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ESTIMATION OF MEASURED DATA OF MICROCHANNEL GAS FLOWS

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ABSTRACT

The measured data of mass flow rates and streamwise pressure distributions at various experimental conditions of microchannels carried out by Pong et al (1994), Harley et al (1995), Shih et al (1996), Arkilic et al (1997, 2001), and Zohar et al (2002) are normalized by the kinetic factors $\dot{M}_c$ and p_k, respectively. The normalized data are compared each other, and they are in excellent agreement, except the few with the small differences. This demonstrates that the measured data available are generally accurate.

NOMENCLATURE

h	microchannel height
Kn	Knudsen number
L	microchannel length
$\dot{M}$	mass flow rate
$\dot{M}_c$	normalized factor of mass flow rate
p	pressure
P	normalized pressure, $P = p/p_o$
dp/dx	streamwise pressure gradient
λ	mean free path
ϑ	ratio of inlet to outlet pressure, $\vartheta \equiv p_i/p_o$

Subscripts

i	Inlet
o	Outlet

INTRODUCTION

Many experimental studies [1-9] on gas flows through micro-channels were carried out to understand the microscale effects that are important for the design and optimization of MEMS devices. The mass flow rates and streamwise pressure distributions were measured at various conditions as shown in Table 1. The dimensions were about one micron high by several tens of microns wide and by several thousands microns long. The flow was driven by the pressure differences between the inlet and outlet, with a typical inlet velocity of about 0.2 m/s [10]. The flows are two dimensional because of the negligible spanwise effect for the large with-to-height ratio, while the isothermal assumption is valid under the low subsonic conditions without external heating.

Comparing these experimental data each other is helpful to assess their accuracy, and reveal the features of microchannel gas flows. Due to the differences between the experimental conditions, we have to normalize the measured data firstly. Let us image to slice up microchannels a cross section by cross section. Every cross section may be localized as the Poiseuille flow. The mass flow rate may be nicely related to the Knudsen number based on the channel height (see, e.g. Fig.6 in Ref. [11]) when a following normalization factor is used

$$\dot{M}_c = \frac{2h^2}{v_m}\frac{dp}{dx}, \tag{1}$$

where h is the channel height, $v_m = \sqrt{2RT}$ is the most probable thermal speed, and dp/dx is the pressure gradient.

Experiments [1,3,4,9] showed that the streamwise pressure distributions of gas flows through microchannel were nonlinear. This means that dp/dx is not constant, which differs from the Poiseuille flow. Therefore, we have to obtain the solution of dp/dx, before the normalized factor $\dot{M}_p$ may be extended to microchannels.

Table 1. Experimental conditions of microchannel gas flows

Source	Gas	Height (μm)	Width (μm)	Length (μm)
Pong et al [1]	N_2, He	1.2	40	3000
Harley et al [2]	N_2, He, Ar	0.51 ~ 19.79	100 ~ 200	10000
Shih et al [3,4]	N_2, He	1.2	40	4000
Arkilic et al [5-8]	He, Ar, N_2, CO_2	1.33	52.3	7490
Zohar et al [9]	He, Ar, N_2	0.53 ~ 0.97	40	4000

CONSERVATION OF MASS FLOW RATE THROUGH MICROCHANNELS

Consider a cross section of microchannel. The mass flow rate through it may be written as

$$\dot{M} \big/ \dot{M}_{N-S} = \phi(Kn)\,, \qquad (2)$$

with

$$\dot{M}_{N-S} = \frac{2h^3}{3\mu RT} p \frac{dp}{dx}\,, \qquad (3)$$

is the non-slip Navier-Stokes solution of mass flow rate for the Poiseuille flows, and with

$$\phi(Kn) \cong 1 + 6\alpha Kn + \frac{12}{\pi} Kn \ln(1+\beta Kn)\,, \qquad (4)$$

where $\alpha = 1.318889$, $\beta = 0.387361$. $\phi(Kn)$ reflects the local deviation from the N-S solution owing to the microscale effect. Eq. (4) is fitted based on the numerical solution of the linearized Boltzmann equation [12,13], under the tangential momentum accommodation coefficient $\sigma = 1$.

The mass flow rate conservation through microchannels requires

$$\frac{d\dot{M}}{dx} = 0\,. \qquad (5)$$

Substituting Eq. (2) into (5) and eliminating the constant term $2h^3/(3\mu RT)$ give rise to a simple relation between p and Kn

$$\frac{d}{dx}\left\{\left[1 + 6\alpha Kn + \frac{12}{\pi} Kn \ln(1+\beta Kn)\right] p \frac{dp}{dx}\right\} = 0\,, \qquad (6)$$

or

$$\left[1 + 6\alpha Kn + \frac{12}{\pi} Kn \log(1+\beta Kn)\right] P \frac{dP}{dX} = C\,, \qquad (7)$$

where C is a constant undetermined, $P = p/p_o$, $X = x/L$, p_o is the outlet pressure, and L is the microchannel length.

Eq. (6) may be regarded alternatively as a special case of the generalized Reynolds equation with the bearing number $\Lambda = 0$. The generalized Reynolds equation was firstly derived by Fukui and Kaneko [14] from the linearized Boltzmann equation, and it works quite well for air slider bearings. Recently C. Shen [15] suggested to apply it to microchannels. Eq. (6) is valid over the entire flow regime from continuum to free molecular, because its kernel $\phi(Kn)$ is obtained based on the linearized Boltzmann equation.

NORMALIZED FACTORS OF PRESSURE AND MASS FLOW RATE

For hard-sphere molecules, the mean free path $\lambda = kT/\sqrt{2}\sigma_T p$, where the collision cross section σ_T is constant. Consequently, the Knudsen number along a microchannel may be expressed as follows

$$Kn \equiv \frac{\lambda}{h} = \frac{\lambda_o}{h} \cdot \frac{\lambda}{\lambda_o} = \frac{Kn_o}{P}\,, \qquad (8)$$

where the subscript o denotes the outlet.

Substitution of Eq. (8) into (7) yields

$$\left[P + 6\alpha Kn_o + \frac{12 Kn_o}{\pi} \log(1+\beta Kn_o/P)\right] dP = C dX\,, \qquad (9)$$

Integrating Eq. (9) from the inlet X=0 and $P = \vartheta \equiv p_i/p_o$, we have

$$\begin{aligned} &\frac{1}{2}P^2 + 6\alpha Kn_o P + \frac{12 Kn_o}{\pi}[P \ln(1+\frac{\beta Kn_o}{P}) + \beta Kn_o \cdot \\ &\ln(P+\beta Kn_o)] - \frac{1}{2}\vartheta^2 - 6\alpha Kn_o \vartheta + \frac{12 Kn_o}{\pi} \cdot \\ &[\vartheta \ln(1+\beta Kn_o/\vartheta) + \beta Kn_o \ln(\vartheta + \beta Kn_o)] = CX \end{aligned} \qquad (10)$$

At the outlet X=1, P=1, therefore

$$\begin{aligned} C = &\frac{1}{2}(1-\vartheta^2) + 6\alpha Kn_o(1-\vartheta) + \frac{12 Kn_o}{\pi} \cdot \\ &[\ln(1+\beta Kn_o) + \beta Kn_o \ln(1+\beta Kn_o) \\ &- \vartheta \ln(1+\beta Kn_o/\vartheta) - \beta Kn_o \ln(\vartheta+\beta Kn_o)] \end{aligned} \qquad (11)$$

The normalized factor of mass flow rate $\dot{M}c$ through microchannels may be obtained using Eqs. (1), (7) and (11)

$$\dot{M}_c = \frac{2h^2}{v_m} \cdot \left.\frac{dp}{dx}\right|_i = \frac{2h^2}{v_m} \cdot \frac{C p_o^2}{\phi(Kn_i) p_i L}\,, \qquad (12)$$

where the subscript I denotes the inlet.

The kinetic solution of the streamwise pressure distribution p_k may be numerically solved from Eq. (10) that depends upon the parameters ϑ and Kn_o.

COMPARISON OF NORMALIZED MEASURED MASS FLOW RATES AND STREAMWISE PRESSURE DISTRIBUTIONS

Figure 1 compares the normalized mass flow rates at different conditions carried out by Harley et al [2], Shih et al

[4], Arkilic et al [6-8], and Zohar et al [9], respectively. Generally they agree well each other, whereas the small differences between the helium cases of Shih et al [4] and Arkilic et al [6,7] are observed. A relation of the normalized mass flow rate to the inlet Kn_i may be simply fitted as

$$\dot{M} = \dot{M}_c \Big/ \left(a + bKn_i + cKn_i^2 + dKn_i^3 + eKn_i^4\right), \qquad (13)$$

with a=0.021998, b=12.48288, c=-87.95779, d=397.432, and e=-716.724.

Figure 2 and Table 2 show the ratio of the measured pressure distributions to the kinetic solution p_k. Cases A1-A5 correspond to the inlet pressure of 25, 20, 15, 10 and 5 psig, respectively, for nitrogen at the experimental conditions of Pong et al [1]; cases B1-B3 to the inlet pressure of 19, 13.6 and 8.7 psig, respectively, for helium at the experimental conditions of Shih et al [4]; cases C1-C3 to the inlet pressure of 2.1, 2.8 and 3.4atm for nitrogen, while cases D1-D4 to 1.9, 2.6, 3.3 and 4atm for argon, respectively, at the experimental conditions of Zohar et al [9]. The four series of cases have the same outlet pressure of 1 atm, but the different inlet pressures and channel heights. The comparison is very satisfactory. All the values are close to unity, and the differences are within 5%, except three measured points (case A4: x = 1800 μm, p_{exp}/p_k =0.907; case B1: x = 3300 μm, p_{exp}/p_k =1.054; case D4: x=3300 μm, p_{exp}/p_k =1.092).

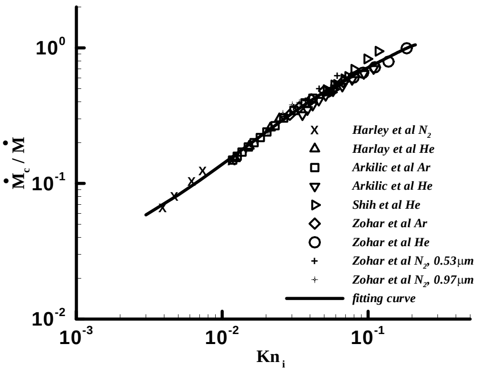

Figure 1. Comparison of normalized measured mass flow rates through microchannels versus the inlet Knudsen number.

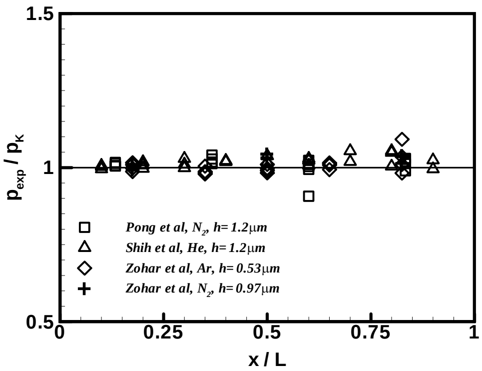

Figure 2. Comparison of normalized measured streamwise pressure distributions.

Table 2. The values of p_{exp}/p_k at different conditions.

Case	x (μm)	p_i (psig)	p_{exp}/p_k
A1	400	25	1.013
	1100	25	1.015
	1800	25	0.994
	2500	25	0.989
A2	400	20	1.005
	1100	20	1.018
	1800	20	1.010
	2500	20	1.013
A3	400	15	1.005
	1100	15	1.028
	1800	15	1.004
	2500	15	1.013
A4	400	10	1.006
	1100	10	1.012
	1800	10	0.907
	2500	10	1.030
A5	400	5	1.017
	1100	5	1.041
	1800	5	1.024
	2500	5	1.027
B1	400	19	1.006
	800	19	1.018
	1200	19	1.029
	1600	19	1.022
	2000	19	1.038
	2400	19	1.029
	3200	19	1.054
	3600	19	1.024
B2	400	13.6	1.003
	800	13.6	1.008
	1200	13.6	1.011
	1600	13.6	1.021
	2000	13.6	1.038
	2400	13.6	1.023
	2800	13.6	1.054
	3200	13.6	1.049
B3	400	8.7	0.997
	800	8.7	0.997
	1200	8.7	0.999
	1600	8.7	1.019
	2000	8.7	1.018

	2400	8.7	1.007
	2800	8.7	1.019
	3200	8.7	1.004
	3600	8.7	0.995
C1	700	16.1	1.004
	2000	16.1	1.045
	3300	16.1	1.039
C2	700	26.4	0.992
	2000	26.4	1.044
	3300	26.4	1.014
C3	700	35.2	1.011
	2000	35.2	1.032
	3300	35.2	1.036
D1	700	13.2	1.016
	1400	13.2	1.005
	2000	13.2	0.991
	2600	13.2	0.993
	3300	13.2	0.982
D2	700	23.4	1.009
	1400	23.4	0.979
	2000	23.4	0.997
	2600	23.4	1.015
	3300	23.4	1.035
D3	700	33.7	0.987
	1400	33.7	0.987
	2000	33.7	0.983
	2600	33.7	1.008
	3300	33.7	1.010
D4	700	44.0	0.998
	1400	44.0	0.983
	2000	44.0	1.010
	2600	44.0	1.012
	3300	44.0	1.092

CONCLUSIONS

The measured data of mass flow rates and streamwise pressure distributions through microchannels at various experimental conditions are normalized by the kinetic factors $\dot{M}_c$ and p_k, respectively. The normalized comparison is satisfactory, except the few that show the small differences. This demonstrates that the measured data available are generally accurate. Consequently, the fitting formula of the measured mass flow rates and the kinetic solution of streamwise pressure distribution may be reliably applied to the design and optimization of MEMS devices.

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REFERENCES

1. K. C. Pong, C. M. Ho, J. Q. Liu, and Y. C. Tai. Non-linear pressure distribution in uniform micro-channels. ASME-FED, 197, 51 (1994)
2. J. C. Harley, Y. Huang, H. Bau, and J. N. Zemel. Gas flow in micro-channels. *J. Fluid Mech.* **248**, 257 (1995)
3. J. C. Shih, C. M. Ho, J. Liu, and Y. C. Tai. Non-linear pressure distribution in uniform microchannels. *ASME-AMD*, 238 (1995)
4. J. C. Shih, C. M. Ho, J. Q. Liu, and Y. C. Tai. Monatomic and polyatomic gas flow through uniform microchannels. *ASME-DSC*, **59**, 197 (1996)
5. E. B. Arkilic, K. Breuer, and M. A. Schmidt. Gaseous slip flow in long micro-channels. *J. MicroElectroMechanical Systems*, **6**, 167 (1997)
6. E. B. Arkilic, M. A. Schmidt, and K. S. Breuer. Measurement of the TMAC in silicon microchannels. in Rarefied Gas Dynamics, edited by C. Shen, Peking University Press, p. 983 (1997)
7. E. B. Arkilic. Measurement of the mass flow and tangential momentum accommodation coefficient in silicon microchannels, Ph.D. thesis, MIT, FDRL TR 97-1 (1997)
8. E. B. Arkilic, K. S. Breuer and M. A. Schmidt. Mass flow and tangential momentum accommodation in silicon micromachined channels. *J. Fluid Mech.*, **437**, 29 (2001)
9. Y. Zohar, S. Y. K. Lee, W. Y. Lee, L. Jiang and P. Tong. Subsonic gas flow in a straight and uniform microchannel. *J. Fluid Mech.*, **472**, 257 (2002)
10. C. M. Ho and C. Y. Tai. Micro-electro-mechanical-systems (MEMS) and fluid flows. *Annu. Rev. Fluid Mech.*, **30**, 579 (1998).
11. J. Fan, and C. Shen, Statistical simulation of low-speed rarefied gas. *J. Comp. Phys.*, **167**, 393 (2001)
12. F. J. Alexander, A. L. Garcia, and B. J. Alder. Direct simulation Monte Carlo for thin-film bearings. *Phys. Fluids*, **6**, 3854 (1994)
13. S. Fukui, and R. Kaneko, A database for interpolation of Poiseuille flow rates for high Knudsen number lubrication problem. *J. Tribology*, **112**, 78 (1990)
14. S. Fukui, and R. Kaneko. Analysis of ultrathin gas film lubrication based on linearized Boltzmann equation: First report derivation of a generalized lubrication equation including thermal creep flow. *J. Tribology*, **110**, 253 (1988)
15. C. Shen. A strict kinetic solution of the finite length microchannel flow. to appear in the 3rd International Conference on Microchannels and Minichannels, June 13-15, 2005, Toronto, Ontario, Canada