Analytic Models for Gain Saturation and Performance Calculation of the Flowing Chemical Oxygen-Iodine Laser

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ABSTRACT

A new gain saturation model of chemical oxygen-iodine laser (COIL) is deduced from the conservational equations of population number of upper and lower lasing levels. The present model is compared with both the Voigt profile function (VPF) model and its low-pressure limit model, and the differences between different models are pointed out. These differences are of use to the optimization of COILs' adjustable parameters.

Key Words: chemical oxygen-iodine laser (COIL) homogeneous broadening inhomogeneous broadening Voigt profile function (VPF) model performance analysis

1. INTRODUCTION

The experimental, theoretical and numerical researches of flowing chemical oxygen-iodine laser (COIL) have been paid extensive attention and developed rapidly in recent twenty years. The spectral line broadening (SLB) model is a basic factor for both the prediction of the COIL's performance and the optimization of adjustable parameters. By optimization of adjustable parameters, the output power was raised from a few watts of multi-mode to several thousand watts of near-diffraction limit during the development of flowing HF chemical laser [1]. There are also great disparities between the chemical efficiencies of supersonic COIL experiments [2]. Here SLB model is a key factor in explaining the disparities besides flow factors; an appropriate SLB model can play great role in the optimization of adjustable parameters, thus it is important to examine and develop different SLB models.

A well-known SLB model, which is called the Voigt profile function model [1, 3], is usually ultilized. The Voigt profile function is a convolution integral of the Lorentzian profile times Gausian profile with respect to the frequency. In the case that gas pressure is not high in the laser cavity, a low pressure limit expression of the Voigt profile function is also used [4,5], refer to fig.1. These two SLB models imply that all lasing particles can interact with monochromatic laser radiation and they can estimate reasonably the output power and extraction efficiency. But neither of them can predict correctly the inhomogeneous broadening effects and the spectral content which requires consideration of finite translational relaxation rates. However, it is rather difficult to solve simultaneously the Navier-Stokes (NS) equations and the conservational equations of the population number of lasing particles per unit volume and per unit frequency interval, i.e., the distribution function of velocity. Fortunately, in the operating condition of flowing COIL, both the translational relaxation rate k_T and the characteristic radiation rate k_v are larger than the characteristic flow rate u/L, thus we may introduce two small parameters u/Lk_T(<<1) and u/Lk_v(<<1), and seek a perturbational solution of the conservational equation of the population number of lasing particles.

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2. ANALYSIS

In order to illustrate the effects of inhomogeneous broadening on the performance of COIL with a mathematical model, it had better simplify some aspects of the COIL, such as the flow field condition and the chemical reaction scheme. As did in paper [4-5], the cavity flow field is assumed as a premixed one-dimensional flow, the iodine molecule have dissociated completely upstream the cavity and the chemical kinetic processes are greatly simplified, just the near-resonance exchange between iodine atom and oxygen molecule and the lasing radiation process are considered:

$$O_2(^1\Delta) + I \underset{k_r}{\overset{k_f}{\leftrightarrow}} O_2(^3\Sigma) + I^* \tag{1}$$

$$I^* + h\nu \rightarrow I + 2h\nu \tag{2}$$

where $O_2(^3\Sigma)$ and $O_2(^1\Delta)$ are the ground and excited level of oxygen, and I and I^* are the ground and excited level of iodine atom, respectively. $h\nu$ is the energy of a photo of frequency ν. The conservational equations of the velocity distribution functions of upper and lower energy levels are[3]

$$u\frac{\partial f_2}{\partial x} = rf_1 + k_T(f_2^0 - f_2) - k_p f_2 - \frac{h\nu B\varphi}{4\pi}(f_2 - \alpha f_1)f_\nu \tag{3}$$

$$u\frac{\partial f_1}{\partial x} = -rf_1 + k_T(f_1^0 - f_1) + k_p f_2 + \frac{h\nu B\varphi}{4\pi}(f_2 - \alpha f_1)f_\nu \tag{4}$$

where f_2 and f_1 are the velocity distribution function of upper and lower lasing levels, and f_2^0, f_1^0 are the velocity distribution function of upper and lower lasing levels of equilibrium Maxwellian state. $r = k_f n_\Delta$ and $k_p = k_r n_\Sigma$ are the pumping and quenching rate of the upper lasing level, respectively. B is the Einstein excited radiant coefficient, α is a constant related to level degeneracy, f_ν is the distribution function of photons, n_Δ、n_Σ are the population number of $O_2(^1\Delta)$、$O_2(^3\Sigma)$, respectively. φ is the Lorentz profile and u is the velocity of the flow in the streamwise direction. In the direction of lasing flux (in vertical to the flow direction), φ is

$$\varphi = \frac{\Delta\nu_N / 2\pi}{[\nu - \nu_0(1 + \nu_{Ty}/\nu_0)]^2 + (\Delta\nu_N/2)^2} \tag{5}$$

where ν_{Ty} is the thermal motion translation velocity of laser particles along the centerline of the resonator mirrors, ν_0 is the central frequency of the spectral line profile, and $\Delta\nu_N$ is the whole width of half height of homogeneous broadening line profile. Because r, k_T, k_p and the radiating characteristic rate $h\nu B\varphi f$ are all much larger than the characteristic transit time L/u, a double-parameter perturbational method is used[3]. Then we obtain

$$g = \int\frac{B\varphi}{4\pi}(f_2 - \alpha f_1)d\nu \cong \int\frac{B\varphi}{4\pi}[1 + \frac{B\varphi I}{4\pi}(1+\alpha)]^{-1}(f_2^0 - \alpha f_1^0)d\nu$$
$$\cong \frac{g_{on}\psi(\xi,\eta,\bar{I})}{1+\bar{I}} \tag{6}$$

$$\psi(\xi,\eta,\bar{I}) = \frac{\eta^2(1+\bar{I})}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{\eta^2(1+\bar{I}) + (\xi - t)^2} dt$$

where the broadening parameter $\eta = \Delta\nu_N / \Delta\nu_D \sqrt{\ln 2}$; frequency-shift parameter $\xi = 2(\nu - \nu_0)\sqrt{\ln 2} / \Delta\nu_D$, $t = 2(\nu - \nu_0)\sqrt{\ln 2} / \Delta\nu_D$ is a temporal integral variable. If the light frequency is coinciding with the central frequency of line profile ν_0, the equation (6) can be simplified as

$$g = K\sigma n \frac{\eta\sqrt{\pi}}{\sqrt{1+\bar{I}}} \cdot \exp[(1+\bar{I})\eta^2] \cdot \mathrm{erfc}(\eta\sqrt{1+\bar{I}}) \tag{7}$$

$$K = (k_f n_\Delta - \alpha k_r n_\Sigma)/(k_f n_\Delta + k_r n_\Sigma), \qquad \bar{I} = I / I_S \tag{8}$$

$$I = h\nu f_\nu, \quad I_S = \{6k_r[O_2][(K_e - 1)Y + 1] + 4k_T\} h\nu / 9\sigma \tag{9}$$

where $[O_2](= n_\Delta + n_\Sigma)$ and n are the total number density of oxygen molecule and iodine atom, respectively. $\sigma (= B/4\pi)$ is the stimulated radiative area and $Y(= n_\Delta /[O_2])$ is the yield of excited oxygen. I and I_S are the intensity and the saturation intensity of the present model, respectively, and $\bar{I}$ is the dimensionless intensity. By expanding the error function, the equation (7) can be approximately simplified as

$$g = K\sigma n/(1+\bar{I}) \ , \qquad \eta >> 1 \tag{10}$$

$$g = K\sigma n\eta\sqrt{\pi} / \sqrt{1+\bar{I}} \ , \qquad \eta << 1 \tag{11}$$

The gain-saturation relations (10) and (11) are in agreement with the well-known theory in the gas laser[6]. For the model of the Voigt profile function (VPF) the gain-saturation relation is [1]

$$g = K\sigma n \cdot \frac{\eta\sqrt{\pi}\,\mathrm{erfc}\,\eta \exp(\eta^2)}{1 + \bar{I}_\nu \eta\sqrt{\pi}\,\mathrm{erfc}\,\eta \exp(\eta^2)} \tag{12}$$

The gain-saturation relations corresponding to the formulae (10) and (11) are, respectively

$$g = K\sigma n/(1+\bar{I}_\nu) \ , \qquad \eta >> 1 \tag{13}$$

$$g = K\sigma n\eta\sqrt{\pi} /(1+\eta\sqrt{\pi}\bar{I}_h), \ \bar{I}_\nu = \bar{I}_h \ \text{when} \ \eta << 1 \tag{14}$$

where $\bar{I}_\nu = I_\nu / I_{S\nu}$, I_ν is the laser intensity of the VPF model, $I_{S\nu}$ the saturation intensity and $\bar{I}_\nu$ the dimensionless intensity, and $\bar{I}_\nu$ is substituted with $\bar{I}_h$ when $\eta << 1$. In addition, $I_{S\nu} = I_S$ if f_i (i=1,2) are very closed to f_i^0 (i=1,2). Obviously, the gain-saturation relation (14) of low-pressure limit model follows the same negative power law as that of the high pressure case of formulation (13).

3. RESULTS AND DISCUSSION

The power of the COIL is deduced by the same way as done in paper [4], and thus a comparison of results of different models are obtained. Fig.1. indicates the correlations between the Voigt profile function model and its low pressure limit model. The latter is the tangent of the former at $\eta = 0$, which simplifies considerably the treatment of problems with a good approximation if the pressure is not high. The correlations between $\bar{I}$ and $\bar{I}_v$ are shown in Fig.2, $\bar{I}$ and $\bar{I}_v$ are nearly the same when $\eta >> 1$, and smaller is the value of η, the larger is the difference between $\bar{I}$ and $\bar{I}_v$. Fig.3 shows the optical intensity decreases with the distance along flow direction, but the decrement predicted by present model is slower than the low pressure limit of VPF model, and longer extraction length is needed, this difference is important to the design of optical cavity. Fig. 4 is a comparison of the results obtained by using present model and low-pressure limit of VPF model with the RotoCOIL experimental data. It is shown that agreements are quite well.

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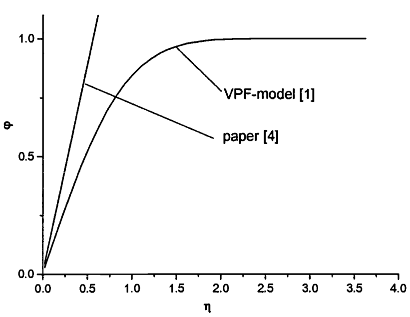

Fig.1. Voigt profile function[1] and its low pressure limit.

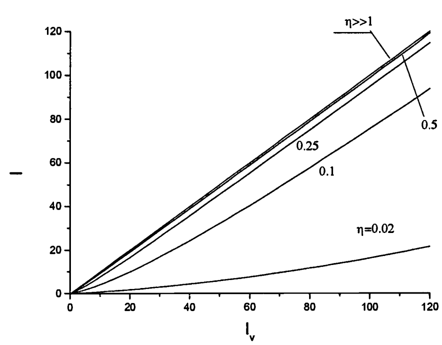

Fig.2. Relations between dimensionless intensity $\bar{I}$ of Eq. (7) and $\bar{I}_v$ of Eq. (12).

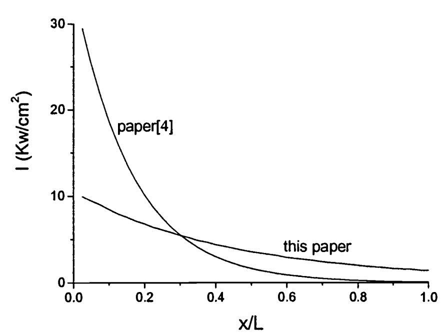

Fig.3. Variation of light intensity of this paper and paper [4] along flow direction.

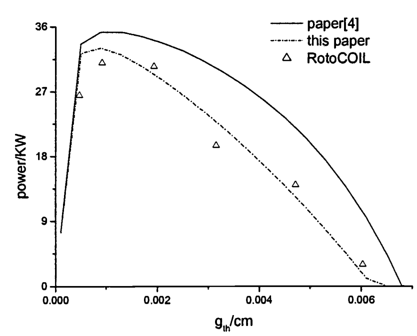

Fig.4. Comparison of extracted power of this paper, low pressure limit model in paper[4] and RotoCOIL experiment data.

REFERENCES

1. R. W. F. Gross and J. F. Bott, Handbook of Chemical Lasers, New York: Wiley, 1976.
2. J. F. Hon, D. N. Plummer, P. G. Crowell, J. Erkkila, G. D. Hager, C. A. Helms, and K. A. Truesdell, "A heuristic method for evaluating COIL performance," presented at the AIAA 25th Plasmadynamics and Laser Conf., Colorado Springs, CO, USA, June 20-23, 1994, paper 94-2422.
3. Z.Gao, Collisional and inhomogeneous broadening effects in gas-flow and chemical lasers—the theoretical models, ACTA physical Sinica (in Chinese), Vol.12, No.12, pp.1591-1602, Dec., 1981
4. G. D. Hager, C. A. Helms, K. A. Truesdell, D. Plummer, J. Erkkila, and P. Crowell, "A simplified analytic model for gain saturation and power extraction in the flowing chemical oxygen-iodine laser," IEEE Journal of Quantum Electronics, Vol. 32, No.9, pp. 1525-1536, 1996.
5. B. D. Barmashenko, and S. Rosenwaks, "Analysis of the optical extraction efficiency in gas-flow lasers with different types of resonator," Applied Optics, Vol.35, No.36, pp. 7091-7101, 1996.
6. Chen Yuqing, and Wang, Jinghuan, "Principles of the laser", The Zhejiang University Press, pp.299-300, 1992, Hangzhou(in Chinese).