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Integrate Pipe-Soil Interaction Model with the Vector Form Intrinsic Finite Element Method-Nonlinear Analysis of Free-Span

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ABSTRACT

A practical model is developed by integrating pipe-soil interaction model with the vector form intrinsic finite element (VFIFE) method. The soil resistance in axial direction of pipeline is taken into account by elasto-frictional method. The soil reactions in the lateral and vertical directions of pipeline are simulated by a force-resultant model (so-called bubble model). An automatic method is developed to simulate pipe-soil contacting and separating phenomenon in dynamical analysis. A series of cases are studied. The displacements of simulation compare well with DNV. The static moment formula of DNV is modified by contrast with simulation results, which are appropriate for interaction of the studied pipeline with sandy seabed.

KEY WORDS: pipeline; tension; bending moment; VFIFE; bubble model; soil resistance; pipe-soil contacting and separating.

INTRODUCTION

Submarine pipeline is a critical infrastructure in exploration and transportation of marine oil and gas. There are always free-span submarine pipelines due to unevenness of seabed, scouring or pipelines crossing (Bijker, Staub et al. 1991, Choi 2001, Soreide, Paulsen et al. 2001, DNV-RP-F105 2006, Jin 2011). The concerned pipeline in South China sea passes through very irregular topography and has span length larger than 200m(L/D>300). However, the general formulas recommended by DNV are just feasible in situations of L/D<120 (Soreide, Paulsen et al. 2001). Therefore, the present study aims at developing a robust numerical model with which the following several points are taken into consideration in this paper to accurately simulate response of free-span pipeline. 1. The geometry nonlinearity and extra-large displacement of the long-span pipeline (DNV-RP-F105 2006). 2. Soil reaction force to pipeline as the pipeline embedded into soil. 3. The axial soil resistance in long-span case. 4. The pipe-soil contacting and separating phenomenon in dynamical analysis.

PIPE-SOIL INTERACTON MODEL

As mentioned above, the response of free-span pipeline is affected by pipe-soil interaction. Traditionally, restraint of seabed to pipeline is assumed as pinned-pinned, fixed-fixed, springs (Nielsen, Kvarme et al. 2002) or frictional (Lyons 1973, Wantland, O'Neill et al. 1979, Lambrakos 1985, Hong, Liu et al. 2004). But the influence of seabed to response of pipeline is just partly reflected by the above simplifications.

Three increasingly sophisticated force-resultant models (Elasto-plastic Model, Bounding Surface Model and Bubble Model) are introduced by Tian and Cassidy(2008). The bubble model is adopted in this paper because it simulates the force-displacement constitution best. Issues about numerically implementing several kinds of force-resultant pipe-soil interaction models in the analysis of pipelines are discussed in Tian and Cassidy(2010). The efficiency and accuracy of several time integration methods are discussed. The Euler-Modified method is proved to be the most time-saving one meeting engineering accuracy and is therefore chosen by us with adopted model accuracy of 10⁻⁴.

Bubble model is integrated with ABAQUS by Tian and Cassidy(2008) and Tian, Cassidy et al.(2010). The integrating of a serial of bubble models with structure nodes are proved to be successful. Nevertheless, both the two papers have not considered the axial soil resistance which should be taken into account for cases with L/D>100(DNV-RP-F105 2006). In both papers, nodes in touch with soil are assumed to be always in touch with soil and nodes separating from soil are assumed to be always separating from soil. Actually, as change of external forces, the displacement of pipeline will also change. It is not known to us when and where the pipeline will contact with or separate from soil. The model in Tian and Cassidy(2008) and Tian, Cassidy et al. (2010) is infeasible in this case. Therefore, besides the geometry nonlinearity and influence of tension force, the axial soil resistance and pipe-soil contacting and separating phenomenon are taken into consideration in this paper

Soil Reaction to Pipeline

The reaction force of soil to pipeline have components in three directions. The two components in lateral and vertical directions of pipeline are modeled by bubble model. The reaction force of soil to pipeline have components in three directions. The two components in lateral and vertical directions of pipeline are modeled by bubble model. The constitutive relation of the bubble model is derived as follows by Tian and Cassidy(2010).

$$\dot{F} = \begin{cases} \dot{V} \\ \dot{H} \end{cases} = D^{ep} \dot{U} = \left(D^e - \frac{D^e \frac{\partial g}{\partial F} \frac{\partial f}{\partial F}^T D^e}{K + \frac{\partial f}{\partial F}^T D^e \frac{\partial g}{\partial F}} \right) \begin{cases} \dot{w} \\ \dot{u} \end{cases}$$
(1)

In which, F is the soil reaction force and U is pipeline displacements; D^{ep} is the elasto-plastic matrix D^e is the elastic matrix; g and f are the equations of the bubble surface and bounding surface separately; K is the plastic modulus. Review (Tian and Cassidy 2008, Tian and Cassidy 2010, Tian, Cassidy et al. 2010, Tian and Cassidy 2011) for more details.

The axial soil resistance is assumed to be elasto-frictional as recommended by DNV-RP-F105(2006). For a node contacting with soil, the maximum soil resistance is the maximum frictional force determined by Eq.2, in which μ is friction coefficient and V_n is vertical soil reaction in time step n. In time step n, assuming that the axial displacement of the node is ds, on the linear elastic theory, we can determine the increment of soil resistance by Eq.3, in which, K_A is the axial soil stiffness with recommended value K_A=K_L by DNV-RP-F105(2006). The soil resistance should not be larger than the maximum friction force. Therefore, we need to check it by Eq. 4 in which sign(a,b) has absolute value of a and sign of b.

$$f_{\max} = \mu \cdot V_n \tag{2}$$

$$df_{axial} = K_A \cdot ds$$

$$f_{axial} = f_{axial} + df_{axial}$$
(3)

$$if |f_{axial}| \ge f_{max}, \ f_{axial} = sign(f_{max}, f_{axial})$$

$$\tag{4}$$

Pipe-Soil Contacting and Separating

The pipeline have different displacement and contacting situation with soil when subjected to different load distribution in dynamical or quasi-dynamical analysis. In dynamical simulation, contacting and separating of pipeline with soil will appear repeatedly and have some kind of nonlinear influence on response of pipeline. Therefore, it is necessary to develop a pipe-soil contacting and separating model with which we can update the contacting situation automatically in quasi-dynamical and dynamical analysis.



Fig. 1 flow chart for simulation of pipe-soil contacting and separating

The contacting criteria is determined by Eq.5, in which, z_{node} is the vertical coordinate of a node and z_{touch} is the vertical coordinate of soil surface corresponding to the same node in the same time step. The initial value of z_{touch} is determined by Eq.6, in which $w_p^{initial}$ is the initial embedment of the corresponding node and z_{seabed} is the vertical coordinate of local seabed. If the pipeline separates from soil in time step n, the soil surface coordinate is updated by Eq. 7 and Eq. 8, in which V^{n-1} is the vertical soil reaction, k_{ve} is the vertical elastic soil stiffness and z^{n-1} is the vertical coordinate of the corresponding node in the last time step, dw is the vertical displacement increment of the corresponding node p and e are critical variables in the integration process of bubble model which have value between 0 and 1 (Tian and Cassidy 2010). The separation criteria is determined by Eq.9. If the vertical soil reaction has negative value, we assume that the corresponding node will separate from soil. The flow chart of pipe-soil contacting and separating method is shown in Fig. 1.

$$z_{Node} \le z_{touch} \tag{5}$$

$$z_{touch}^{initial} = z_{seabed} - w_p^{initial} \tag{6}$$

In the bubble surface:

$$z_{touch}^{n} = z^{n-1} + V^{n-1} / k_{ve}$$
⁽⁷⁾

Cross the bubble surface:

$$z_{touch}^{n} = z^{n-1} - dw \cdot (e + p \cdot (1.0 - e))$$
(8)

$$V_n \le 0 \tag{9}$$

PRPPONDERANCE OF VFIFE

Ting, Shih et al.(2004) for the first time proposed a vector form intrinsic finite element (VFIFE) procedure to calculate motions of a system of rigid and deformable bodies. The VFIFE method models the analyzed domain to be composed by finite particles and the Newton's second law is applied to describe each particle's motion. By tracing the motions of all the mass particles in the space, it can simulate the large geometrical and material nonlinear changes during the motion of structure without using geometrical stiffness matrix and iterations (Ting, Shih et al. 2004). The VFIFE method includes 4 main procedures: (1) construct the equation of motion using Newton's law at the mass points, (2) update the material frame, (3) compute the fictitious reverse rotations, and (4) determine the deformation coordinates (Wang, Chen et al. 2011). Compared with traditional FEM, the VFIFE method have the following three advantages for problems we are concerned about.

(1) We must take into consideration the geometric nonlinearity for long-span pipeline. The rigid motion of long pipeline should also be taken into account in the laying process and in global buckling analysis. The conventional FEM is an energy-based method. It does not specifically require the balance of forces within each element. Since these unbalanced residual forces will do some work under rigid body motions, they will cause inaccuracy and un-convergence of the computed results (Wang, Chen et al. 2011). Hence, there are fundamental difficulties in using traditional FEM to treat large rigid body motions. When dealing with nonlinear problems, we have to handling with nonlinear matrix and iterations are necessary for FEM. The VFIFE performs better in both circumstances and is more easily programing. The analysis procedure is vastly simple, accurate and versatile and no extra-work is needed for nonlinearity. The programs for nonlinear problem and for linear problem are identical in VFIFE method.

(2) The traditional finite element method gives the displacements and inner forces by solving linear equation constituted of mass matrix, stiffness matrix and displacement vector which are assembled from element matrixes. Though it's a ripe method in dealing with contacting problem and complex boundary condition, we still have to deal with the stiff matrix and the force vector. In contract, the first step of VFIFE is to disperse the studied body into particles instead of establishing equations. Force on the particle should be analyzed at next step. So the acceleration of particle is obtained. The position of every particle at any time can be calculated by discrete-time method (Zhong 2011). Therefore, the VFIFE method performs much like a 'mesh-less' method. There are no complex matrixes and we need only to control the motion state of the standalone mass particles. Therefore the VFIFE method is a more intuitive procedure and needs less memory.

(3) FEM is actually a method for numerical solving of the governing equation. Therefore, FEM have different procedures for different problems which increases the workload of programing and reduces the universality of programs. We know that the governing equations are just an approximation describing of the physical problem. For example, there's no absolute static mechanical problem in real physical world and the so-called 'static' is just an idealized approximation. The VFIFE method is a method used to simulate the physical problem instead of the ideal governing equations. As a result, not only the programs for linear problem and geometric nonlinear problem and identical, but the programs for dynamic problem and static problem are also same. Thus, the VFIFE method will be a more practical method and will beloved by engineers and programmers. As a result, we can only approximate the static answer by a 'dynamic analysis' with a large structural damping ratio and a gentle loading process (see Fig. 4). Correspondingly, VFIFE method may be inevitably more time-consuming for linear static analysis compared with FEM. Emphasis should be placed on this point when only using it to do simple linear static analysis. While with consideration of all the above advantages and the characteristics of the concerned problem, the only shortcoming can be readily accepted. With the above superiority, the VFIFE method is therefore chosen as structural analysis method in this paper

For cases of short span it is advisable to integrate FEM with bubble model. A 100m pipeline subjected to periodic horizontal load with span length of 20m or 30m is analyzed in Tian and Cassidy(2008) by integrating bubble model with ABAQUS with uniform element length of 5m. The same simulations are implemented by VFIFE and good agreement is obtained (see Fig. 2). Refer to Tian and Cassidy(2008) for details of the comparison analysis.

In the last case, because the soil is very soft compared with the stiffness of the concerned pipeline, there is no separation of pipe with soil and the initial assumed contacting situation has not changed in the simulation. Thus the model in Tian and Cassidy(2008) is feasible. For cases of long span with relatively harder seabed, the geometric nonlinearity of pipeline and contacting situation changing of pipe with soil should be considered. Then the VFIFE method is superior to FEM. A 400m pipeline with single span of 100m is studied to verify the applicability of combined model in this paper. Details of the case will be introduced in the next section. The initial seabed and locations of pipeline at different time are shown in Fig. 3. The embedment at span shoulder and the seperation of pipeline from soil are predicted successfully.

The concerned span is symmetric and thus only half of the free-span pipeline is shown in Fig. 3(b). The separation zone and the contacting zone are marked out. In Fig. 3(c) is the locations of initial seabed and pipeline at span-shoulder at time t/T=0.3,0.6,1.0. Embedment, inclination and contacting of pipeline with soil at different time are illustrated. In Fig. 3(a) is the locations of initial seabed and pipeline in separation zone at different time. Because of inclination of pipeline at span shoulder, pipeline farer away from the span separates from seabed respond like a 'seesaw'. Though the gap between pipe and soil here is at the order of centimeter, it should be noted that the resulted considerable influence on soil reaction is of great importance and false result will be obtained if this effect is neglected.



Fig. 2 predicted displacement of 20m span and 30m span compare with Tian and Cassidy(2008)

CASE STUDY

The pipeline studied in this paper is a real one in South China of which the parameters are listed in Table 1. During pipe laying the vertical force pushing the pipe into the soil is larger than the truly submerged weight (Tian and Cassidy 2008). A low load concentration factor of 2 is used for simplifying of laying process (Tian, Cassidy et al. 2010). In all of the cases, pipeline underwent vertical loads of twice self-weight 2W and then unloaded to W (see Fig. 4). The loading process has a total time of T=300s. An initial tension of 30kN is given before weight loading. A smooth initial seabed is given by Eq.10. A series of cases with different span lengths from 10m to 200m ($19 \le L/D \le 383$) and different tension forces ($0 \le S_{eff}/P_{cr} \le 6.8$) are considered. In this article the element length is 1m. Though larger than the DNV recommended value of 1D, it is proved to be shorter enough by simulation results. Parameters of interaction-model for the studied pipeline are listed in Table 2.

$$z = \begin{cases} -\frac{L}{20} (1 + \cos(2\pi \frac{x}{L})), |x| \le L/2\\ 0, |x| > L/2 \end{cases}$$
(10)



Fig. 3 line shape of pipeline at span-shoulder and separation zone





Fig. 5 computation model of free-span

Comparison of Simulation Results with DNV

Results of 60 cases are compared with DNV formulas to study application range of DNV formulas. The static deflection of mid-span DEF is recommended as Eq. 11, in which PW is the buoyant weight of pipeline per meter (DNV-RP-F105 2006).

$$DEF = \frac{1}{384} \frac{PW \cdot L_{eff}^4}{EI(1 + S_{eff} / P_{cr})}$$
(11)

 L_{eff} is the effective span length determined by Eq.12 (DNV-RP-F105 2006).

$$\frac{L_{eff}}{L} = \begin{cases} \frac{4.73}{-0.066\beta^2 + 1.02\beta + 0.63}, \beta \ge 2.7\\ \frac{4.73}{0.036\beta^2 + 0.61\beta + 1.0}, \beta < 2.7 \end{cases}$$
(12)

$$\beta = \log_{10} \left(\frac{K \cdot L^4}{(1 + CSF)EI} \right)$$
(13)

 S_{eff} is the effective tension force which is the mid-span tension of simulation by model in this article. P_{cr} is the critical buckling load determined by Eq.14 (DNV-RP-F110 2007).

$$P_{cr} = (1 + CSF)C_2 \pi^2 EI / L_{eff}^2$$
(14)

The effective tension force S_{eff} and the non-dimensional effective tension force S_{eff}/P_{cr} are shown in Fig. 6 and Fig. 7 respectively.

Outer diameter of steel	
pipe	0.508 m
inner diameter of steel	
pipe	0.4794 m
thickness of steel pipe	0.0143 m
thickness of outer	
concrete weight coating	0.04 m
outer diameter of	
concrete coating	0.516 m
Young's module of steel	2.07E+11 Pa
Shear Module of steel	8.00E+10 Pa
Density of steel	7850 kg/m ³
Density of concrete	3044 kg/m ³
Density of water	1025 kg/m ³
Weight Per Unit Length	1296 N/m

Table 1 parameters of studied pipeline

Table 2 parameters for bubble model

Plastic stiffness of	$95 \text{ kN/m}^2(\text{loose sand})$
vertical loading per	175 kN/m^2 (medium sand)
unit length(k_{vp})	410 kN/m ² (dense sand)
Elastic stiffness of	
vertical loading(K_v)	20 <i>k</i> _{vp} (Zhang 2001)
Elastic stiffness of	$K_{he}=0.75 K_{ve}$
horizontal loading(KL)	(DNV-RP-F105 2006)
Elastic stiffness of	$K_{Ae} = K_{he}$
axial loading(K_A)	(DNV-RP-F105 2006)
Initial size of	
bounding	0.1 kN
$surface(V_0^{initial})$	
	$V_{a}^{initial} \cdot (k - k)$
Initial pipeline	$\frac{r_0}{r_{ve}}$ $\left(\frac{n_{vp}}{n_{vp}}\right)$
embedment(wp ^{initial})	$K_{ve} \cdot K_{vp}$

The recommended static bending moment formula at mid-span and span shoulder by DNV-RP-F105(2006) is determined by Eq. 15 in which C_5 is factor of boundary condition and have value of $C_5=1/[18(L_{eff}/L)^2-6]$ for span shoulder and $C_5=1/24$ for mid-span. The simulation results are compared with Eq. 15, as shown in Fig. 9 and Fig. 10.

$$M_{DNV} = C_5 \frac{PW \cdot L_{eff}^2}{(1 + \frac{S_{eff}}{P_{cr}})}$$
(15)



Fig. 6 Seff for different span lengths and sand types

The interaction model in this paper predicts very close mid-span deflection (see Fig. 8) and diverging bending moment especially at span shoulder with DNV (see Fig. 9 and Fig. 10). Actually, the moment is corresponding to the deflection curve of pipeline. The reason of close deflections and diverse moments may be that the two methods predicts different deflection curves.

As illustrated in Fig. 9, the moments at mid-span have the same tendency for the both two methods. Nevertheless, the DNV formula predicts larger moment for cases with extro-long spans. As shown in Fig. 10, DNV estimate smaller

moment for cases with short span and larger moment for cases with long span. Actually, the reason of the diversion of bending moments in the two methods is different for short-span case and long-span case. The diversion is dominated by non-dimensional seabed stiffness for short-span case and non-dimensional tension force for long-span case. It will be quantitatively analyzed in the following section.



Fig. 7 Seff/Pcr for different span lengths and sand types



Fig. 8 DEF for different span lengths and sand types



In Fig. 9 and Fig. 10 DNV predicts decreasing results with increasing *L*. It illustrates a false appearance that pipeline with longer span will be more secure when L>120m. Nevertheless,

the actual reason of the decreasing is the considerable large tension force for pipelines with extro-long span (see Fig. 6 and Fig. 7). As shown in Eq. 15, the non-dimensional effective tension force has competitive relation with effective span length. The bending moment is result of the competition and therefore decreases with exponential growth of S_{eff}/P_{cr} and approximately linear growth of L_{eff} .



Fig. 10 Span-shoulder bending moment for different span lengths and sand types

Modification of DNV Moment Formula

The goodness of comparison of simulation bending moments with DNV changes with span length and sand type. Non-dimensional seabed stiffness and non-dimensional tension force also change with span length and sand type. Soreide, Paulsen et al.(2001) states that DNV recommendation is just reliable for case of L/D<120(Soreide, Paulsen et al. 2001). For extreme conditions the nonlinear numerical tools are recommended by DNV and the semi-empirical formulas of DNV are just secondary choice to accurate numerical simulation (DNV-RP-F105 2006). Therefore, the results of this paper are supposed to be superior to DNV results. In order to further study the reasons of difference between the predicted bending moments of the two method quantitatively, a ratio $C_{Modify}=M/M_{DNV}$ is introduced. The relationships between C_{Modify} and non-dimensional effective tension force or non-dimensional seabed stiffness are shown in Fig. 11 and Fig. 12 respectively. It is illustrated in Fig. 11 that for large effective tension force, Cmodify has the same tendency with three types of sands. Cmodify is therefore assumed to be a single-variable function of non-dimensional tension force for extro-long-span case. In Fig. 12, with small β values, C_{modify} has the same tendency for three types of sands. C_{modify} is therefore assumed to be a single-variable function of non-dimensional seabed stiffness for short-span case. In conclusion, Eq.15 couldn't effectively express the influences of tension force and seabed stiffness for short-span case and long-span case respectively. Therefore, C_T and C_β , which represent the affection of tension and seabed stiffness respectively, are brought in to modify the DNV formula. The modified formula is expressed as Eq.16. By results of simulations, C_T and C_β are fitted and have values determined by Eqs.17~20.

$$M_{Modify} = C_{Modify} \cdot M_{DNV} = C_{\beta} \cdot C_{T} \cdot C_{5} \frac{PW \cdot L_{eff}^{2}}{(1 + \frac{S_{eff}}{P_{eff}})}$$
(16)

At span shoulder:

$$C_{\beta} = \begin{cases} 2.53 - 0.383\beta, & \text{for } \beta < 4\\ 1.0, & \text{for } \beta \ge 4 \end{cases}$$
(17)

$$C_{T} = \begin{cases} 0.414\overline{T}^{2} + 0.192\overline{T} + 1.0, & for & \overline{T} > 0\\ 1.0, & for & \overline{T} \le 0 \end{cases}$$
(18)

At mid-span:

$$C_{\beta} = \begin{cases} 0.588 + 0.225\beta, & for \quad \beta < 1.83\\ 1.0, & for \quad \beta \ge 1.83 \end{cases}$$
(19)

$$C_{T} = \begin{cases} -0.05\overline{T}^{2} - 0.18\overline{T} + 0.868, & for & \overline{T} > -1 \\ 1.0, & for & \overline{T} \le -1 \end{cases}$$
(20)

In which,

$$\overline{T} = \log_{10} \left(\frac{S_{eff}}{P_{cr}} \right)$$
(21)

The modified results compare well with the simulation results, as shown in Fig. 13 and Fig. 14. However, it should be noted that Eqs.17~20 are obtained by fitting the numerical results of studied pipeline with three types of sandy seabed. Therefore, the fitted formulas could only be used in the above condition. For pipelines with different diameter, different sectional features or on clay seabed, it is necessary to verify the applicability of the formulas.



Fig. 11 C_{Modify} for different S_{eff}/P_{cr} and soil types



Fig. 12 C_{Modify} for different β and soil types



Fig. 13 Modified mid-span bending moment and simulation results for different span lengths and sand types



Fig. 14 Modified span-shoulder bending moment and simulation results for different span lengths and sand types

SUMMARY

The main contents of this article are as follows.

1. VFIFE method has the advantages of easy programing, robustness and simplicity in handling the geometric nonlinear problems. With the above advantages, VFIFE method is for

the first time integrated with pipe-soil interaction model and verified to be superior to FEM in several conditions.

2. Soil reactions in the lateral and vertical directions of pipeline are simulated by bubble model. Axial soil resistance is assumed to be elasto-frictional in the model. It's crucial for long span analysis. A pipe-soil contacting and separating method is developed to extend the model applicability.

3. The parameters of the model for a pipeline in South China Sea with three types of sandy seabed are reasonably estimated. Results of a series of cases with different sand types and tension forces are obtained and compared with DNV. DNV moment formula is modified to take the influence of tension force and seabed stiffness into account. The modification is applicable for the studied pipeline with sandy seabed.

There are still much work to do to further study pipe-soil interaction and develop the combined model proposed in this paper.

1. To simulate response of free-span pipeline more accurately, the laying process have to be taken into consideration. The influence of laying condition, current, waves and unevenness of seabed to response of pipeline should be evaluated.

2. In shallow water region, currents and waves have strong impact on free-span pipeline. The dynamic response and the vortex induced vibration of pipeline are much more significant than the buoyant weight of pipeline. In situation of vibration analysis, the damping effect of seabed is of great significance. The interaction model in this paper is only applicable to low frequency response of pipeline currently. In order to apply this model to vibration analysis of pipeline in wide frequency range the soil damping effect should be taken into consideration.

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