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# THE ATTENUATION OF CONCENTRATION SHOCKWAVE IN A FLUIDIZED BED WITH PULSING INLET FLOW

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## ABSTRACT

Although the Local Equilibrium Model (LEM) is simple, it can predict the change of the bed height and the dynamic concentration distribution in a bed with pulsing inlet flow, and its numerical results coincide well with the experimental results. In order to go into the study of the flow character and attenuation of concentration shockwave in the bed, the numerical results are not enough. This paper presents the analytic solution of one-dimensional LEM, gets the law of attenuation of concentration shockwave from the solution and, at the same time, gives the leading factors that affect the law of attenuation.

**Keywords:** Local Equilibrium Model, the analytic solution, attenuation of concentration shockwave

## INTRODUCTION

Pulsed fluidization, an operation in which the fluidizing velocity $U(t)$ pulsates as rectangular wave or any other wave patterns, is an effective method to eliminate slugs and gas channeling, to reduce the size of bubbles, and thus to improve fluidization quality. When the fluidizing velocity $U(t)$ changes alternately between $U_1$ and $U_2$ ($u_T > U_2 > U_1 > U_{mf}$, $u_T$ is particle terminal velocity and $U_{mf}$ is minimum fluidizing velocity), series of dilatation waves and concentration shockwaves will come into being in the bed, they transmit up to the bed and, as the result, the particle concentration changes alternately. The interaction of the concentration shockwave and dilatation wave will decrease the concentration change extent, and therefore will affect the quality of fluidization.

The numerical results of the LEM coincide quite well with the experimental results (See Fig.1), it shows that the LEM can predict the change of the bed height and the dynamic distribution of concentration. Although the LEM equations are much more simple than that of Two-Fluid Model, it is very useful to have research on the attenuation law thoroughly and quantitatively if we get the analytic solution of the LEM.

The paper gives the analytic solution of LEM, which fits quite well with the numerical results of LEM. It is demonstrated that the analytic solution is right. In order to calculate the concentration distribution, it is more convenient and accurate to use the analytic solution than to use the numerical method. (See Fig.1).

## HYDRODYNAMIC EQUATIONS OF LOCAL EQUILIBRIUM MODEL (LEM) AND ITS BOUNDARY CONDITIONS

With pulse period $T$ as the characteristic time, particle terminal velocity $u_T$ as the characteristic velocity and ($Tu_T$) as the characteristic length, the one-dimensional dimensionless LEM equations and the boundary conditions can be written as [1,2]:

**Hydrodynamic Equations:**

$$\frac{\partial \alpha_p}{\partial t}+\frac{\partial(\alpha_p u_p)}{\partial x}=0 \quad (1)$$

$$\alpha_p u_p+\alpha_f u_f=U(t)=\begin{cases}U_2 & 0\le t-N<T_2\\ U_1 & T_2\le t-N<1\end{cases} \quad (2)$$

$$U(t)-u_p=\alpha_f^n \quad (3)$$

$$\alpha_p+\alpha_f=1 \quad (4)$$

**Boundary Conditions:**

$$u_p(t,0)=0 \quad (5)$$

where $t$ is time coordinate(s), $x$ is space coordinate up the bed, $\alpha_p(t, x)$ and $\alpha_f(t, x)$ are the particle volume fraction and the fluid volume fraction, $u_p(t, x)$ and $u_f(t, x)$are solid phase velocity and fluid phase velocity, $g$ is gravity acceleration, $n$ is the Richardson-Zaki number ($n$=2.414), $N$ is an integer, indicating the number of pulses. There are four dimensionless parameters $U_m$, $U_1$, $U_2$, and $T_2$ in the Eqs.1-5, but only three of them are independent. The four dimensionless parameters meet with $U_m=(1-T_2)U_1+T_2U_2$, $U_m$ is the average fluidizing velocity.

From the Eq.3 we can get the volume fraction $\alpha_{f1}$ (or $\alpha_{f2}$) and the particle concentration $\alpha_{p1}$ (or $\alpha_{p2}$) that under conditions of $U_1$ (or $U_2$) and the solid phase velocity is zero:

$$1-\alpha_{p1}=\alpha_{f1}=U_1^{1/n} \quad (6)$$

$$1-\alpha_{p2}=\alpha_{f2}=U_2^{1/n} \quad (7)$$

## THE ANALYTIC SOLUTION OF CONCENTRATION SHOCKWAVE ATTENUATION AND CONCENTRATION DISTRIBUTION IN A FLUIDIZED BED WITH PULSING INLET FLOW

As it is shown in Fig.1c and Fig.2d, particle concentration $\alpha_p$ changes just between the two curves, $\alpha_{p,F}$ $(t,(x_S))$ and $\alpha_{p,B}$ $(t,(x_S))$, which show the particle concentration before and after the shockwaves respectively. When we get the $\alpha_{p,F}(t)$, $\alpha_{p,B}(t)$ and $x_S(t)$ from the analytic solution, where $x_S(t)$ is the position of shockwave in the bed, the law of concentration shockwave attenuation is determined. Underside the analytic solution of the attenuation of concentration shockwave is given under the two different instances that $T_2<\Psi$ and $T_2>\Psi$, in which $\Psi$ and $x_R(t)$ are defined as follows:

$$\Psi=\frac{\alpha_{f1}^{n-1}\alpha_{p1}^2}{\alpha_{f1}^{n-1}\alpha_{p1}^2-\alpha_{f2}^{n-1}\alpha_{p2}^2}\left[1-\frac{\alpha_{f2}^{n-1}\alpha_{p2}^2}{\alpha_{f2}^n-\alpha_{f1}^n}\frac{n(\alpha_{f2}-\alpha_{f1})}{\alpha_{p1}\alpha_{p2}}\right] \quad (8)$$

$$x_R(t)=\int_{T_2}^{t}\left[U(t')-U_m\right]dt' =\begin{cases}-(1-T_2)(T_2-t)(U_2-U_1) & 0\le t-N<T_2\\ -T_2(t-T_2)(U_2-U_1) & T_2\le t-N<1\end{cases} \quad (9)$$

$x_R(t)$ changes periodically between 0 and $[-T_2(1-T_2)(U_2-U_1)]$.

### 1) the case of $T_2<\Psi$

Shockwaves come into being at the distributor when time $t=N+T_2$ ($N$=1,2,...), they will not intersect with the dilatation wave before time $t=N+T_2+\tau_P$, thus their intensity and transmitting velocity will not change; At time $t=N+T_2+\tau_P$ they begin intersect with the dilatation waves that come into being at the distributor at time $t=N$, the voidage before shockwave $\alpha_{f,F}$ decreases, the intensity of the shockwave weakens and the wave velocity changes gradually. From $t=N+T_2+\tau_D$ the shockwave will intersect with two series of dilatation waves that comes into being at the distributor at time $t=N$ and $t=N+1$ respectively, the voidage before shockwave $\alpha_{f,F}$ decrease and the voidage after shockwave $\alpha_{f,B}$ increase gradually, so the attenuation of concentration shockwave is accelerated. Based on the $\alpha_{f1}$, $\alpha_{f2}$, $T_2$ and $U_m$, we can get $\tau_P$, $\tau_D$ and $\alpha_{f,FD}$ (the voidage before shockwave at time $t=N+T_2+\tau_D$) from Eq.(10a) and Eq.(11a).

$$\tau_P=\frac{T_2(\alpha_{f2}^n-\alpha_{f1}^n)}{(\alpha_{f2}^n-\alpha_{f1}^n)-n\alpha_{f2}^{n-1}\alpha_{p2}(\alpha_{f2}-\alpha_{f1})/\alpha_{p1}} \quad (10a)$$

$$\begin{cases}\tau_D=\dfrac{\alpha_{f1}^{n-1}\alpha_{p1}^2}{\alpha_{f1}^{n-1}\alpha_{p1}^2-\alpha_{f,FD}^{n-1}\alpha_{p,FD}^2}\\ U_m=\dfrac{\alpha_{f,FD}^{n-1}\alpha_{f1}^{n-1}(\alpha_{f,FD}-\alpha_{f1})[1-\alpha_{f,FD}\alpha_{f1}-n\alpha_{p,FD}\alpha_{p1}]}{\alpha_{f1}^{n-1}\alpha_{p1}^2-\alpha_{f,FD}^{n-1}\alpha_{p,FD}^2}\end{cases} \quad (11a)$$

According to $t$, $\tau_P$ and $\tau_D$, the volume fractions $\alpha_{f,F}(t)$ and $\alpha_{f,B}(t)$ and the shockwave position $x_S(t)$ can be expressed as:

$$\begin{cases}\alpha_{f,F}=\alpha_{f2}\\ \alpha_{f,B}=\alpha_{f1}\\ x_S(t)=\left[U_m-(\alpha_{p1}\alpha_{f1}^n-\alpha_{p2}\alpha_{f2}^n)/(\alpha_{p1}-\alpha_{p2})\right](t-T_2)+x_R(t)\end{cases} \quad (0<t\le\tau_P) \quad (12a)$$

$$\begin{cases}t=\dfrac{T_2(\alpha_{f2}^n-\alpha_{f1}^n)\alpha_{p1}}{(\alpha_{f,F}^n-\alpha_{f1}^n)\alpha_{p1}-n\alpha_{f,F}^{n-1}\alpha_{p,F}(\alpha_{f,F}-\alpha_{f1})}\\ \alpha_{f,B}=\alpha_{f1}\\ x_S(t)=x_R(t)+\displaystyle\int_{T_2}^{t}\left[U_m-\frac{\alpha_{p1}\alpha_{f1}^n-\alpha_{p,F}\alpha_{f,F}^n}{\alpha_{p1}-\alpha_{p,F}}\right]dt'\end{cases} \quad (\tau_P<t\le\tau_D) \quad (13a)$$

$$\begin{cases}\dfrac{\alpha_{f,B}^{n-1}\alpha_{p,B}^2}{\alpha_{f,B}^{n-1}\alpha_{p,B}^2-\alpha_{f,F}^{n-1}\alpha_{p,F}^2}=t\\ \dfrac{\alpha_{f,F}^{n-1}\alpha_{f,B}^{n-1}(\alpha_{f,F}-\alpha_{f,B})[1-\alpha_{f,F}\alpha_{f,B}-n\alpha_{p,F}\alpha_{p,B}]}{\alpha_{f,B}^{n-1}\alpha_{p,B}^2-\alpha_{f,F}^{n-1}\alpha_{p,F}^2}=U_m\\ x_S(t)=x_R(t)+\displaystyle\int_{T_2}^{t}\left[U_m-\frac{\alpha_{p,B}\alpha_{f,B}^n-\alpha_{p,F}\alpha_{f,F}^n}{\alpha_{p,B}-\alpha_{p,F}}\right]dt'\end{cases} \quad (\tau_D<t) \quad (14a)$$

When $U_1$=0.284, $U_2$=0.765 and $T_2$=0.25, then $U_m$=0.404, $\Psi$=0.756, $1-\alpha_{p1}=\alpha_{f1}$=0.594, $1-\alpha_{p2}=\alpha_{f2}$=0.895, $\tau_P$=0.375, $\tau_D$=2.224, $1-\alpha_{p,FD}=\alpha_{f,FD}$=0.743. Fig.2b shows the trace of shockwave that comes into being at the distributor at time $t=T_2$-4, $t=T_2$-3, $t=T_2$-2, $t=T_2$-1 and $t=T_2$. Fig.2b also shows the dilatation waves that come into being from the distributor at time $t$=-4, $t$=-3, $t$=-2, $t$=-1 and $t$=0. Each dilatation wave corresponds to a certain voidage, so the analytic solution of the concentration distribution $\alpha_p(t,x)$ at any time can be gotten. Fig.2c shows the concentration distribution at time $t$=0.65 and Fig.2d shows the concentration distribution at time $t$=0.05. Fig.1 and Fig.2 show the concentration distribution at different time $t$=0.05, 0.25 and 0.65 that gotten from the analytic solution, as well as the concentration distribution from experiment.

### 2) the case of $T_2>\Psi$

When $T_2>\Psi$, the shockwave coming into being at the distributor at time $t=N+T_2$ ($N$=1,2,$\cdots$) will intersect with the dilatation waves that come into being at the distributor at time $t=N$ firstly and then intersect with the dilatation waves that come into being at the distributor at time $t=N+T_2+\tau_D$, so the expressions are different with those in case $T_2<\Psi$.

$$\tau_p = \frac{n\alpha_{f1}^{n-1}\alpha_{p1}(\alpha_{f2}-\alpha_{f1})/\alpha_{p2} - T_2(\alpha_{f2}^n - \alpha_{f1}^n)}{n\alpha_{f1}^{n-1}\alpha_{p1}(\alpha_{f2}-\alpha_{f1})/\alpha_{p2} - (\alpha_{f2}^n - \alpha_{f1}^n)} \qquad (10b)$$

$$\begin{cases} \tau_D = \dfrac{\alpha_{f,BD}^{n-1}\alpha_{p,BD}^2}{\alpha_{f,BD}^{n-1}\alpha_{p,BD}^2 - \alpha_{f2}^{n-1}\alpha_{p2}^2} \\ U_m = \dfrac{\alpha_{f2}^{n-1}\alpha_{f,BD}^{n-1}(\alpha_{f2}-\alpha_{f,BD})[1-\alpha_{f2}\alpha_{f,BD} - n\alpha_{p2}\alpha_{p,BD}]}{\alpha_{f,BD}^{n-1}\alpha_{p,BD}^2 - \alpha_{f2}^{n-1}\alpha_{p2}^2} \end{cases}$$

(11b)

$$\begin{cases} \alpha_{f,F} = \alpha_{f2} \\ \alpha_{f,B} = \alpha_{f1} \\ x_S(t) = x_R(t) + \left[U_m - (\alpha_{p1}\alpha_{f1}^n - \alpha_{p2}\alpha_{f2}^n)/(\alpha_{p1}-\alpha_{p2})\right](t-T_2) \end{cases}$$

$$(0 < t \le \tau_P) \qquad (12b)$$

$$\begin{cases} t = 1 + \dfrac{(1-T_2)(\alpha_{f2}^n - \alpha_{f1}^n)\alpha_{p2}}{n\alpha_{f,B}^{n-1}\alpha_{p,B}(\alpha_{f2}-\alpha_{f,B}) - (\alpha_{f2}^n - \alpha_{f,B}^n)\alpha_{p2}} \\ \alpha_{f,F} = \alpha_{f2} \\ x_S(t) = x_R(t) + \displaystyle\int_{T_2}^{t}\left[U_m - \frac{\alpha_{p,B}\alpha_{f,B}^n - \alpha_{p2}\alpha_{f2}^n}{\alpha_{p,B}-\alpha_{p2}}\right]dt' \end{cases}$$

$$(\tau_P < t \le \tau_D) \qquad (13b)$$

Expression for Eq. (14b) is the same as Eq. (14a).

The analytic solution of the concentration distribution $\alpha_p(t,x)$ is listed in **Appendix A**.

## EFFECT OF THE DIMENSIONLESS PARAMETERS ON THE ATTENUATION OF CONCENTRATION SHOCKWAVE

According to Eqs.(13)-(14), we can observe the effect of the dimensionless parameters on the attenuation of concentration shockwave, such as $U_m$, $U_1$, and $U_2$. Fig.3 and Fig.4 give the main results in the case $T_2 < \Psi$. It is shown in Eq.(14a) and Fig.3 that $\alpha_{f,F}(t)$, $\alpha_{f,B}(t)$ and the main part of $x_S(t)$ are completely dependent on $U_m$ and independent of $U_1$ and $U_2$ when $t > \tau_D$, however, $\tau_D$ is determined by the $U_m$ and $U_1$. $\alpha_{f,F}(t)$, $\alpha_{f,B}(t)$ and main part of $x_S(t)$ are completely dependent on $U_m$ and $U_1$ and independent of $U_2$ when $\tau_P < t < \tau_D$, but $\tau_P$ is a function of $U_m$, $U_1$ and $U_2$. It is similar to the case $T_2 > \Psi$, only the roles of $U_1$ and $U_2$ change with each other.

## CONCLUSIONS

(1) Integrating Eqs. (1)-(4), we can get $\alpha_{p,F}(t)$, $\alpha_{p,B}(t)$ and $x_S(t)$ at any time $t$ during the transmitting process of concentration shockwave (See Eqs. (12)-(14)), and the analytic solutions of the concentration distribution along the bed height.

(2) This paper gets the attenuation law of concentration shockwave from the analytic solution. A) The attenuation height is in proportion to the pulsed period $T$ and terminal velocity $u_T$, which are the main factors. B) $U_m/u_T$ is the secondary factor affecting the attenuation. C) Parameters $U_1/u_T$ and $U_2/u_T$ affect the attenuation only in the shallow section of bed.

## ACKNOWLEDGMENTS

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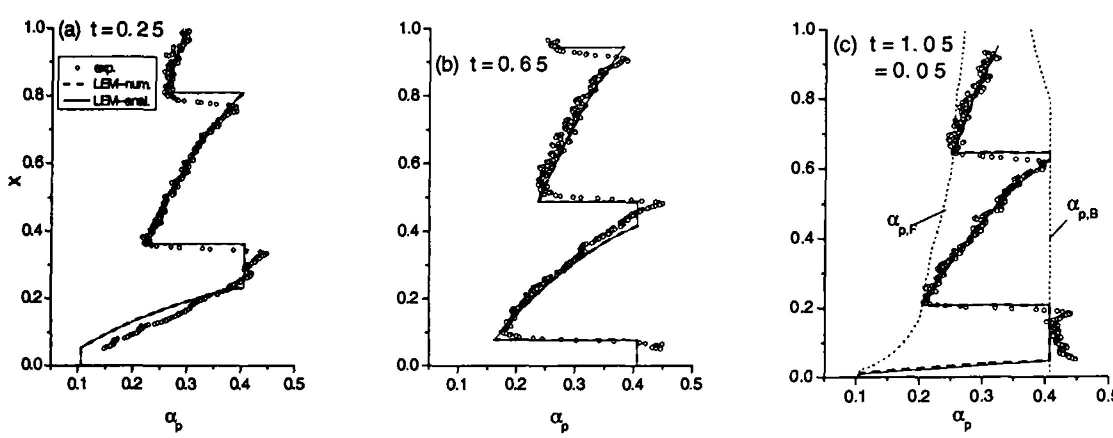


Fig.1 Comparison of the analytic solution of concentration distribution with experimental and numerical results

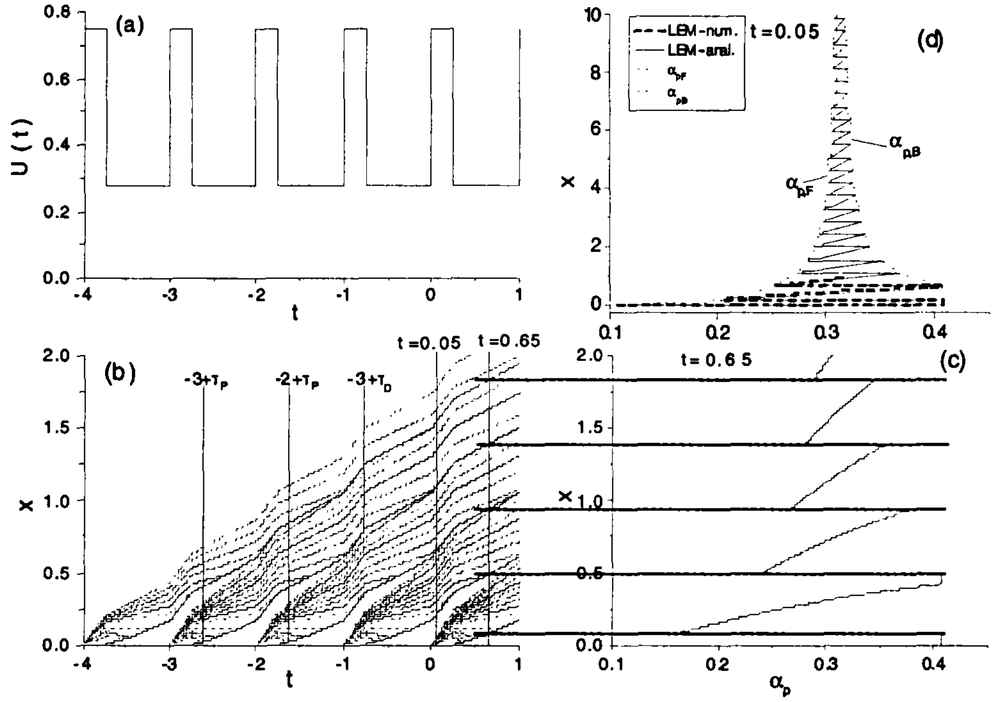


Fig.2 Wave hierarchies in a fluidized bed with pulsing inlet flow

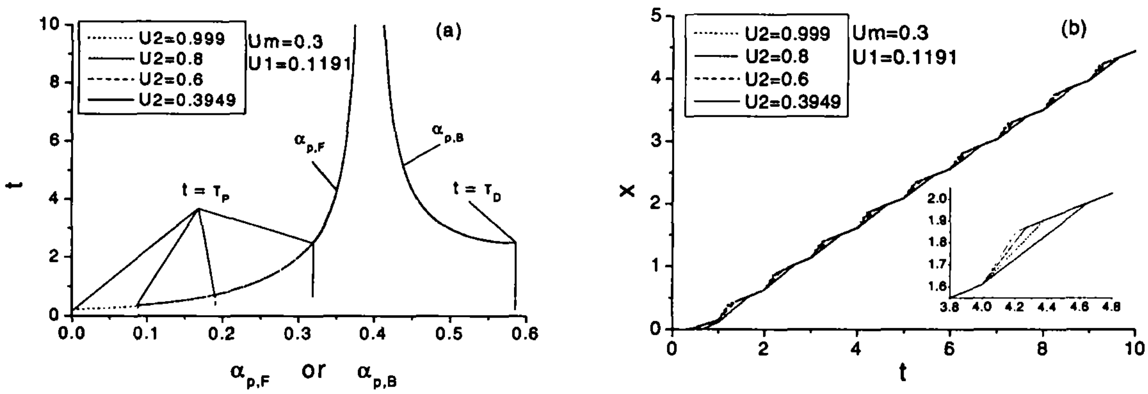


Fig.3 Trace of concentration shockwave $x_s(t)$ and its attenuation law (I)

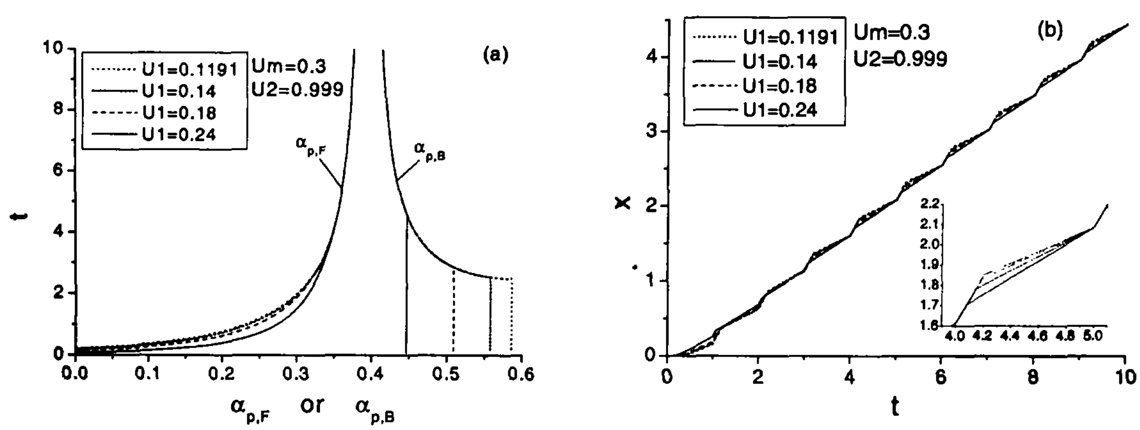


Fig.4 Trace of concentration shockwave $x_s(t)$ and its attenuation law (II)

## APPENDIX A

For $0 \le x^* \le x_S^*(t+1)$ we have

$$\alpha_f(t,x^*) = \begin{cases} \left.\begin{array}{ll} \alpha_{f2} & (0 \le x^* < \psi_1^*(t,\alpha_{f2})) \\ f_1^*(t,x^*) & (\psi_1^*(t,\alpha_{f2}) \le x^* \le \mu_1) \end{array}\right\} \text{if } t < T_2 \\ \left.\begin{array}{l} \left.\begin{array}{ll} \alpha_{f1} & (0 \le x^* \le x_S^*(t)) \\ \alpha_{f2} & (x_S^*(t) \le x^* < \psi_1^*(t,\alpha_{f2})) \\ f_1^*(t,x^*) & (\psi_1^*(t,\alpha_{f2}) \le x^* \le \mu_1) \end{array}\right\} \text{if } x_S^*(t) < \psi_1^*(t,\alpha_{f2}) \\ \left.\begin{array}{ll} \alpha_{f1} & (0 \le x^* \le x_S^*(t)) \\ f_1^*(t,x^*) & (x_S^*(t) \le x^* \le \mu_1) \end{array}\right\} \text{if } x_S^*(t) \ge \psi_1^*(t,\alpha_{f2}) \end{array}\right\} \text{if } t \ge T_2 \quad (0 \le x^* \le \mu_1) \\ \alpha_{f1} \qquad (\mu_1 < x^* \le x_S^*(t+1)) \end{cases}$$

where:

$$x^* = x - x_R$$

$$\mu_1 = \begin{cases} \psi_1^*(t,\alpha_{f1}) & \text{if } \psi_1^*(t,\alpha_{f1}) \le x_S^*(t_1) \\ x_S^*(t+1) & \text{if } \psi_1^*(t,\alpha_{f1}) > x_S^*(t+1) \end{cases}$$

$$\psi_1^*(t,\alpha_f) = (U_2 - U_m)T_2 + \{U_m + [n\alpha_f^{n-1} - (n+1)\alpha_f^n]\}t \quad \text{for} \begin{cases} \alpha_{f2} \ge \alpha_f \ge \alpha_{f,B}(t+1) & \text{if } t < T_2 \\ \alpha_{f,F}(t) \ge \alpha_f \ge \alpha_{f,B}(t+1) & \text{if } t \ge T_2 \end{cases}$$

and $f_1^*(t,x^*)$ is a inverse function of $\psi_1^*(t,\alpha_f)$.

For $x_S^*(t_N) \le x \le x_S^*(t_{N+1})$ and $N \ge 1$ we have

$$\alpha_f(t,x^*) = \begin{cases} \alpha_{f2} & (x_S^*(t_N) \le x^* < \nu_N) \\ f_{N+1}^*(t,x^*) & (\nu_N \le x^* \le \mu_{N+1}) \\ \alpha_{f1} & (\mu_{N+1} < x^* \le x_S^*(t_{N+1})) \end{cases}$$

where

$$t_N = t + N$$

$$x_S^*(t_N) = x_S^*(t+N) = x_S(t+N) - x_R(t)$$

$$\nu_N = \begin{cases} \psi_{N+1}^*(t,\alpha_{f2}) & \text{if } x_S^*(t_N) \le \psi_{N+1}^*(t,\alpha_{f2}) \\ x_S^*(t_N) & \text{if } x_S^*(t_N) > \psi_{N+1}^*(t,\alpha_{f2}) \end{cases}$$

$$\mu_{N+1} = \begin{cases} \psi_{N+1}^*(t,\alpha_{f1}) & \text{if } \psi_{N+1}^*(t,\alpha_{f1}) \le x_S^*(t_{N+1}) \\ x_S^*(t_{N+1}) & \text{if } \psi_{N+1}^*(t,\alpha_{f1}) > x_S^*(t_{N+1}) \end{cases}$$

$\psi_{N+1}^*(t,\alpha_f) = (U_2 - U_m)T_2 + \{U_m + [n\alpha_f^{n-1} - (n+1)\alpha_f^n]\}[t+N]$ for $\alpha_{f,F}(t_N) \ge \alpha_f \ge \alpha_{f,B}(t_{N+1})$ and $f_N^*(t,x^*)$ is a inverse function of $\psi_N^*(t,\alpha_f)$.