Numerical Simulation of Shock-Interface Interaction With Modified Compact Scheme

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Abstract. A modified fourth order accurate compact scheme with group velocity control (GVC) is developed. The scheme is high order accurate with less stencil, simple, less computer consuming, and can capture the shocks with high cross pressure ratio and the contact discontinuity with high density ratio. The developed method is used to simulate Richtmyer-Meshkov (R-M) instability problem driven by a cylindrical shock. The behavior of R-M instability is studied.

1 Introduction

For solving shock-interface interaction with high cross density ratio and pressure ratio it is required that the numerical method can capture both shock and contact discontinuity well. There have been a lot of activities geared towards constructing efficient schemes with high resolution of the shock. These include TVD and ENO type schemes. Development of the shock capturing methods of TVD and ENO types is mainly from the viewpoint of mathematics. In the most existing TVD and ENO type schemes the physical reason of oscillation production is not considered directly. In [2] the relevance of group velocity in numerical solution to the behavior of the solution of finite difference schemes was considered. Schemes were divided into three groups with the group velocity of wave packets: slower (SLW), faster (FST) and mixed (MXD). For the SLW schemes the numerical wave packets are propagated slower than the physical ones, and the oscillations in numerical solutions may appear behind the shock. For the FST schemes the numerical wave packets are propagated faster than the physical ones and the oscillations may appear in front of the shocks. For the MXD schemes in some range of low and middle wave numbers the schemes are FST, but are SLW in the range of high wave numbers. In order to improve the numerical solutions the method of group velocity control (GVC) is presented in [2]. The basic idea of the GVC scheme is that the scheme used must be reconstructed so that it is FST/MXD behind the shock, and is SLW in front of the shock. According to the basic idea of the GVC method a modified fourth order accurate compact scheme is developed. The scheme is used to simulate Richtmyer-Meshkov (R-M) instability problem. The numerical results for interaction between the cylindrical shock and the cylindrical interface with small disturbance are presented. The behavior of R-M instability is studied.

S. W. Armfield et al. (eds.), *Computational Fluid Dynamics 2002*

2 Method Development

2.1 Scheme description

Consider the following model equation and its semi-discrete approximation

$$\frac{\partial u}{\partial t}+\frac{\partial f}{\partial x}=0, f=cu, c=const.>0 \tag{1}$$

$$\frac{\partial u_j}{\partial t}+\frac{F_j}{\Delta x}=0 \tag{2}$$

where $F_j/\Delta x$ is an approximation of $\partial f/\partial x$ and F_j is obtained from

$$\frac{1}{6}F_{j+1}+\frac{2}{3}F_j+\frac{1}{6}F_{j-1}+H_{j+1/2}-H_{j-1/2}=h_{j+1/2}-h_{j-1/2} \tag{3}$$

$$H_{j+1/2}=(\varepsilon_2-\varepsilon_4)_{j+1/2}(F_{j+1}+F_j)$$

$$h_{j+1/2}=(f_j+f_{j+1})/2-2\varepsilon_{4,j+1/2}(f_{j+1}-f_j) \qquad \varepsilon_{2,j+1/2}=\left[g_{j+1/2}(\rho)\right]^3$$

$$\varepsilon_{4,j+1/2}=\left[1+\gamma_0 S_{j+1/2}(\rho)\right]g_{j+1/2}(\rho), \quad S_{j+1/2}(\rho)=\text{sign}\left[\partial\rho/\partial x, \partial^2\rho/\partial x^2\right]$$

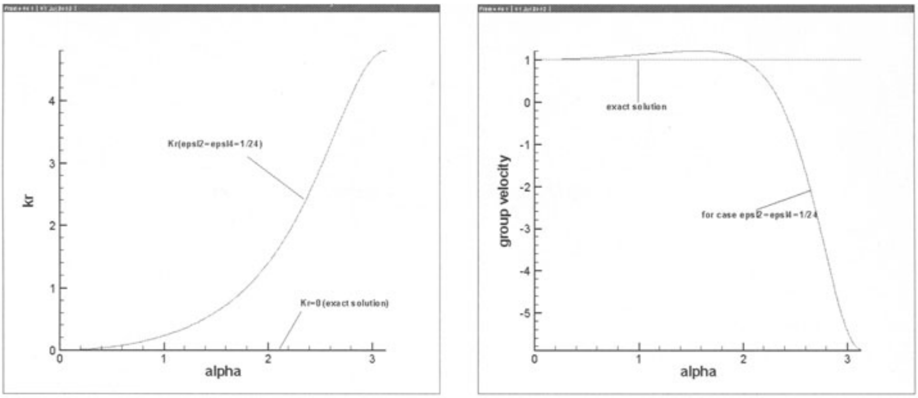
$$g_{j+1/2}(\rho)=\left[g_j(\rho)+g_{j+1}(\rho)\right]/2$$

$$g_j(\rho)=\left[|\rho_{j+1}-2\rho_j+\rho_{j-1}|/(\rho_{j+1}+2\rho_j+\rho_{j-1}\right]^{1/2}$$

The function $g(\rho)$ is used to control the accuracy and dissipativity, $S(\rho)$ is used to control the group velocity. The 3-stage TVD R-K method is used for time discretization. Here not any limiter and minmod function are used.

2.2 Accuracy and behavior analysis of numerical solution

According to the analysis in Ref.[2] for improvement of shock resolution the scheme should be constructed so that it is SLW in front of the shock and FST or MXD behind the shock. Fourier analysis shows that scheme (3) is dissipative for $\varepsilon_2>0$ and $\varepsilon_4>0$, SLW for small ε_2 and ε_4, and MXD for $\varepsilon_4>\varepsilon_2$ and $\varepsilon_4>\varepsilon_0$ (ε_0 is a positive constant). In fig.1 is given variation of $\partial k_i(\alpha)/\partial\alpha$ related to the group velocity of wave packet and $k_r(\alpha)$ related to the numerical dissipativity as function of α. The accuracy of the scheme is $\mathrm{O}(\Delta x\varepsilon_2+\Delta x^3\varepsilon_4)$. $D(\alpha)$ is the group velocity of wave packet and $\alpha=k\Delta x$, k is wave number. It is obvious that the constructed scheme is fourth order accurate in the smooth region. Numerical experiments show that the scheme has high resolution of the shock with high cross pressure ratio and high density difference across the interface.



a) b)

Fig. 1. (a) Variation of $K_r(\alpha)$ for $\varepsilon_2 = \varepsilon_4 = \frac{1}{24}$. (b) Variation of $dk_i/d\alpha$

3 Numerical Experiments

3.1 Model problems

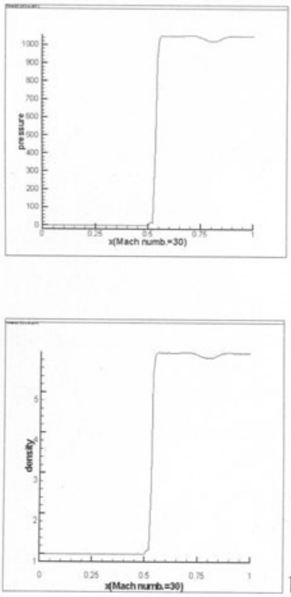
The scheme is used to solve the Euler equations for simulation of 1-D shock tube and Riemann problems. Numerical results of 1-D shock tube with steady shock for incoming Mach number M_a=30 are given in fig.2. Numerical results for 1-D Riemann problems at $t = 0.4$ with initial condition $(t = 0) : u_1 = u_2 = 0, p_1/p_2 = 10, \rho_1/\rho_2 = 800$ and 8 are given in fig.3. u, p and ρ are velocity, pressure and density, respectively, the lower index 1 and 2 related to parameters in two sides of discontinuity. From fig.3.2 it can be seen that numerical results agree well with exact solutions.

For 2-D Riemann problem the initial conditions are taken as shown in fig.4: lower index corresponds to the sub domain. The computational region is $-0.5 \le x \le 0.5, -0.5 \le y \le 0.5$ with grid points 400×400. Numerical results at $t = 0.6$ are given in fig.5 for density counters and in fig.6 for pressure counters. From fig.5 and fig.6 it can be seen that the double Mach reflection is formed, and obtained results including contact discontinuity, position of the mushroom cap and fine structures agree well with results obtained in ref. [1]. It should be noted that in our results there is not nonphysical discontinuity produced due to nonphysical oscillations.

3.2 Richtmyer-Meshkov instability

The developed scheme with three stage R-K method is used to solve the 2-D N-S equations for simulation of Richtmyer-Meshkov(R-M) instability problem. A cylindrical shock collides a cylindrical interface with small sinusoid disturbances. The shock wave implodes from heavy fluid to light fluid. The initial interface is located as follows

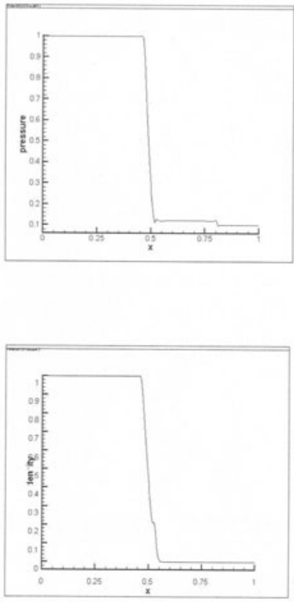
$$r = 1 + a_0 cos\,(n\theta)$$



a. pressure

b. density

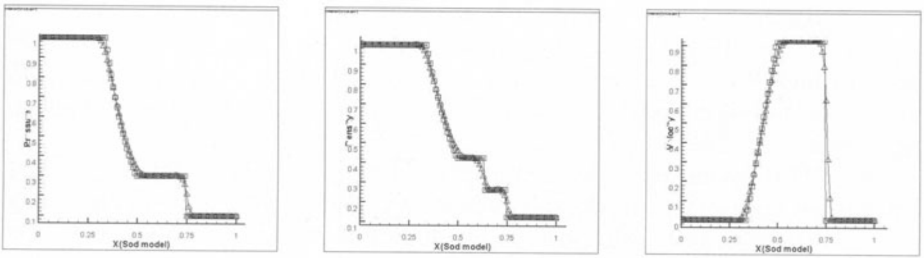
Fig. 2. Shock tube $Ma_\infty = 30$



a. pressure

b. density

Fig. 3.1. Results of Sod problem $\rho_1/\rho_2 = 800$



pressure density velocity

Fig. 3.2 Results of Sod model with $\rho_1/\rho_2 = 8$ and comparison with exact solution

where $r = \bar{r}/R_0$, $a = \bar{a}/R_0$ are dimensionless radius and perturbation amplitude, R_0 is the mean radius of the interface at $t = 0.0$ (the radius of the unperturbed interface). In present computation the initial dimensionless perturbation amplitude at the interface is a_0=0.033, and the Mach number of the shock is Ma=1.2. The ratio between outside and inside densities of the interface is ρ_1/ρ_2=5 and 20. The Reynolds number is Re=5000. For getting fine structures in computation non-uniform mesh grid system near the disturbed region is used. The density counters at different times are given in fig.7 for ρ_1/ρ_2=5, the pressure and the density distributions along a radial direction of 45 degrees at the corresponding times are also given. From figures it can be seen when the incident shock collides the interface at t =0.034 (where t is dimensionless time defined as

$t = \bar{t}\, R_0/U$, and U is the speed of the incident shock), the shock bifurcates into a transmitted shock which is moving radially inward, two reflected rarefaction waves which are moving radially outward. After bifurcation the phase inversion at the interface is produced due to interaction, and the interface is flattened. There are a series of wave interactions behind the transmitted shock. At $t = 1.0$ the phase inversion is completed, and the transmitted shock is reflected back from the center of the cylinder. The interface is re-shocked twice by the reflected back shock, second bifurcation is produced, and the interface is moving toward to the center. For second bifurcation the incoming shock from the center explodes from light fluid to heavy, so there is no phase inversion at the interface, and the reflected wave is reflected shock, which will move away from the center. In this case there are several re-shocks at the interface. From above results it can be seen that the numerical method used in present paper can capture shocks, reflected waves, interface and fine structures well.

References

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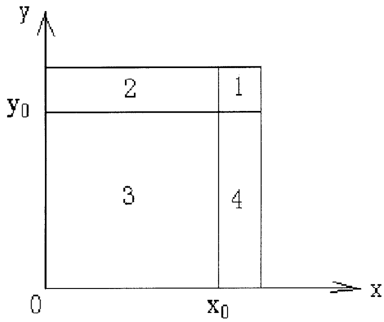

$$\rho_1 = 1.5, p_1 = 1.5, u_1 = v_1 = 0.0$$
$$\rho_2 = 0.5323, p_2 = 0.3, u_2 = 1.206, v_2 = 0.0$$
$$\rho_3 = 0.1379, p_3 = 0.029, u_3 = 1.206, v_3 = 1.206$$
$$\rho_4 = 0.5323, p_4 = 0.3, u_4 = 0.0, v_4 = 1.206$$

Fig. 4. Initial conditions ($x_0 = y_0 = 0.8$)