perturbation enters the nonlinear regime. In the weakly nonlinear growth regime,[24–39] within the framework of the third-order weakly nonlinear theory,[24–28] the interface function is $\eta(x,t) = \eta_1\cos(kx) + \eta_2\cos(2kx) + \eta_3\cos(3kx)$, where η_n ($n = 1, 2, 3$) is the amplitude of the nth harmonic

$$\eta_1 = \eta_L - \frac{1}{16}(3A^2+1)k^2\eta_L^3, \tag{2a}$$

$$\eta_2 = -\frac{1}{2}Ak\eta_L^2, \tag{2b}$$

$$\eta_3 = \frac{1}{2}\left(A^2 - \frac{1}{4}\right)k^2\eta_L^3. \tag{2c}$$

It should be noted that the amplitude of the second harmonic is negative for arbitrary A, and that of the third is negative for $A < 1/2$, and positive for $A > 1/2$. For $A = 1/2$, the third harmonic vanishes. Here, the negative amplitude denotes the corresponding phase being opposite to the initial cosine modulation's (anti-phase). For problems with large Atwood number, $A \to 1$, Equations (2a)–(2c) are reduced to

$$\eta_1 = \eta_L - \frac{1}{4}k^2\eta_L^3, \tag{3a}$$

$$\eta_2 = -\frac{1}{2}k\eta_L^2, \tag{3b}$$

$$\eta_3 = \frac{3}{8}k^2\eta_L^3. \tag{3c}$$

As can be seen in Equation (2a) or (3a), at the third-order, the linear growth of the fundamental mode is reduced by the nonlinear mode-coupling effect, i.e., third-order negative feedback to the fundamental mode. Weakly nonlinear behaviors in planar RTI have become a field of theoretical,[24–34] experimental[35–37] and numerical[38–41] interest.

Strictly speaking, perturbation growth driven only by buoyant force is named as RTI, while the modifications of perturbation behavior by compression and geometrical

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convergence are usually referred to as the BP effects.[42,43] Both the RTI and the BP effects are important to the outcome of implosion experiments in the ICF.[44,45] Epstein[46] discussed that the BP effects based on a simple model are formulated in terms of the mass amplitude of interface perturbations, and work by Mikaelian[47] gives the differential equations in cylindrical and spherical coordinates for the stability of the cylindrical and spherical shells, presents a linearized analysis for different geometries, and the case of concentric shells of fluid undergoing implosion or explosion is considered. This linearized analysis is extended to the case of compressible fluids by Yu and Livescu.[48] Since then, a number of papers by Forbes[49–51] have examined the effects of cylindrical and spherical geometry on an axi-symmetric Rayleigh-Taylor growth.

The term "accelerationless growth" is also used for the BP effects appearing without the effects of acceleration.[52] To be more precise, "undriven growth" will denote compression and convergence effects in the absence of the buoyant force driving the RTI. The term "accelerationless" suggests the more restricted case of zero acceleration.[53] Generally, the net perturbation growth does not separate naturally into an acceleration-driven RTI contribution and an undriven (i.e., BP effects) contribution. Nevertheless, the chosen formulation clarifies the physical distinction between the RTI and BP effects. As Plesset noted for the spherical case, a source or a sink should exist at the spherical center to allow that region surrounded by the interface to expand or contract while maintaining a constant fluid density.

Because of the complexity of the spherical RTI in analytic calculations, we just consider the case $A=1$. In this paper, development of the first four harmonics including the zeroth, first, second, and third harmonics in the spherical RTI for irrotational, incompressible, and inviscid fluid is investigated analytically. It should be pointed out that we do not assume a source or a sink to exist at the spherical center, while the region surrounded by the spherical interface is still in constant pressure. In the following expression, we use the phrase "spherical RTI" to express the compositive effects containing the net RTI and the BP effects.

II. THEORETICAL FRAMEWORK AND EXPLICIT RESULTS

This section is devoted to the detailed description of the theoretical framework in the present work, and analytic expressions of amplitudes of the first four harmonics are demonstrated.

In spherical polar coordinates (r, φ, θ), in which r is the radial coordinate, φ is the angle measured down from the z-axis, and θ is the azimuthal angle in the $x-y$ plane, there is a gas with constant pressure and a fluid, respectively, occupying both sides of the spherical interface $r=r_0$. For some reason, there always exist perturbations on the material interface. According to the relations of the acceleration direction and fluid distribution, two cases can motivate the spherical RTI. The first means the acceleration pointing to the center of the spherical system and the fluid occupying the outer space of the interface, the second is in complete antithesis to the first. Here, we focus mainly on the first case, and assume that interface perturbations are only related with the azimuthal angle θ. In other words, there are perturbations with the same mode number at arbitrary declination angle φ. This is similar to the configuration of the one-dimensional planar RTI. In order to contrast conveniently with the planar results, we consider perturbations in the equatorial $x-y$ plane, and this is achieved by setting the angle $\varphi = \pi/2$. The detailed reason that we select $\varphi = \pi/2$ will be discussed in the section of the initial perturbation as the following. Assuming the fluid in a gravitational field $-g\boldsymbol{e}_r$ to be irrotational, incompressible, and inviscid, the governing equations for this system are

$$r^2\frac{\partial^2\phi}{\partial r^2}+2r\frac{\partial\phi}{\partial r}+\frac{1}{\sin^2\varphi}\frac{\partial^2\phi}{\partial\theta^2}=0 \quad \text{in the fluid}, \tag{4a}$$

$$\frac{\partial s}{\partial t}+\frac{1}{r^2\sin^2\varphi}\frac{\partial s}{\partial\theta}\frac{\partial\phi}{\partial\theta}-\frac{\partial\phi}{\partial r}=0 \quad \text{on} \quad r=s(\theta,t), \tag{4b}$$

$$\frac{\partial\phi}{\partial t}+\frac{1}{2}\left(\frac{\partial\phi}{\partial r}\right)^2+\frac{1}{2r^2\sin^2\varphi}\left(\frac{\partial\phi}{\partial\theta}\right)^2+gr+f(t)=0 \quad \text{on} \quad r=s(\theta,t), \tag{4c}$$

where the $f(t)$ is an arbitrary function of time, $\phi(r,\theta,t)$ is the velocity potential of the fluid, and the perturbation interface $r=s(\theta, t)$ corresponds to $y=\eta(x, t)$ in Cartesian geometry.

Laplace equation (4a) comes from the incompressibility condition of the fluid. Equation (4b) represents the kinematic boundary condition that a fluid particle initially situated on the material interface remains on the interface afterwards. Bernoulli equation (4c) represents the dynamic boundary condition in which the pressure is continuous across the material interface.

We consider an initial perturbation in the form

$$r=s(\theta,t=0)=r_0+\varepsilon\cos(\kappa\theta), \tag{5}$$

where r_0 is a positive constant, mode number $\kappa=2\pi r_0/\lambda$ which must be an integer and the amplitude of the perturbation $\varepsilon\ll\lambda$. In fact, for arbitrary declination angle φ, the mode number should be $\kappa=2\pi r_0\sin\varphi/\lambda$. Under the condition of the same perturbation wavelength for the planar and the spherical RTI, there is a relational expression $\kappa=kr_0\sin\varphi$ with the wave number being $k=2\pi/\lambda$. Take $\tilde{r}_0=r_0\sin\varphi$, then $\tilde{r}_0\le r_0$. From this point, the spherical effect is the weakest when the angle φ is set as $\pi/2$. In addition, at the same wavelength, whether the spherical result in the limit of the large r_0 (i.e., $r_0\to\infty$) recovers the corresponding planar result, depends on the angle φ. When $\varphi=\pi/2$, the former will tend to the latter. We also know that for the spherical perturbations with the same mode number, the condition of $\varepsilon\ll\lambda$ can be satisfied when $\varphi=\pi/2$, while it cannot when $\varphi\neq\pi/2$, especially $\varphi\to0$ or $\varphi\to\pi$. This will result in the failure of the expansion method with a small parameter, particularly for $\varphi\to0$ or $\varphi\to\pi$. Accordingly, we just take the case of the perturbations in the equatorial plane, namely $\varphi=\pi/2$, into consideration in the present paper. As regards to the φ effect related to axi-symmetric solutions, we plan to investigate it in the next work.

This perturbed interface is prone to RTI, and the higher harmonics (i.e., the second harmonic, the third harmonic, and so on) are subsequently generated by the nonlinear mode-coupling process. Hence, the evolution interface $s(\theta, t)$ and velocity potential $\phi(r, \theta, t)$ can be expanded into a power series in $\hat{\varepsilon}$ as

$$s(\theta, t) = r_0\zeta(t) + \sum_{n=1}^{N} s^{(n)}(\theta, t) + O(\hat{\varepsilon}^{N+1}), \tag{6a}$$

$$\phi(r, \theta, t) = \sum_{n=1}^{N} \phi^{(n)}(r, \theta, t) + O(\hat{\varepsilon}^{N+1}), \tag{6b}$$

with

$$\zeta(t) = 1 + \sum_{n=1}^{[\frac{N}{2}]} \hat{\varepsilon}^{2n} e^{2n\beta t} \alpha_{2n,0}, \tag{7a}$$

$$s^{(n)}(\theta, t) = \hat{\varepsilon}^n e^{n\beta t} \sum_{m=0}^{[\frac{n}{2}]-1} \alpha_{n,n-2m} \cos(n-2m)\kappa\theta, \tag{7b}$$

$$\phi^{(n)}(r, \theta, t) = \hat{\varepsilon}^n e^{n\beta t} \sum_{m=0}^{[\frac{n}{2}]} \phi_{n,n-2m} r^{-\frac{1}{2}\left(\sqrt{4\kappa^2(n-2m)^2+1}+1\right)} \times \cos(n-2m)\kappa\theta. \tag{7c}$$

Here, the small parameter $\hat{\varepsilon} = \varepsilon/\lambda \ll 1$, N is set as 3, Gauss's symbol $[n/2]$ denotes a maximum integer that is less than or equal to $n/2$, β is the linear growth rate, and the functions $s^{(n)}(\theta, t)$ and $\phi^{(n)}(r, \theta, t)$ are, respectively, nth-order perturbed interface and nth-order perturbed velocity potential of the fluid. Regarding the $(n-2m)$th Fourier harmonic at the nth-order, when $m=0$, $s^{(n)}_{n-2m} = \hat{\varepsilon}^n e^{n\beta t}\alpha_{n,n-2m}$ $[\phi^{(n)}_{n-2m} = \hat{\varepsilon}^n e^{n\beta t}\phi_{n,n-2m} r^{-(\sqrt{4\kappa^2(n-2m)^2+1}+1)/2}]$ is a generation coefficient of the perturbation interface [the velocity potential]; while $m>0$, it is a feedback coefficient of the nth-order for the perturbation interface [the velocity potential]. Note that the perturbation velocity potential $\phi(r, \theta, t)$ has satisfied Laplace equation (4a) and boundary condition $\nabla\phi|_{r\to+\infty} = 0$, $\alpha_{1,1} = \lambda$ according to the initial condition, and $\zeta(t)$ is a function of time. Equation (6a) shows that the whole evolution interface at time t consists of two sections. One is the net perturbed interface $\sum_{n=1}^{N} s^{(n)}(\theta, t)$ related to θ, the other is the unperturbed interface $r_0\zeta(t)$ which is independent of θ. That is, the perturbations grow on the profile of the radius $r_0\zeta(t)$, while the radius $r_0\zeta(t)$ changes with time. The function $\zeta(t)$ determines whether the unperturbed interface moves with time: the relation $\zeta(t) \equiv 1$ means that the interface keeps resting; otherwise, it moves from the initial position $r(t=0) = r_0$. As a result, the function $\zeta(t)$ represents the radius of the unperturbed interface, rather than the spatial average of the radius of the whole bubble, which can be explained as below. To maintain the constant pressure without a source or sink inside of the interface, the capacity there should remain unchanged, and then the spatial average of the radius of the whole bubble should be invariable (i.e., the initial radius r_0). The coupling factors in the amplitudes of the Fourier harmonic, $\alpha_{n,n-2m}$ ($n=2,\ldots, N$, and $m=0, 1,\ldots, [n/2]$), and β are undetermined quantities.

On the basis of the method employed in the work of Ref. 8, this system can be solved successively from the first order to the third one. Note that the zeroth-order equations are satisfied due to the arbitrary function $f(t)$. The linear growth rate and coupling factors of the first four harmonics with corrections up to the third order can be expressed as

$$\beta = \sqrt{\frac{g\left(\sqrt{4\kappa^2+1}+1\right)}{2r_0}}, \tag{8a}$$

$$\alpha_{2,0} = -\frac{1}{2r_0^2}, \tag{8b}$$

$$\alpha_{2,2} = \frac{2\kappa^2(K_1 - K_2 + 3) - (K_1+1)(K_2-1)}{2r_0[(K_1+1)(K_2-1) - 4\kappa^2]}, \tag{8c}$$

$$\alpha_{3,1} = \frac{16\kappa^4(-4K_1 + K_2 + 5) + \kappa^2[-83K_2 + 2K_1(K_2+131) + 347] - 33(K_1+1)(K_2-1)}{16r_0^2(K_2-1)[(K_1+1)(K_2-1) - 4\kappa^2]}, \tag{8d}$$

$$\alpha_{3,3} = \frac{K_4}{6r_0^2(K_2-1)[(K_1+1)(K_2-1) - 4\kappa^2]\left[4\kappa^2 - (K_1+1)(K_3-1)\right]} \tag{8e}$$

with $K_1 = \sqrt{4\kappa^2+1}$, $K_2 = \sqrt{16\kappa^2+1}$, $K_3 = \sqrt{36\kappa^2+1}$, and $K_4 = 12\kappa^6(12K_1 - 3K_2 - 12K_3 + 83) + \kappa^4(-361K_3 + 3K_2(19K_3 - 87) + 3K_1(-35K_2 + 3(K_2-17)K_3 + 211) + 949) + 2\kappa^2(-67K_3 + 3K_2(9K_3 - 17) + K_1(-57K_3 + K_2(17K_3 - 41) + 81) + 91) + 10(K_1+1)(K_2-1)(K_3-1)$.

The linear growth rate on the spherical interface [see Eq. (8a)] is different from that on the planar one [see Eq. (1) where A should be selected as 1]. Keeping acceleration g and mode number κ invariable, the smaller the initial radius of the interface r_0 is, the larger the linear growth rate in the spherical geometry is. In addition, expressions (8c)–(8e) demonstrate that coupling factors are influenced by not only κ but also r_0. If the constant λ is considered in both the spherical and Cartesian geometries [i.e., $\kappa/r_0 = k$], and r_0 is large [i.e., $r_0 \to +\infty$], the interface constructed by the above results will be reduced to the planar one. That is, the first three harmonics in spherical RTI will be simplified as those [see Eqs. (3a)–(3c)] in planar RTI. In this configuration, it should be noted that the second-order feedback to the zeroth harmonic will vanish away, which can be easily confirmed in Eq. (8b). The generation of $\alpha_{2,0}$ is an essential character completely different from the result in Cartesian geometry where $\eta_{2,0} = 0$.

Accordingly, the interface function at the framework of the third-order theory takes the form $r \doteq \zeta r_0 + \sum_{n=1}^{3} s_n \cos(n\kappa\theta)$, and the function ζ and the amplitude of the nth harmonic, s_n, are

$$\zeta = 1 + \alpha_{2,0}\eta_{Ls}^2, \tag{9a}$$

$$s_1 = \eta_{Ls}(1 + \alpha_{3,1}\eta_{Ls}^2), \tag{9b}$$

$$s_2 = \alpha_{2,2}\eta_{Ls}^2, \tag{9c}$$

$$s_3 = \alpha_{3,3}\eta_{Ls}^3, \tag{9d}$$

where $\eta_{Ls} = \varepsilon e^{\beta t}$ is the linear growth amplitude of the fundamental mode. It should be pointed out that just the amplitude of the fundamental mode is corrected by the third order, while the second and the third harmonics are not. As discussed above, an essential character different from the Cartesian case is that the zeroth harmonic appears [see Equation (9a)]. This means that the position of the initial unperturbed interface $r = r_0$ changes into $r = \zeta(t) r_0$ with the development of the perturbation, differing completely from that in Cartesian space where the initial unperturbed interface keeps resting all the time. Additionally, it is found that both the unperturbed interface $\bar{r} = \zeta r_0$ and the net perturbed interface $\tilde{r} = \sum_{n=1}^{3} s_n \cos(n\kappa\theta)$ are closely related with g (in linear growth rate β) and r_0. This means that the net perturbation growth and the BP effects affect the spherical RTI collectively.

III. HARMONIC DEVELOPMENT

Because of the nonlinear mode-coupling, high harmonics will be generated quickly and the initial interface developing in linear growth will be reduced. The evolutional interface includes two sections: the initial unperturbed interface known as the zeroth harmonic and the perturbed interface. Within the third-order theory, the zeroth harmonic has a second-order correction, the fundamental mode (the first harmonic) is just corrected by the third one, and the second and third harmonics have no higher-order feedback. For unity, we use the characteristic quantities λ and g to normalize the initial radius and the time. The development of the amplitude of these four harmonics is investigated in this paper. Figures 1–4 show the evolution of the zeroth, first, second, and third harmonics. To better understand the spherical effect, we take the initial radius as $r_0/\lambda = 10/2\pi$, $20/2\pi$, $100/2\pi$, and $1000/2\pi$, as well as the planar results (i.e., the amplitudes of the first, second, and third harmonics expressed by Eqs. (3a), (3b), and (3c), and the amplitude of the zeroth harmonic which is zero in the planar case), respectively. The initial amplitude of the perturbed interface is set as $\varepsilon/\lambda = 1/1000$.

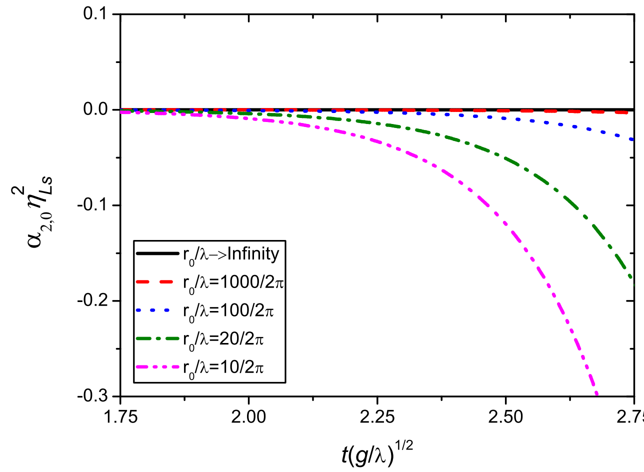

FIG. 1. The amplitude development of the second-order feedback to the zeroth harmonic for different initial radii versus time. The initial amplitude is $\varepsilon/\lambda = 1/1000$.

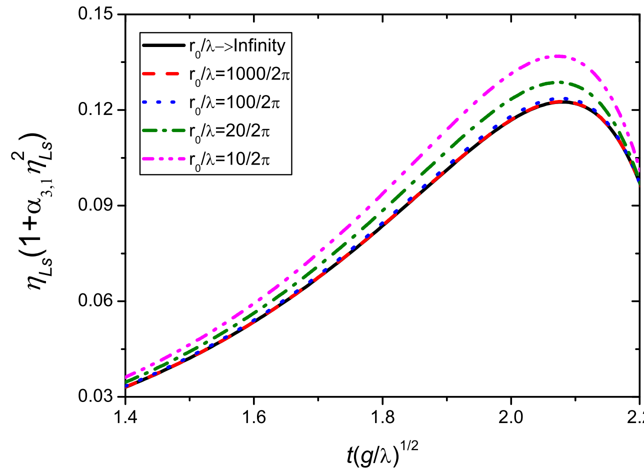

FIG. 2. The amplitude development of the fundamental mode versus time. The initial amplitude is $\varepsilon/\lambda = 1/1000$.

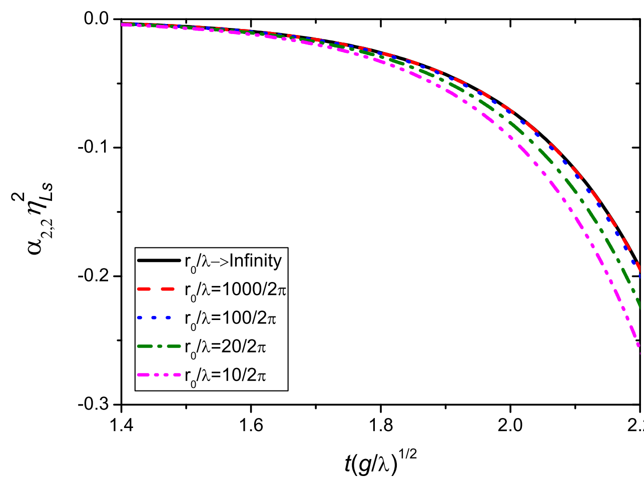

FIG. 3. The amplitude development of the second harmonic versus time. The initial amplitude is $\varepsilon/\lambda = 1/1000$.

Figure 1 shows that for the case of large r_0, the amplitude of the zeroth harmonic in spherical RTI tends to be zero (i.e., the planar result). With r_0 decreasing, the amplitude of

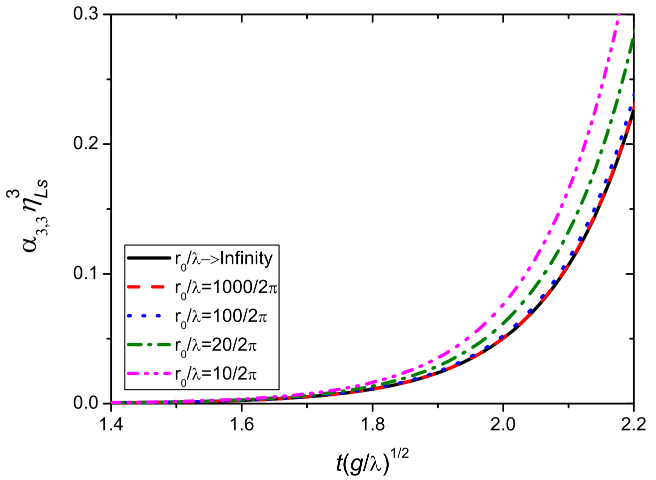

FIG. 4. The amplitude development of the third harmonic versus time. The initial amplitude is $\varepsilon/\lambda = 1/1000$.

the zeroth harmonic quickly increases. That is to say, the spherical effect inspires the appearance of the zeroth harmonic which vanishes in the planar RTI. The negative amplitude of the zeroth harmonic indicates that the unperturbed interface starts moving towards the spherical center. The unperturbed interface in planar RTI, nevertheless, keeps rest. Accordingly, the phenomenon that the unperturbed interface evolves to the spherical center is an innate character in the spherical RTI.

Figure 2 denotes that the fundamental mode has the same trend in spherical RTI as the planar one. With time, the amplitude of the fundamental mode mildly increases first, and then rapidly decreases. It should be noted that the smaller the radius is, the larger the amplitude is. That is, the spherical effect has a great influence on the fundamental mode.

The amplitude of the second harmonic in Fig. 3 is found to grow negatively for the different initial radii. When the normalized radius tends to be infinity, the amplitude goes to the result of the planar RTI. With decreasing radius, the amplitude grows fast.

In Fig. 4, one finds that the spherical effect strengthens the positive growth of the third harmonic: the smaller the radius is, the faster the amplitude grows.

From the above figures, one also finds that for the first, second, and third harmonics, their amplitude growth is dramatically strengthened when the initial interface radius is less than $100\lambda/2\pi$. That is, when the initial interface radius is more than $100\lambda/2\pi$, the spherical effect on their amplitude growth vanishes. However, for the zeroth harmonic, when the initial radius is between $100\lambda/2\pi$ and $1000\lambda/2\pi$, the spherical effect still plays a role.

IV. CONCLUSION

In this paper, we use the method of the small parameter expansion with nonlinear corrections up to the third order to analytically explore the amplitude evolution of the first four harmonics in the classical RTI (irrotational, incompressible, and inviscid fluid) for spherical geometry. Take the same initial wavelength and large initial radius, then our spherical results will tend to those in the planar RTI.

Unlike the planar RTI, the second-order feedback to the zeroth harmonic is always negative for the varying initial radius, leading the initial unperturbed interface to move towards the center of the spherical system. The smaller the initial radius is, the faster the initial unperturbed interface moves. In other words, the spherical effect has a great influence on the initial interface. In the spherical RTI, the first, second, and third harmonics have the same development trend as the planar RTI. However, the smaller the radius is, the faster they grow. Hence, the spherical effect plays a key role in the amplitude development of the harmonics.

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[1] L. Rayleigh, Proc. London Math. Soc. **14**, 170 (1883).
[2] G. Taylor, Proc. R. Soc. London, Ser. A **201**, 192 (1950).
[3] Q. Zhang and M. J. Graham, Phys. Fluids **10**, 974 (1998).
[4] M. A. Sweeney and F. C. Perry, J. Appl. Phys. **52**, 4487 (1981).
[5] S. T. Weir, E. A. Chandler, and B. T. Goodwin, Phys. Rev. Lett. **80**, 3763 (1998).
[6] P. Amendt, Phys. Plasmas **13**, 042702 (2006).
[7] C. Matsuoka and K. Nishihara, Phys. Rev. E **73**, 026304 (2006).
[8] W. H. Liu, C. P. Yu, W. H. Ye, L. F. Wang, and X. T. He, Phys. Plasmas **21**, 062119 (2014).
[9] B. A. Remington, R. P. Drake, and D. D. Ryutov, Rev. Mod. Phys. **78**, 755 (2006).
[10] I. Hachisu, T. Matsuda, K. Nomoto, and T. Shigeyama, Astrophys. J. **358**, L57 (1990).
[11] J. Nuckolls, L. Wood, A. Thiessen, and G. Zimmerman, Nature **239**, 139 (1972).
[12] S. Bodner, Phys. Rev. Lett. **33**, 761 (1974).
[13] H. Takabe, K. Mima, L. Montierth, and R. L. Morse, Phys. Fluids **28**, 3676 (1985).
[14] M. Tabak, D. H. Munro, and J. D. Lindl, Phys. Fluids B **2**, 1007 (1990).
[15] V. N. Goncharov, R. Betti, R. L. McCrory, P. Sorotokin, and C. P. Verdon, Phys. Plasmas **3**(4), 1402 (1996).
[16] S. G. Glendinning, S. N. Dixit, B. A. Hammel, D. H. Kalantat, M. H. Key, J. D. Kilkenny, J. P. Knauer, D. M. Pennington, B. A. Remington, R. J. Wallace, and S. V. Weber, Phys. Rev. Lett. **78**, 3318 (1997).
[17] K. Shigemori, H. Azechi, M. Nakai, M. Honda, K. Meguro, N. Miyanaga, H. Takabe, and K. Mima, Phys. Rev. Lett. **78**, 250 (1997).
[18] A. R. Piriz, Phys. Plasmas **8**(3), 997 (2001).
[19] W. H. Ye, W. Y. Zhang, and X. T. He, Phys. Rev. E **65**, 057401 (2002).
[20] S. Atzeni and J. Meyer-ter-Vehn, *The Physics of Inertial Fusion: Beam Plasma Interaction Hydrodynamics, Hot Dense Mater* (Oxford University, Oxford, 2004).
[21] J. D. Lindl, P. Amendt, R. L. Berger, S. G. Glendinning, S. H. Glenzer, S. W. Haan, R. L. Kauffman, O. L. Landen, and L. J. Suter, Phys. Plasmas **11**(2), 339 (2004).
[22] X. T. He and W. Y. Zhang, Eur. Phys. J. D **44**, 227 (2007).
[23] W. H. Liu, C. P. Yu, and X. L. Li, Phys. Plasmas **21**(11), 112103 (2014).
[24] D. Layzer, Astrophys. J. **122**, 1 (1955).
[25] J. W. Jacobs and I. Catton, J. Fluid Mech. **187**, 329 (1988).
[26] S. W. Haan, Phys. Fluids B **3**, 2349 (1991).
[27] M. Berning and A. M. Rubenchik, Phys. Fluids **10**, 1564 (1998).
[28] W. H. Liu, L. F. Wang, W. H. Ye, and X. T. He, Phys. Plasmas **19**, 042705 (2012).
[29] S. Hasegawa and K. Nishihara, Phys. Plasmas **2**(12), 4606 (1995).

[30] J. Sanz, J. Ramìrez, R. Ramis, R. Betti, and R. P. J. Town, Phys. Rev. Lett. **89**, 195002 (2002).
[31] T. Ikegawa and K. Nishihara, Phys. Rev. Lett. **89**, 115001 (2002).
[32] J. Garnier and L. Masse, Phys. Plasmas **12**, 062707 (2005).
[33] S. I. Sohn, Phys. Rev. E **67**, 026301 (2003).
[34] V. N. Goncharov and D. Li, Phys. Rev. E **71**, 046306 (2005).
[35] B. A. Remington, S. W. Haan, S. G. Glendinning, J. D. Kilkenny, D. H. Munro, and R. J. Wallace, Phys. Fluids B **4**(4), 967 (1992).
[36] B. A. Remington, S. V. Weber, M. M. Marinak, S. W. Haan, J. D. Kilkenny, R. J. Wallace, and G. Dimonte, Phys. Plasmas **2**(1), 241 (1995).
[37] V. A. Smalyuk, V. N. Goncharov, T. R. Boehly, J. P. Knauer, D. D. Meyerhofer, and T. C. Sangster, Phys. Plasmas **11**(11), 5038 (2004).
[38] M. J. Dunning and S. W. Haan, Phys. Plasmas **2**(5), 1669 (1995).
[39] L. F. Wang, W. H. Ye, and Y. J. Li, Phys. Plasmas **17**, 052305 (2010).
[40] C. Mügler and S. Gauthier, Phys. Rev. E **58**, 4548 (1998).
[41] A. Rikanati, U. Alon, and D. Shvarts, Phys. Rev. E **58**, 7410 (1998).
[42] G. I. Bell, Los Alamos National Laboratory, Report LA-1321, 1951.
[43] M. S. Plesset, J. Appl. Phys. **25**, 96 (1954).
[44] V. N. Goncharov, P. McKenty, S. Skupsky, R. Betti, R. L. McCrory, and C. Cherfils-Clrouin, Phys. Plasmas **7**, 5118 (2000).
[45] P. Amendt, J. D. Colvin, J. D. Ramshaw, H. F. Robey, and O. L. Landen, Phys. Plasmas **10**, 820 (2003).
[46] R. Epstein, Phys. Plasmas **11**, 5114 (2004).
[47] K. O. Mikaelian, Phys. Fluids **17**, 094105 (2005).
[48] H. Yu and D. Livescu, Phys. Fluids **20**, 104103 (2008).
[49] L. K. Forbes, J. Eng. Math. **70**, 205 (2011).
[50] K. Chambers and L. K. Forbes, Phys. Plasmas **19**, 102111 (2012).
[51] L. K. Forbes, ANZIAM J. **53**, 87 (2011).
[52] W. W. Hsing, C. W. Barnes, J. B. Beck *et al*., Phys. Plasmas **4**, 1832 (1997).
[53] J. B. Beck, Ph.D. thesis, Purdue University, 1996.