REAL TIME ANALYSIS ON EXPERT SYSTEMS

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Abstract - In some comprehensive systems, as decisive loops, expert systems consist in the whole system, so much more studies on its reasoning time are necessary. The reasoning time in expert system relates to its reasoning mode, its knowledge base architecture, the performance of computer and the support languages. This paper takes the knowledge base architecture as an example to study the reasoning time. At last, temporally homogeneous Markov chain is used to model for the knowledge base. The time estimating mathematics model and the computation method are also given, which can provide proofs for the comprehensive system controlling and real time simulation.
Keywords:Expert System; Real Time; Markov Chain; Reasoning Time

INTRODUCTION

After an expert system is developed, it is necessary to test and evaluate it. Testing is used to identify if the reasoned results comply with the correct results in qualitative, evaluating is used to judge how the entire system performances are in quantitative; Evaluating is based on testing and follows the testing work. We are able to make use of the tested results to observe if the expert system is better or worse. Most important, it is the basis to modify and improve the expert system.

Evaluating an expert system is to see if the reasoned results are correct, if its interface is friendly, how its reasoning efficiency is and how much its developing expense is, in which the reasoning efficiency of the expert system depends on its reasoning time [1]. When the reasoning process is fast to complete, the results can be achieved within a given time, otherwise not, hence study on the reasoning time model is an efficient path to study the efficiency of the expert system.

In most expert systems, it is rare to consider reasoning time [2-4], for they work off line. But in the comprehensive or complex system [5], the expert system, acting as a decisive loop, runs synchronously with the entire system, thus it is necessary to consider the reasoning time. The reasoning time relates to the reasoning mode of the expert system, the structure of its knowledge base, the used computer performance and the support language, so this paper takes the structure of the knowledge base as an example to study its time estimating model. Because the structure of the knowledge base relates to the quantity and organization mode of the knowledge in the expert system, this paper will study the relation amongst the time-estimating model, the quantity of the knowledge and the alignment in the knowledge base.

1 TIME ESTIMATING MODEL

Knowledge in the knowledge base consists of facts and rules. Assuming that there are s pieces of knowledge in an expert system, which are denoted by the capital letters $K_i, i \in \mathbf{S}$, respectively, where the capital letter $\mathbf{S}$ is a integer subset, and $\mathbf{S} = \{1,2,\cdots,s\}$. The alignment sequence is $(K_1, K_2, \cdots, K_s)$, i is the order number.

The corresponding reasoning time from

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the first knowledge to itself is $\tau_1, \tau_2, \cdots, \tau_s$. Assuming that the initially used probability of each knowledge during the reasoning process is $(p_1, p_2, \cdots, p_s)$, respectively. The dependent relation amongst all pieces of knowledge can be denoted by finite state Markov chain, and the difference that the reasoning mode is affected by time is not generally considered, so the Markov chain is also temporally homogeneous. Its transition relation can be denoted by one-step transition matrix $\mathbf{P}$, and $\mathbf{P} \stackrel{\Delta}{=} \{p_{ij}$, as showed in (1). Where p_{ij} is the condition probability of the reasoning process from the i th knowledge to the j th knowledge.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1s} \\ p_{21} & p_{22} & \cdots & p_{2s} \\ & \cdots & \cdots & \\ p_{s1} & p_{s2} & \cdots & p_{ss} \end{bmatrix} \tag{1}$$

If $i < j$, the reasoning time from K_i to K_j is $(\tau_{K_j} - \tau_{K_i})$; if $i > j$, the reasoning time from K_i to K_j is $(\tau_{K_s} + \tau_{K_j} - \tau_{K_i})$. The universal formulation is:

$$\tau_{ij} = \begin{cases} \tau_{K_s} + \tau_{K_j} - \tau_{K_i} & i > j \\ \tau_{K_j} - \tau_{K_i} & i < j \end{cases} \tag{2}$$

The time-estimating model about the reasoning time is further analyzed as follows.

If the reasoning step $\mu = 1$, then the mean reasoning time $\overline{T}_1$ is

$$\overline{T}_1 = \sum_{i=1}^{s} p_i \cdot \tau_{K_i} \tag{3}$$

If the reasoning step $\mu = 2$, the mean reasoning time $\overline{T}_2$ is

$$\begin{aligned}
\overline{T}_2 &= \sum_{i=1}^{s}(p_i \cdot \tau_{K_i} + \sum_{j=1}^{s} p_i \cdot p_{ij} \cdot \tau_{ij}) \\
&= \sum_{i=1}^{s} p_i \cdot \tau_{K_i} + \sum_{i=1}^{s}\sum_{j=1}^{s} p_i \cdot p_{ij} \cdot \tau_{ij} \\
&= \sum_{i=1}^{s} p_i \cdot \tau_{K_i} + \sum_{i=1}^{s}\sum_{j=1}^{i} p_i \cdot p_{ij} \cdot (\tau_{K_s} + \tau_{K_j} - \tau_{K_i}) + \sum_{i=1}^{s}\sum_{j=i+1}^{s} p_i \cdot p_{ij} \cdot (\tau_{K_j} - \tau_{K_i}) \\
&= \overline{T}_1 + \sum_{i=1}^{s}\sum_{j=1}^{i} p_i \cdot p_{ij} \cdot \tau_{K_s} + \sum_{i=1}^{s}\sum_{j=1}^{s} p_i \cdot p_{ij} \cdot (\tau_{K_j} - \tau_{K_i})
\end{aligned} \tag{4}$$

If the reasoning step $\mu = 3$, the mean reasoning time $\overline{T}_3$ is

$$\begin{aligned}
\overline{T}_3 &= \sum_{i=1}^{s} (p_i \cdot \tau_{K_i} + \sum_{j=1}^{s} (p_i \cdot p_{ij} \cdot \tau_{ij} + \sum_{k=1}^{s} p_i \cdot p_{ij} \cdot p_{jk} \cdot \tau_{jk})) \\
&= \overline{T}_2 + \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j} p_i \cdot p_{ij} \cdot p_{jk} \cdot (\tau_{K_s} + \tau_{K_k} - \tau_{K_j}) \\
&+ \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=j+1}^{s} p_i \cdot p_{ij} \cdot p_{jk} \cdot (\tau_{K_k} - \tau_{K_j}) \\
&= \overline{T}_2 + \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{i} p_i \cdot p_{ij} \cdot p_{jk} \cdot \tau_{K_s} \\
&+ \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s} p_i \cdot p_{ij} \cdot p_{jk} \cdot (\tau_{K_k} - \tau_{K_j})
\end{aligned} \tag{5}$$

Similarly, if the reasoning step $\mu = n+2$, then the mean reasoning time $\overline{T}_{n+2}$ is

$$\begin{aligned}
\overline{T}_{n+2} &= \overline{T}_{n+1} \\
&+ \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j} p_i \cdot p_{ij}^{(n)} \cdot p_{jk} \cdot (\tau_{K_s} + \tau_{K_k} - \tau_{K_j}) \\
&+ \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=j+1}^{s} p_i \cdot p_{ij}^{(n)} \cdot p_{jk} \cdot (\tau_{K_k} - \tau_{K_j}) \\
&= \overline{T}_{n+1} + \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j} p_i \cdot p_{ij}^{(n)} \cdot p_{jk} \cdot \tau_{K_s} \\
&+ \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s} p_i \cdot p_{ij}^{(n)} \cdot p_{jk} \cdot (\tau_{K_k} - \tau_{K_j})
\end{aligned} \tag{6}$$

Where $p_{ij}^{(n)}$ is the reasoning conditional probability from the i th knowledge to the j th knowledge past by n step.

Next (7) can be inferred

$$\bar{T}_{m+2}=\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j}p_i\cdot(1+p_{ij}^{(1)}+p_{ij}^{(2)}+\cdots+p_{ij}^{(m)})\cdot p_{jk}\cdot\tau_{K_s}$$
$$+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s}p_i\cdot(1+p_{ij}^{(1)}+p_{ij}^{(2)}+\cdots+p_{ij}^{(m)})\cdot p_{jk}\cdot(\tau_{K_k}-\tau_{K_j})$$
$$+\sum_{i=1}^{s}p_i\cdot\tau_{K_i}+\sum_{i=1}^{s}\sum_{j=1}^{i}p_i\cdot p_{ij}\cdot\tau_{K_s}+\sum_{i=1}^{s}\sum_{j=1}^{s}p_i\cdot p_{ij}\cdot(\tau_{K_j}-\tau_{K_i}) \tag{7}$$

So:

$$\bar{T}_{m+2}=\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j}\sum_{l=1}^{n}p_i\cdot p_{ij}^{(l)}\cdot p_{jk}\cdot\tau_{K_s}$$
$$+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s}\sum_{l=1}^{n}p_i\cdot p_{ij}^{(l)}\cdot p_{jk}\cdot(\tau_{K_k}-\tau_{K_j})$$
$$+\sum_{i=1}^{s}p_i\cdot\tau_{K_i}+\sum_{i=1}^{s}\sum_{j=1}^{i}p_i\cdot p_{ij}\cdot\tau_{K_s}+\sum_{i=1}^{s}\sum_{j=1}^{s}p_i\cdot p_{ij}\cdot(\tau_{K_j}-\tau_{K_i}) \tag{8}$$

(8) can be proved to be universal as followings.

Proof

The method of proof by induction can be used for (8).

If $n=1$, then:

$$\bar{T}_3=\sum_{i=1}^{s}p_i\cdot\tau_{K_i}+\sum_{i=1}^{s}\sum_{j=1}^{i}p_i\cdot p_{ij}\cdot\tau_{K_i}+\sum_{i=1}^{s}\sum_{j=1}^{s}p_i\cdot p_{ij}\cdot(\tau_{K_j}-\tau_{K_i})$$
$$+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j}p_i\cdot p_{ij}\cdot p_{jk}\cdot\tau_{K_s}+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s}p_i\cdot p_{ij}\cdot p_{jk}\cdot(\tau_{K_k}-\tau_{K_j} \tag{9}$$

It satisfies the requirement of (5).

Assume $n=m$, it can also satisfy the requirement of (8).

$$\bar{T}_{m+2}=\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j}\sum_{l=1}^{m}p_i\cdot p_{ij}^{(l)}\cdot p_{jk}\cdot\tau_{K_s}$$
$$+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s}\sum_{l=1}^{m}p_i\cdot p_{ij}^{(l)}\cdot p_{jk}\cdot(\tau_{K_k}-\tau_{K_j})$$
$$+\sum_{i=1}^{s}p_i\cdot\tau_{K_i}+\sum_{i=1}^{s}\sum_{j=1}^{s}p_i\cdot p_{ij}\cdot\tau_{K_j} \tag{10}$$

Next, if $n=m+1$, then

$$\bar{T}_{m+3}=\bar{T}_{m+2}+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s}p_i\cdot p_{ij}^{(m+1)}\cdot p_{jk}\cdot\tau_{jk} \tag{11}$$

So:

$$\bar{T}_{m+3}=\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j}\sum_{l=1}^{m}p_i\cdot p_{ij}^{(l)}\cdot p_{jk}\cdot\tau_{K_s}$$
$$+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s}\sum_{l=1}^{m}p_i\cdot p_{ij}^{(l)}\cdot p_{jk}\cdot(\tau_{K_k}-\tau_{K_j})$$
$$+\sum_{i=1}^{s}p_i\cdot\tau_{K_i}+\sum_{i=1}^{s}\sum_{j=1}^{i}p_i\cdot p_{ij}\cdot\tau_{K_s}+\sum_{i=1}^{s}\sum_{j=1}^{s}p_i\cdot p_{ij}\cdot(\tau_{K_j}-\tau_{K_i})$$
$$+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s}p_i\cdot p_{ij}^{(m+1)}\cdot p_{jk}\cdot\tau_{jk} \tag{12}$$

So:

$$\bar{T}_{m+3}=\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j}\sum_{l=1}^{m}p_i\cdot p_{ij}^{(l)}\cdot p_{jk}\cdot\tau_{K_s}$$
$$+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s}\sum_{l=1}^{m}p_i\cdot p_{ij}^{(l)}\cdot p_{jk}\cdot(\tau_{K_k}-\tau_{K_j})$$
$$+\sum_{i=1}^{s}p_i\cdot\tau_{K_i}+\sum_{i=1}^{s}\sum_{j=1}^{i}p_i\cdot p_{ij}\cdot\tau_{K_s}+\sum_{i=1}^{s}\sum_{j=1}^{s}p_i\cdot p_{ij}\cdot(\tau_{K_j}-\tau_{K_i})$$
$$+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j}p_i\cdot p_{ij}^{(m+1)}\cdot p_{jk}\cdot(\tau_{K_s}-\tau_{K_j}+\tau_{K_k})$$
$$+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=j+1}^{s}p_i\cdot p_{ij}^{(m+1)}\cdot p_{jk}\cdot(\tau_{K_k}-\tau_{K_j}) \tag{13}$$

The following is given:

$$\overline{T}_{m+3} = \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j}\sum_{l=0}^{m+1} p_i \cdot p_{ij}^{(l)} \cdot p_{jk} \cdot \tau_{K_s}$$

$$+\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s}\sum_{l=0}^{m+1} p_i \cdot p_{ij}^{(l)} \cdot p_{jk} \cdot (\tau_{K_k} - \tau_{K_j})$$

$$+\sum_{i=1}^{s} p_i \cdot \tau_{K_i} + \sum_{i=1}^{s}\sum_{j=1}^{i} p_i \cdot p_{ij} \cdot \tau_{K_s}$$

$$+\sum_{i=1}^{s}\sum_{j=1}^{s} p_i \cdot p_{ij} \cdot (\tau_{K_j} - \tau_{K_i}) \tag{14}$$

Thus (8) is proved.

It can be seen from (8), the reasoning process increases by one step, the additional time $\Delta\overline{T}_n$ increases by the following formula (15).

$$\Delta\overline{T}_{n+2} = \overline{T}_{n+2} - \overline{T}_{n+1}$$

$$= \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j} p_i \cdot p_{ij}^{(n)} \cdot p_{jk} \cdot \tau_{K_s}$$

$$+ \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s} p_i \cdot p_{ij}^{(n)} \cdot p_{jk} \cdot (\tau_{K_k} - \tau_{K_j}) \tag{15}$$

Where $\Delta\overline{T}_{n+2}$ is the additional time during the $(n+2)$ th reasoning process, and it can be analyzed from (15). Generally, $p_{ij}^{(n)}$ varies with n, thus the additional time $\Delta\overline{T}_{n+2}$ is variable.

Assuming that the Markov chain has its stationary distribution , and it is $[\pi_1 \ \ \pi_2 \ \ \cdots \ \ \pi_s]$, thus if the reasoning step $\mu \to +\infty$:

$$\lim_{n\to+\infty} \Delta\overline{T}_{n+2} = \lim_{n\to+\infty} \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{j} p_i \cdot p_{ij}^{(n)} \cdot p_{jk} \cdot \tau_{K_s}$$

$$+ \lim_{n\to+\infty} \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s} p_i \cdot p_{ij}^{(n)} \cdot p_{jk} \cdot (\tau_{K_k} - \tau_{K_j}) \tag{16}$$

So:

$$\lim_{n\to+\infty} \Delta\overline{T}_{n+2} = \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{j=1}^{i} p_i \cdot \pi_j \cdot p_{jk} \cdot \tau_{K_s}$$

$$+ \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s} p_i \cdot \pi_j \cdot p_{jk} \cdot (\tau_{K_k} - \tau_{K_j})$$

$$= \sum_{j=1}^{s}\sum_{k=1}^{i} \pi_j \cdot p_{jk} \cdot \tau_{K_s} + \sum_{j=1}^{s}\sum_{k=1}^{s} \pi_j \cdot p_{jk} \cdot (\tau_{K_k} - \tau_{K_j}) \tag{17}$$

It can be seen from (17), if the reasoning step n is much bigger number, which makes the Markov chain realizes stationary distribution, the additional time of each step is identical.

2 CONCLUSION

The above time-estimating model can describe how much time expert systems execution consume in quantity and it is useful the real-time simulation in complex system [8].

REFERENCES

[1] Guan Shouping. Real Time Expert System Technology. Changsha(15)Computer Engineering and Science, Vol.18, No.4, 1996.10. (in Chinese).

[2] Chen Jiafa. Prolog Program Design for Expert System. Guangzhou: Publication House of Zhongshan University, 1992.12 (in Chinese).

[3] Wu Xindong. Expert System Design. Hefei: Publication House of China University of Science and Technology, 1990.04 (in Chinese).

[4] Coombs, M.J.(ed.), Developments of Expert Systems, New York, Academic Press, 1984

[5] Wen Chuanyuan. Comprehensive System and its Modeling and Simulation. In: CASS. Proceedings of BICSC'92. Beijing: CASS, 1992.

[6] Fan Jiakun. Stochastic Process. Zhenzhou: Publication House of Henan University, 1988. (in Chinese).

[7] He Shengwu. Introduction of Stochastic Process. Shanghai: Publication House of Huadong Teacher College, 1986. (in Chinese).

[8] MingRao and Tsung-Shan Jing, Integrated Intelligent Simulation Environment, Simulation, PP291-295, June, 1990