Non-Fourier effect and inertia effect analysis of a strip with an induced crack under thermal shock loading

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ABSTRACT

In this paper, the transient temperature fields and the dynamic stress intensity factors of a thermo-elastic strip containing an inner crack parallel to the heated surface under thermal shock are studied. The Biot number of the crack gap, hyperbolic heat conduction theory and equation of motion are considered to investigate the behavior of the temperature fields around the crack and the stress intensity factors. Fourier transform and Laplace transform are used to reduce this mixed boundary value problem. Numerical methods are used to solved the singular integrate equations. Finally, the numerical results are presented illustrating the influence of Biot number, non-Fourier effect and inertia effect on temperature field and stress intensity factors. It is found that the Biot number strongly affect the uniformity of the temperature field and the magnitude of the stress intensity factors. The stress intensity factors have higher amplitude and an oscillating feature comparing to those obtained under conventional Fourier thermal conduction condition and quasi-static hypothesis, which can help to better understand the crack behaviors of advanced materials under thermal impact loading.

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1. Introduction

Engineering thermo-mechanical structure components are widely used in server thermal loading such as extremely powerful laser impact or thermal barrier coatings (TBCs) used in gas-turbine engines [1,2]. When materials function in the presence of thermal gradients and high heat flux, they are susceptible to submit delamination and fracture. Many articles have studied the failure of such materials after suffering thermal loading [1–5]. In general, authors pay more attention on the cracks parallel to the heating surface (we call it CPHS in short), which play a key role in the spallation failure of materials. Most existing studies about CPHS are based on thermal expansion mismatch between the two different types of the materials or microgeometry defects of materials [2,3,6], both of which rely on the presence of interface. However, CPHS are found not only near the interface of coating materials but also in the coatings far from the interface and non-layered materials [1,7]. There is no doubt that we need an in-depth study to find the source and magnitude of the force which leading to the CPHS.

Classical thermos-elastic stress analysis in a homogeneous body cannot provide tensile stress leading to fractures parallel to the heating surfaces. Therefore, additional driving forces should be considered to explain this class of failures. These driving forces may include the non-Fourier thermal conduction effect [8–10] or the inertia force effect [11–13]. Moreover, the

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Nomenclature

a	thermal diffusivity
$\mathbf{A}_{i,j}$	function defined in Eq. (A2)
Bi	Biot number
B_j	function defined in Eq. (39)
C_p	specific heat capacity
C_i	unknown coefficients
D_i	unknown coefficients
e	mathematical constant
E	Young's modulus
f_1, f_2	dislocation function
h	heat transfer coefficient
i	imaginary unit
i, j	subscript
k	thermal conductivity
$k()$	function defined in Eq. (24)
K_{ij}	function defined in Eq. (A4)
l_a, l_b	distance between the crack and the boundary
La, Lb	dimensionless distance between the crack and the boundary
m	variable defined in Eq. (18)
m_0	variable defined in Eq. (22)
p	Laplace variable
$\vec{q}$	heat flux vector
r	crack length
t	time
T	temperature
T_0	initial temperature
T_∞	temperature of the surrounding external environment of the strip
u, v	displacement
$\bar{u}, \bar{v}$	displacement in Laplace space
$\tilde{u}, \tilde{v}$	displacement in Laplace–Fourier space
V	elastic wave velocity
W_i	function defined in Eq. (A4)
x, y	coordinate
X, Y	dimensionless coordinate
∇	Laplace operator

Greek symbols

α	coefficient of linear thermal expansion
β_i	function defined in Eq. (34)
γ_j	function defined in Eq. (A2)
$\delta()$	Dirac delta function
ε	strain
η	integral variable
θ	dimensionless temperature
λ_i	function defined in Eq. (37)
ν	Poisson's ratio
ξ	Fourier variable
π	circumference ratio
ρ	density
σ_x, σ_y	normal stress
τ	dimensionless time
τ_0	thermal relaxation time
τ_1	Fourier factor
τ_2	inertia factor
τ_{xy}	shear stress
$\varphi()$	dislocation density function
$\Phi()$	function defined in Eq. 25
ω_i	function defined in Eq. (A1)

initial heterogeneities such as micro-cracks or inclusions can also develop into macro-cracks under thermal shock [2,3]. In fact, these micro-cracks may act as heat conduction resistance and create temperature gradient around the heterogeneities, favorable to the crack initiation and growth in the material [14–16]. However, the interaction of all these factors leads to a very complex initial-boundary value problem and has not been thoroughly investigated in the literature so far [1].

The main objective of the present work is to perform a theoretical-numerical analysis that allows the assessment of all these driving forces in the thermal stress induced fracture process, especially in the propagation of a CPHS. To this end, we focus our attention on stress analysis in a cracked bi-dimensional strip under a thermal load. The Laplace and Fourier transforms were used as principal theoretical tool in this analysis. First of all, for the boundary conditions, heat transfer coefficient is introduced to the crack surfaces in order to modulate the heat transfer across the crack. A dimensionless parameter, the Biot number is therefore introduced to describe the influence of the heat insulation of the crack. Secondly, a higher-order time-derivative term specifying the non-Fourier effect is added to the Fourier heat conduction equation. Consequently, a hyperbolic heat conduction equation should be resolved instead of the classical parabolic equations. Finally, for the stress field, equations of motion are considered by introducing the inertia effect instead of the equilibrium equations. The resolution of such an initial-boundary value problem permits us to assess the role of each fracture driving force and their interaction. Then the numerical results on the stress intensity factors of the crack are presented and discussed accordingly, followed by some brief concluding remarks.

2. Statement of the problem and basic equations

We consider a thin plate under transient thermal loading which contains a straight crack of length $2r$ parallel to the free surface, as shown in Fig. 1. A Cartesian coordinate system is established with the x -axis coincided with the crack plane and the origin at the crack center. We assume that the material is homogeneous, isotropic and linearly elastic. In this work, only thermal loading is considered in evaluating the temperature and stress fields. The initial temperature is assumed to be uniform and equals to T_0 in the plate. Then the free surface $y = -l_a$ is suddenly heated to a fixed temperature T_∞ , while the free surface $y = l_b$ stays unchanged, where l_a and l_b are strictly positive real. The heat transfer coefficients of the crack surface are assumed to be h . Additionally, the thermal-elastic coupling is neglected in this study.

In order to assess the influence of the non-Fourier and the dynamic effect of the thermal shock, we adopt the hyperbolic heat conduction equations in temperature field evaluation and introduce the inertia effect in stress field to the equilibrium equations.

In the following, we present the detailed mathematical formulation for the resolution of the above-defined thermal shock problem.

2.1. Heat conductions

The famous Fourier's law of heat conduction presents a linear relationship between the heat flux ($\vec{q}$) through a material and the gradient of temperature (T), whose differential form is $\vec{q} = -k\nabla T$, where k is the material thermal conductivity and ∇ is the gradient operator. In principle, however, the Fourier's law leads to an unphysical infinite heat propagation speed within a continuum for transient heat conduction processes because of its parabolic characteristics. To overcome this contradiction, a hyperbolic model, named the Cattaneo-Vernotte model, has been proposed by introducing a time-dependent term into the Fourier equation [17,18]. The introduced time-derivative term describes a wave nature of heat propagation at a finite speed, which has been proved in both theory and experiments [10,18,19]. The governing equation of non-Fourier thermal conduction writes

$$a \cdot \nabla^2 T = \frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \quad (1)$$

Fig. 1. Model and coordinates.

where $a = \frac{k}{\rho C_p}$ stands for thermal diffusivity, t is the time, ρ and C_p are the mass density and the specific heat capacity, respectively, ∇ is the Laplace differential operator and τ_0 is the thermal relaxation time.

In 2D problems, we introduce the following dimensionless coordinates, time and temperature

$$(X, Y) = \frac{(x, y)}{r}, \quad \tau = \frac{at}{r^2}, \quad \theta = \frac{T - T_0}{T_\infty - T_0} \quad (2)$$

where T_0 is the initial temperature of the plate and the governing Eq. (1) takes the following dimensionless forms

$$\nabla^2 \theta = \frac{\partial \theta}{\partial \tau} + \tau_1 \frac{\partial^2 \theta}{\partial \tau^2} \quad (3)$$

where τ_1 is the normalized thermal relaxation time defined as:

$$\tau_1 = \frac{a\tau_0}{r^2} \quad (4)$$

Here the gradient operator ∇ is related to the dimensionless coordinates X and Y .

Referring to Fig. 1, the hyperbolic heat Eq. (3) is subjected to the following boundary and initial conditions written in dimensionless forms

$$\left\{ \begin{array}{l} (a) : \theta(Y = -La) = 1 \\ (b) : \frac{\partial \theta}{\partial Y} \Big|_{Y=0^-} = \frac{\partial \theta}{\partial Y} \Big|_{Y=0^+} = Bi[\theta(0^+) - \theta(0^-)] \quad |X| \leq 1 \\ (c) : \frac{\partial \theta}{\partial Y} \Big|_{Y=0^-} = \frac{\partial \theta}{\partial Y} \Big|_{Y=0^+} \\ (d) : \theta \Big|_{Y=0^-} = \theta \Big|_{Y=0^+} \\ (e) : \theta(Y = La) = 0 \\ (f) : \theta \Big|_{\tau=0} = 0 \\ (g) : \frac{\partial \theta}{\partial \tau} \Big|_{\tau=0} = 0 \end{array} \right\} \quad |X| > 1 \quad (5)$$

where Bi is the Biot number, which gives a simple index of the ratio of the heat transfer resistances inside of and at the crack surface, is defined as:

$$Bi = \frac{hr}{k} = \frac{1/k}{1/hr} \quad (6)$$

with h being the heat transfer coefficient across the crack, which is the proportionality coefficient between the heat flux and the thermodynamic driving force for the flow of heat. The numerator and the denominator of the right part of Eq. (6) stand for the thermal resistance of the internal material and the thermal resistance of the crack surfaces, respectively.

2.2. Thermal-elastic field equations

In the following, we focus on attention in resolving the plane-stress thermal-elastic problem under the plane stress hypothesis

$$\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0 \quad (7)$$

The basic equations of plane thermal stress problems for nonhomogeneous isotropic elastic body are:

(1) Equations of motion including the inertia effect:

$$\sigma_{ij,j} = \rho \ddot{u}_i \quad (8)$$

(2) Strain-displacement relations:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (9)$$

(3) Thermal-elastic stress-strain relations:

$$\left\{ \begin{array}{l} \sigma_{xx} = \frac{E}{1-\nu^2} [\varepsilon_{xx} + \nu \varepsilon_{yy} - (1+\nu)\alpha(T - T_0)] \\ \sigma_{yy} = \frac{E}{1-\nu^2} [\nu \varepsilon_{xx} + \varepsilon_{yy} - (1+\nu)\alpha(T - T_0)] \\ \sigma_{xy} = \frac{E}{1+\nu} \varepsilon_{xy} \end{array} \right. \quad (10)$$

where E , ν and α are the Young modulus, the Poisson ratio and the coefficient of linear thermal expansion, respectively. The index i, j refers to the coordinate x, y . The index i, j refers to the coordinate x, y . It is to notice that in this paper, the italic letter “ i ” is used as a subscript. It should not be confounded with the regular script “ i ”, which stands for the imaginary unit.

Besides Eq. (2), we introduce some other dimensionless quantities:

$$\sigma_{ij} = \frac{(1-\nu)\sigma_{ij}}{E\alpha(T_\infty - T_0)}, \quad (u, v) = \frac{(u_x, u_y)}{r(1+\nu)\alpha(T_\infty - T_0)} \quad (11)$$

where the index I, J refer to the dimensionless coordinate X, Y .

By introducing the dimensionless variables defined in (11) into (9) and (10), we obtain stress–strain relationships in dimensionless form:

$$\begin{cases} \sigma_{XX} = \frac{\partial u}{\partial X} + \nu \frac{\partial v}{\partial Y} - \theta \\ \sigma_{YY} = \nu \frac{\partial u}{\partial X} + \frac{\partial v}{\partial Y} - \theta \\ \tau_{XY} = \frac{1-\nu}{2} \left(\frac{\partial u}{\partial Y} + \frac{\partial v}{\partial X} \right) \end{cases} \quad (12)$$

The equations of motion become:

$$\begin{cases} \frac{\partial \sigma_{XX}}{\partial X} + \frac{\partial \tau_{XY}}{\partial Y} = \tau_2^2 \frac{(1-\nu)^2}{1-2\nu} \frac{\partial^2 u}{\partial \tau^2} \\ \frac{\partial \tau_{XY}}{\partial X} + \frac{\partial \sigma_{YY}}{\partial Y} = \tau_2^2 \frac{(1-\nu)^2}{1-2\nu} \frac{\partial^2 v}{\partial \tau^2} \end{cases} \quad (13)$$

By substituting the equations of motion (13) with Eq. (12), and by using the dimensionless quantities, we obtain:

$$\begin{cases} \frac{\partial^2 u}{\partial X^2} + \frac{1+\nu}{2} \frac{\partial^2 v}{\partial X \partial Y} + \frac{1-\nu}{2} \frac{\partial^2 u}{\partial Y^2} - \tau_2^2 \frac{(1-\nu)^2}{1-2\nu} \frac{\partial^2 u}{\partial \tau^2} = \frac{\partial \theta}{\partial X} \\ \frac{\partial^2 v}{\partial Y^2} + \frac{1+\nu}{2} \frac{\partial^2 u}{\partial X \partial Y} + \frac{1-\nu}{2} \frac{\partial^2 v}{\partial X^2} - \tau_2^2 \frac{(1-\nu)^2}{1-2\nu} \frac{\partial^2 v}{\partial \tau^2} = \frac{\partial \theta}{\partial Y} \end{cases} \quad (14)$$

where $\tau_2 = \frac{a}{Vr}$ with $V = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$ being the stress wave velocity.

The dimensionless parameter τ_2 , which we named “**inertia factor**”, plays an important role in describing the dynamic behavior of a cracked solid under thermal loading. The physical implication of this parameter in stress concentration near the crack tips will be discussed afterward in this paper.

In the present problem, the mechanical boundary conditions can be expressed as follows:

$$\begin{cases} (a) : \tau_{XY}(X, -La) = \sigma_{YY}(X, -La) = 0 \\ (b) : \tau_{XY}(X, Lb) = \sigma_{YY}(X, Lb) = 0 \\ (c) : \tau_{XY}(X, 0) = \sigma_{YY}(X, 0) = 0 & |X| < 1 \\ (d) : \tau_{XY}(X, 0^+) = \tau_{XY}(X, 0^-) \\ (e) : \sigma_{YY}(X, 0^+) = \sigma_{YY}(X, 0^-) \\ (f) : u(X, 0^-) = u(X, 0^+) \\ (g) : v(X, 0^-) = v(X, 0^+) \end{cases} \quad |X| \geq 1 \quad (15)$$

3. Temperature field

In the following, we present the resolution procedure of the above-defined thermal-elastic equations. The analytical–numerical methods used in [20,21] are essentially followed in this work. Globally speaking, the Laplace and the Fourier transform techniques are used in resolving the problem [22]. Since the inertia effect is introduced in the equilibrium equations, the resolution procedure becomes more complex. Special resolution techniques are developed in this paper.

The temperature field in the cracked plate can be obtained by solving the governing Eq. (3) under initial and boundary conditions (5). Applying the Laplace transform to (3) and (5) leads to:

$$\nabla^2 \bar{\theta} = (p + \tau_1 p^2) \bar{\theta} \quad (16)$$

and

$$\begin{cases} (a) : \bar{\theta}(X, -La) = \frac{1}{p} \\ (b) : \bar{\theta}(X, Lb) = 0 \\ (c) : \frac{\partial \bar{\theta}(X, 0^+)}{\partial Y} = \frac{\partial \bar{\theta}(X, 0^-)}{\partial Y} = Bi[\bar{\theta}(X, 0^+) - \bar{\theta}(X, 0^-)] & |X| < 1 \\ (d) : \bar{\theta}(X, 0^+, p) = \bar{\theta}(X, 0^-, p) \\ (e) : \frac{\partial \bar{\theta}}{\partial Y}|_{Y=0^+} = \frac{\partial \bar{\theta}}{\partial Y}|_{Y=0^-} \end{cases} \quad |X| > 1 \quad (17)$$

where $\bar{\theta}(X, Y, p) = \int_0^\infty \theta(X, Y, \tau) e^{-p\tau} d\tau$ is the Laplace transform of dimensionless temperature.

The general solution of (16) can be solved by using Fourier transform method. Temperature in Laplace–Fourier space can be written as:

$$\tilde{\theta}(\xi, Y, p) = \begin{cases} D_1(\xi)e^{-mY} + D_2(\xi)e^{mY} & -La < Y < 0 \\ D_3(\xi)e^{-mY} + D_4(\xi)e^{mY} & 0 < Y < Lb \end{cases} \quad (18)$$

where $m = \sqrt{\xi^2 + p + \tau_1 p^2}$ and $\tilde{\theta}(\xi, Y, p) = \int_{-\infty}^{\infty} \bar{\theta}(X, Y, p)e^{i\xi X} dx$, $D_i(\xi) (i = 1, \dots, 4)$ are unknown parameters to be determined.

Now we introduce the following density function in Laplace space:

$$\bar{\phi}(X) = \frac{\partial \bar{\theta}}{\partial X} \Big|_{Y=0^+} - \frac{\partial \bar{\theta}}{\partial X} \Big|_{Y=0^-} \quad (19)$$

It is clear that from the boundary condition (c) in (17), we have:

$$\int_{-1}^1 \bar{\phi}(X) dX = 0 \quad (20)$$

and

$$\bar{\phi}(X) = 0, |X| > 1 \quad (21)$$

By applying the boundary conditions (17) and the density function (19), the temperature field in Laplace space can be expressed as:

$$\bar{\theta} = \begin{cases} \frac{1}{2\pi} \int_0^\infty \frac{ie^{-mY}(1+e^{2Lbm})(1-e^{2m(La+Y)})}{(-1+e^{2(La+Lb)m})\xi} \tilde{\phi} \cos(\xi X) d\xi + \frac{e^{m_0(La+2Lb-Y)} - e^{m_0(La+Y)}}{(-1+e^{2(La+Lb)m_0})p} & -La < Y \leq 0 \\ \frac{1}{2\pi} \int_0^\infty \frac{ie^{-mY}(1+e^{2Lbm})(e^{2Lbm}-e^{2mY})}{(-1+e^{2(La+Lb)m})\xi} \tilde{\phi} \cos(\xi X) d\xi + \frac{e^{m_0(La+2Lb-Y)} - e^{m_0(La+Y)}}{(-1+e^{2(La+Lb)m_0})p} & 0 < Y < Lb \end{cases} \quad (22)$$

where $\tilde{\phi} = \int_{-\infty}^\infty (\int_0^\infty \phi e^{-p\tau} d\tau) e^{i\xi X} d\xi$ is the Fourier transform of Laplace transform of ϕ ; $m = \sqrt{\xi^2 + p + \tau_1 p^2}$ and $m_0 = \sqrt{p + \tau_1 p^2}$.

Substituting (22) into (c) in (17), we get the singular integral equation for $\phi(X)$ as follows:

$$\int_{-1}^1 \bar{\phi}(\eta) \left[\frac{1}{\eta - X} + k(X, \eta) \right] d\eta = \frac{2\pi m_0 e^{Lam_0} (1 + e^{2Lbm_0})}{(-1 + e^{2m_0(La+Lb)})p} \quad (23)$$

where

$$k(X, \eta) = \int_0^\infty \left[1 - \frac{2Bi}{\xi} - \frac{(1 + e^{2Lam})(1 + e^{2Lbm})m}{(-1 + e^{2(La+Lb)m})\xi} \right] \sin \xi(X - \eta) d\xi \quad (24)$$

The integral Eq. (23) under the singles-value condition (20) has the following form of solution [23]

$$\bar{\phi}(X) = \frac{\bar{\Phi}(X)}{\sqrt{1 - X^2}} \quad (25)$$

where $\bar{\Phi}(X)$ is bounded and continuous on the interval $[-1, 1]$. Following the numerical techniques proposed by [23], Eqs. (20) and (23) can be solved at discrete points from the algebraic equations system:

$$\begin{cases} \sum_{j=1}^n \frac{\pi}{n} \bar{\Phi}(\eta_j) \left[\frac{1}{\eta_j - X_i} + k(X_i, \eta_j) \right] = \frac{2\pi m_0 e^{Lam_0} (1 + e^{2Lbm_0})}{(-1 + e^{2m_0(La+Lb)})p} \\ \sum_{j=1}^n \frac{\pi}{n} \bar{\Phi}(\eta_j) = 0 \end{cases} \quad (26)$$

where the Gauss–Chebyshev points are defined as

$$\eta_j = \cos \left(\frac{2j-1}{2n} \pi \right) \quad j = 1, 2, \dots, n \quad (27)$$

and

$$X_i = \cos \left(\frac{i\pi}{n} \right) \quad i = 1, 2, \dots, n-1 \quad (28)$$

Once the function $\bar{\Phi}(X)$ is obtained, the temperature fields in p -plane can be calculated by using Eqs. (22) and applying the Chebyshev quadrature for integration.

Temperature in time domain can be given by applying the inverse Laplace transform, as detailed in [20].

4. Thermal stress

Once the temperature field in Laplace space is found, it is introduced to the equations of motion (14) to determine the displacement and stress fields. To this end, we first write the equations of motion (14) and the boundary conditions (15) in Laplace space by means of the Laplace transform. Then the Fourier transform with respect to X coordinate is applied to these equations such that a non-linear ordinary differential equation system in Y -coordinate can be obtained in Fourier–Laplace space. These differential equations are resolved by using appropriate numerical methods. Inverse Fourier transform then inverse Laplace transform of these solutions provide the displacement and stress fields with which some important measures such as the dynamic stress intensity factors at the crack tips, the crack opening distance etc. can be obtained.

4.1. Integral transformation

Firstly, using the Laplace transform to Eqs. (14) and (15), we obtain:

$$\begin{cases} \frac{\partial^2 \bar{u}}{\partial X^2} + \frac{1+\nu}{2} \frac{\partial^2 \bar{v}}{\partial X \partial Y} + \frac{1-\nu}{2} \frac{\partial^2 \bar{u}}{\partial Y^2} - \tau_2^2 \frac{(1-\nu)^2}{1-2\nu} p^2 \bar{u} = \frac{\partial \bar{\theta}}{\partial X} \\ \frac{\partial^2 \bar{v}}{\partial Y^2} + \frac{1+\nu}{2} \frac{\partial^2 \bar{u}}{\partial X \partial Y} + \frac{1-\nu}{2} \frac{\partial^2 \bar{v}}{\partial X^2} - \tau_2^2 \frac{(1-\nu)^2}{1-2\nu} p^2 \bar{v} = \frac{\partial \bar{\theta}}{\partial Y} \end{cases} \quad (29)$$

$$\begin{cases} (a) : \bar{\tau}_{XY}(X, -La) = \bar{\sigma}_{YY}(X, -l) = 0 \\ (b) : \bar{\tau}_{XY}(X, Lb) = \bar{\sigma}_{YY}(X, Lb) = 0 \\ (c) : \bar{\tau}_{XY}(X, 0) = \bar{\sigma}_{YY}(X, 0) = 0 & |X| < 1 \\ (d) : \bar{\tau}_{XY}(X, 0^+) = \bar{\tau}_{XY}(X, 0^-) \\ (e) : \bar{\sigma}_{YY}(X, 0^+) = \bar{\sigma}_{YY}(X, 0^-) \\ (f) : \bar{u}(X, 0^+) = \bar{u}(X, 0^-) \\ (g) : \bar{v}(X, 0^+) = \bar{v}(X, 0^-) \end{cases} \quad |X| \geq 1 \quad (30)$$

where $(\bar{u}, \bar{v}) = \int_0^\infty (u, v) e^{-p\tau} d\tau$ and $\bar{\sigma}_{ij} = \int_0^\infty \sigma_{ij} e^{-p\tau} d\tau$ are the Laplace transform of the displacement components and stress components, respectively.

Then applying the Fourier transform to Eqs. (29) and (30) gives

$$\begin{cases} \frac{d^2 \tilde{u}}{dY^2} + \beta_1 \frac{d\tilde{v}}{dY} + \beta_2 \tilde{u} = \beta_3 \tilde{\theta} \\ \frac{d^2 \tilde{v}}{dY^2} + \beta_4 \frac{d\tilde{u}}{dY} + \beta_5 \tilde{v} = \frac{d\tilde{\theta}}{dY} \end{cases} \quad (31)$$

$$\begin{cases} (a) : \widetilde{\tau}_{XY}(-La) = \widetilde{\sigma}_{YY}(-La) = 0 \\ (b) : \widetilde{\tau}_{XY}(Lb) = \widetilde{\sigma}_{YY}(Lb) = 0 \\ (c) : \widetilde{\tau}_{XY}(0) = \widetilde{\sigma}_{YY}(0) = 0 & |X| \leq 1 \\ (d) : \widetilde{\tau}_{XY}(0^+) = \widetilde{\tau}_{XY}(0^-) \\ (e) : \widetilde{\sigma}_{YY}(0^+) = \widetilde{\sigma}_{YY}(0^-) \\ (f) : \tilde{u}(0^+) = \tilde{u}(0^-) \\ (g) : \tilde{v}(0^+) = \tilde{v}(0^-) \end{cases} \quad |X| > 1 \quad (32)$$

$$\begin{cases} \widetilde{\sigma}_X = -i\zeta \tilde{u} + \nu \frac{d\tilde{v}}{dY} - \tilde{\theta} \\ \widetilde{\sigma}_Y = -i\zeta \nu \tilde{u} + \frac{d\tilde{v}}{dY} - \tilde{\theta} \\ \widetilde{\tau}_{XY} = \frac{1-\nu}{2} \left(\frac{d\tilde{u}}{dY} - i\zeta \tilde{v} \right) \end{cases} \quad (33)$$

where $\tilde{u}$ and $\tilde{v}$ are the Fourier transform of $\bar{u}_x$ and $\bar{u}_y$ respectively and β_i ($i = 1, 2, 3, 4, 5$) are given by:

$$\begin{cases} \beta_1 = -\frac{1+\nu}{1-\nu} i\zeta \\ \beta_2 = -\frac{2}{1-\nu} \left(\zeta^2 + \tau_2^2 \frac{(1-\nu)^2}{1-2\nu} p^2 \right) \\ \beta_3 = -\frac{2i\zeta}{1-\nu} \\ \beta_4 = -\frac{1+\nu}{2} i\zeta \\ \beta_5 = -\left(\frac{1-\nu}{2} \zeta^2 + \tau_2^2 \frac{(1-\nu)^2}{1-2\nu} p^2 \right) \end{cases} \quad (34)$$

4.2. Displacement field and stress field

By considering the boundary conditions (c), (d) in (32) and the temperature fields, the solution of Eqs. (31), i.e., the displacement field in Laplace–Fourier space can be expressed as:

$$\begin{cases} \begin{cases} \tilde{u}(\xi, Y) = \sum_{i=1}^4 C_i(\xi) e^{\lambda_i Y} + \omega_1 e^{-mY} + \omega_2 e^{mY} \\ \tilde{v}(\xi, Y) = \sum_{i=1}^4 B_i C_i(\xi) e^{\lambda_i Y} + \omega_3 e^{-mY} + \omega_4 e^{mY} \end{cases} & -La < Y < 0 \\ \begin{cases} \tilde{u}(\xi, Y) = \sum_{i=5}^6 C_i(\xi) e^{\lambda_i Y} + \omega_5 e^{-mY} + \omega_6 e^{mY} \\ \tilde{v}(\xi, Y) = \sum_{i=5}^6 B_i C_i(\xi) e^{\lambda_i Y} + \omega_7 e^{-mY} + \omega_8 e^{mY} \end{cases} & 0 < Y < Lb \end{cases} \quad (35)$$

where $C_i(\xi)$ ($i = 1, \dots, 6$) are unknown functions to be determined. The expressions of the coefficients ω_i ($i = 1, \dots, 8$) are given in Eq. (A1). Substituting Eq. (35) into Eq. (31) results in the following equations:

$$\begin{cases} \beta_1 \lambda_j B_j + \lambda_j^2 + \beta_2 = 0 \\ (\lambda_j^2 + \beta_5) B_j + \beta_4 \lambda_j = 0 \end{cases} \quad (36)$$

so λ_i ($i = 1, \dots, 4$) can be given by the roots of the following equation:

$$\lambda^4 + (\beta_2 + \beta_5 - \beta_1 \beta_4) \lambda^2 + \beta_2 \beta_5 = 0 \quad (37)$$

and the coefficients λ_j ($j = 5, 6, 7, 8$) are given by:

$$\lambda_j = \lambda_{j-4} \quad (j = 5, 6, 7, 8) \quad (38)$$

Straightforwardly, the coefficients B_j ($j = 1, 2, \dots, 8$) are given by:

$$B_j = -\frac{\lambda_j^2 + \beta_2}{\beta_1 \lambda_j} \quad (39)$$

By substituting the displacement field (35) and temperature field (18) into (33), the stress can be obtained as following:

$$\begin{cases} \widetilde{\sigma_X} = \sum_{i=i_0}^{i_0+3} (B_i \lambda_i v - i \xi) C_i e^{\lambda_i Y} - i \xi (\omega_1 e^{-mY} + \omega_2 e^{mY}) + v (-m \omega_3 e^{-mY} + m \omega_4 e^{mY}) \\ \quad - (D_{i_1} e^{-mY} + D_{i_1+1} e^{mY}) \\ \widetilde{\sigma_Y} = \sum_{i=i_0}^{i_0+3} (B_i \lambda_i - i \xi v) C_i e^{\lambda_i Y} - i v \xi (\omega_1 e^{-mY} + \omega_2 e^{mY}) - m \omega_3 e^{-mY} + m \omega_4 e^{mY} \\ \quad - (D_{i_1} e^{-mY} + D_{i_1+1} e^{mY}) \\ \widetilde{\tau_{XY}} = \frac{1-v}{2} \left[\sum_{i=i_0}^{i_0+3} (\lambda_i - i \xi B_i) C_i e^{\lambda_i Y} - m \omega_1 e^{-mY} + m \omega_2 e^{mY} - i \xi (\omega_3 e^{-mY} + \omega_4 e^{mY}) \right] \end{cases} \quad (40)$$

where $i_0 = 1, i_1 = 1$ for $-La < Y < 0$ and $i_0 = 5, i_1 = 3$ for $0 < Y < Lb$.

4.3. Integral equations

We now introduce two dislocation functions $\bar{f}_i(X)$ ($i = 1, 2$) along the crack line which can be written as follows:

$$\begin{cases} \bar{f}_1(X) = \frac{\partial \langle \bar{u} \rangle}{\partial X} \\ \bar{f}_2(X) = \frac{\partial \langle \bar{v} \rangle}{\partial X} \end{cases} \quad (41)$$

where $\langle \bar{u} \rangle$ and $\langle \bar{v} \rangle$ denote the jumps of displacements across the crack lips $y = 0$ in Laplace space:

$$\begin{cases} \langle \bar{u} \rangle = \bar{u}(X, 0^+) - \bar{u}(X, 0^-) \\ \langle \bar{v} \rangle = \bar{v}(X, 0^+) - \bar{v}(X, 0^-) \end{cases} \quad (42)$$

From the conditions (30), we know that $\bar{f}_i(X)$ ($i = 1, 2$) also satisfy the single-value conditions:

$$\int_{-1}^1 \bar{f}_i(X) dX = 0 (i = 1, 2) \quad (43)$$

and

$$\bar{f}_i(X) = 0 (i = 1, 2) |X| \geq 1 \quad (44)$$

Physically, Eq. (44) means that the displacements are single-valued for the uncracked portion along $y = 0$. By applying the Fourier transforms to Eq. (41), we obtain:

$$\begin{cases} \tilde{f}_1(\xi) = -i\xi [\tilde{u}(0^+) - \tilde{u}(0^-)] \\ \tilde{f}_2(\xi) = -i\xi [\tilde{v}(0^+) - \tilde{v}(0^-)] \end{cases} \quad (45)$$

where by definition and from (43), $\tilde{f}_1$ and $\tilde{f}_2$ are given by:

$$\tilde{f}_i = \int_0^\infty \left(\int_{-1}^1 f_i \cdot e^{i\xi X} dX \right) \cdot e^{-p\tau} d\tau \quad i = 1, 2 \quad (46)$$

By applying the boundary conditions (a)–(e) in (32) and (45), C_i can be expressed as

$$C_i = \mathbf{A}_{ij}^{-1} \gamma_j + \frac{1}{i\xi} \mathbf{A}_{i,7}^{-1} \tilde{f}_1 + \frac{1}{i\xi} \mathbf{A}_{i,8}^{-1} \tilde{f}_2 \quad i = 1, 2, \dots, 8 \quad (47)$$

where the terms in matrix $\mathbf{A}_{ij}$ and γ_j are given in Eqs. (A2) and (A3), respectively.

By substituting (35) into the boundary conditions (c) in (32) and by applying the relationships in (33), it can be shown that $f_i(x)$ ($i = 1, 2$) satisfy the following equations:

$$\sum_{j=1}^2 \int_{-1}^1 \bar{f}_j(\eta) K_{ij}(X, \eta) d\eta = W_i \quad (i = 1, 2; j = 1, 2) \quad (48)$$

where K_{ij} and W_i ($i = 1, 2; j = 1, 2$) are given in Eq. (A4).

The solution of the integral equations of $f_i(x)$ ($i = 1, 2$) can be expressed in the following form:

$$\bar{f}_i(X) = \frac{\bar{F}_i(X)}{\sqrt{1-X^2}} \quad (i = 1, 2) \quad (49)$$

Using the Lobatto-Chebyshev method [24–26], we can reduce the integral equations to the following algebraic equations:

$$\begin{cases} \sum_{j=1}^2 \left[\sum_{k=1}^n A_k K_{1j}(X_j, \eta_k) \bar{F}_j(\eta_k) \right] = W_1(x_j) \\ \sum_{j=1}^2 \left[\sum_{k=1}^n A_k K_{2j}(X_j, \eta_k) \bar{F}_j(\eta_k) \right] = W_2(x_j) \\ \sum_{k=1}^n A_k \bar{F}_1(\eta_k) = 0 \\ \sum_{k=1}^n A_k \bar{F}_2(\eta_k) = 0 \end{cases} \quad (50)$$

where

$$\begin{cases} \eta_k = \cos \frac{(k-1)\pi}{n-1}, \quad k = 1, 2, \dots, n; \\ X_j = \cos \frac{(2j-1)\pi}{2(n-1)}, \quad j = 1, 2, \dots, n-1; \\ A_1 = A_n = \frac{\pi}{2(n-1)}; \\ A_k = \frac{\pi}{n-1}, \quad k = 2, 3, \dots, n-1. \end{cases} \quad (51)$$

The functions F_1 and F_2 at the crack line can be given by resolving $\bar{F}_1$ and $\bar{F}_2$ from (50) and performing the inverse Laplace transform. The dynamic stress intensity factors $K_I(p)$ and $K_{II}(p)$ are defined as [27]

$$\begin{cases} K_I(t) = \mathbf{L}^{-1} \left[-\frac{(1-v^2)\sqrt{\pi}}{8} \bar{F}_2(1, p) \right] \\ K_{II}(t) = \mathbf{L}^{-1} \left[-\frac{(1+v)\sqrt{\pi}}{4} \bar{F}_1(1, p) \right] \end{cases} \quad (52)$$

where the operator $\mathbf{L}^{-1}[\]$ denotes the inverse Laplace transform

5. Numerical result and discussions

The coupled effect of the non-Fourier heat conduction and the inertia forces provides complex stress fields near the crack tips, which depend additionally on the crack position with respect to the boundary of the strip. The influence of both these factors on the principal crack stress concentration characterized mainly by the stress intensity factors (SIFs) and by the crack opening distances (CODs) will be evaluated and discussed in the following sections.

5.1. Temperature field

In the analysis of temperature fields and stresses intensity factors, the dimensionless position of the crack in the strip are fixed to be $L_a = 1$ and $L_b = 2$. The temperature field can be obtained consecutively. The temperatures in different conditions are depicted in Figs. 2–5.

Fig. 2 shows the influence of the Biot number and the non-Fourier effect on the temperature field. In Fig. 2(a)–(d), thermal relaxation time τ_1 is set to be 0, 0.2, 0.4 and 0.6, respectively.

When $Bi = 0$, as shown in Fig. 2(a), the temperature field corresponding to the case of perfectly insulate crack surfaces, at which condition the temperature differences of the crack surfaces are maximum. When $Bi = \text{Infinite}$, as shown in Fig. 2(d), the temperature curves of the upper and lower crack surfaces overlap. In the last case, the temperature field degenerates to that of a strip without crack.

The curves in Fig. 2 show also the influence of the non-Fourier parameter τ_1 on the temperature field. The red curves denote $\tau_1 = 0$, which represents the condition of the Fourier thermal conduction. As the increase of τ_1 , some wave properties appear in the temperature evolution. The peak values of the temperature increase as τ_1 increases. Particularly when $\tau_1 > 0$, the maximum normalized temperature could higher than 1 and the minimum normalized temperature could lower than 0, which is a characteristic phenomenon of the non-Fourier heat conduction. These curves converge to those obtained with the condition of Fourier thermal conduction. The bigger the value of τ_1 is, the slower the curve convergence will be.

Fig. 3 shows the Maximum temperature differences between the midpoints of crack surfaces versus τ_1 for different Biot number. The increasing of the Biot number would improve the thermal conductivity of the crack gap, which would decrease the temperature differences between the crack surfaces. Whereas the non-Fourier effect increases the peak value of the temperature, especially for the crack surface close to the heating boundary, as shown in Fig. 2, so the maximum temperature difference increasing with τ_1 .

Fig. 2. Temperature evolution at the midpoints of the crack surfaces for different τ_1 when (a) $Bi = 0$, (b) $Bi = 0.2$, (c) $Bi = 1$ and (d) $Bi = \text{Infinity}$.

Fig. 3. Temperature differences between the midpoints of the crack surfaces versus τ_1 for different Biot number.

Fig. 4. Normalized stress intensity factors (a) for K_I and (b) for K_{II} for versus dimensionless time for different τ_1 when $\tau_2 = 0$.

5.2. Stress intensity factors (SIFs)

In this work, the Poisson's ratio is set to be 0.3 for all computations which is close to those of most of the brittle or quasi-brittle materials used in engineering applications.

The dynamic stress intensity factors K_I and K_{II} can be evaluated from (52) once we obtain the solutions of the algebraic Eqs. (50) and perform the numerical inverse of Laplace transform. Their normalized values $(K_I^*, K_{II}^*) = \frac{(K_I, K_{II})(1-\nu)}{E\alpha(1_\infty - 1_0)\sqrt{r}}$ are presented in Figs. 4–7.

Fig. 5. Normalized stress intensity factors (SIFs) versus dimensionless time for different τ_2 when $\tau_1 = 0$.

Fig. 6. Variation of SIFs versus dimensionless time for different Biot number when $\tau_1 = \tau_2 = 0$.

Let us first examine separately the influence of each parameter on the stress intensity factors. Fig. 4 illustrates the evolutions of the SIFs under classical quasi-static condition by taking the non-Fourier thermal conduction effect into account, i.e. $\tau_2 = 0$ and $\tau_1 \neq 0$.

In this case, the model is identical to the theory developed by Hu and Chen [22] except the thermal shock boundary conditions. In their article both the upper and lower surfaces of the infinite strip suffer from the thermal shock loading with the dimensionless temperature differences to be 2 and 1, respectively. We remark that the trend of the temperature field and SIFs shown in Figs. 2 and 4 exhibit an oscillating feature and agree well with the results reported in the literature [8,22,28].

The influence of the thermal inertia factor τ_2 on the normalized dynamic SIFs is depicted in Fig. 5 by neglecting the non-Fourier effect, i.e. by fixing $\tau_1 = 0$. From this figure, we can clearly observe the influence of this parameter on the dynamic

Fig. 7. Variation of maximum SIFs versus τ_2 for different τ_1 when Biot number is set to be 0.

SIFs: Firstly, the impact of τ_2 is much stronger on K_I^* than on K_{II}^* in the range of $\tau_2 \in [0, 0.3]$. Secondly, as expected, the amplitudes of the SIFs increase with τ_2 . Moreover, similar to the case when $\tau_1 > 0$, the time evolution of the SIFs presents an oscillating features when τ_2 is large. It is to notice that the dynamic SIF K_I^* can be positive or negative when $\tau_2 > 0$. This anomaly comes from the fact that the crack lip contact under compression is not taken into account in this study.

Fig. 6 shows the effect of the Biot number on the dynamic SIFs in the case of Fourier heat conduction ($\tau_1 = 0$) and quasi-static thermal stress ($\tau_2 = 0$). It is clear that both the K_I and K_{II} have the peak value when the crack gap is heat insulating. As the growing of the Biot number, the temperature difference between the crack surfaces decreases, as shown in Fig. 2. Consequently, the absolute values of SIFs gradually decrease and finally approach to zero when the Biot number tends to infinity.

In practice, the peak values of the dynamic SIFs during the time evaluation are most significant in predicting the crack growth. The peak values of K_I^* evaluated for different τ_1 and τ_2 are plotted in Fig. 7, in which the Biot number is set to be 0. Overall, the maximum absolute value increase with τ_1 and τ_2 . The non-Fourier effect has bigger impact to K_{II} while the inertial effect affects more on K_I . In particular, the maximum K_I increases more than 10 times as τ_2 grows from 0 to 0.3.

In most engineering problems the length of engineering cracks is considered to be larger than 1 mm. In this condition the value of τ_2 is normally less than 0.01 and the magnitude of τ_2 is small enough to be ignored. However, according to the definition of τ_2 , its value will be greatly increased as the decreasing of the crack length. For example, the τ_2 value of gold will reach up to 0.115 when the crack length is 1 μm . In fact, the micro-cracks smaller than 1 μm do exist in some coating materials. [29,30], the inertia effect should be considered and will play an important role in the thermal shock damage process.

6. Conclusions

In this paper, the insulation of the crack gap, the non-Fourier heat conduction condition and the inertia effect are considered to investigate the transient thermal stress problem of an insulated crack parallel to the boundary of a thermal-elastic plate under thermal shock loading. The mixed boundary value problem is formulated in terms of Laplace and Fourier transforms techniques. The established singular integral equations are solved by using the Gauss integrate method. Temperature and dynamic stress intensity factors in the time domain are obtained by using numerical inverse Laplace transform. The Temperature field and dynamic stress intensity factors are evaluated for different values of parameters Bi , τ_1 and τ_2 .

The magnitude of Biot number of the crack gaps has great influence for both the temperature field and stress intensity factors. The numerical results reveal that the influence of the crack to the temperature and stress field can be neglect when $Bi > 100$. In other words, the increasing of Biot number would increase the resistance of cracks growth. This result might be useful in designing new materials.

The magnitude of thermal relaxation time and the strength of non-Fourier effect have been investigated by many authors. General speaking, the non-Fourier effect is unneglectable in some non-homogeneous materials [17] and biological materials, such as human tissue [31].

The inertia effect is negligible when the crack length is in millimeter scale or larger. However, when the scale of interest is small to micron scale, the inertia effect will play an important role in the thermal shock damage of materials.

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Appendix A

$$\begin{cases}
 \omega_1 = \frac{-(1+e^{2Lbm})\ddot{\phi}p+2i\zeta e^{(La+2Lb)m}\delta(\zeta)}{2p(1-e^{2(La+Lb)m})(m^2-\zeta^2-p^2\tau_2^2)} \\
 \omega_2 = \frac{e^{Lam}[(1+e^{2Lbm})e^{Lam}p\ddot{\phi}-2i\zeta\delta(\zeta)]}{2p(1-e^{2(La+Lb)m})(m^2-\zeta^2-p^2\tau_2^2)} \\
 \omega_3 = \frac{im[(1+e^{2Lbm})p\ddot{\phi}-2i\zeta e^{(La+2Lb)m}\delta(\zeta)]}{2p\zeta(1-e^{2(La+Lb)m})(m^2-\zeta^2-p^2\tau_2^2)} \\
 \omega_4 = \frac{ie^{Lam}m[(1+e^{2Lbm})e^{Lam}\ddot{\phi}p-2i\zeta\delta(\zeta)]}{2p\zeta(1-e^{2(La+Lb)m})(m^2-\zeta^2-p^2\tau_2^2)} \\
 \omega_5 = \frac{e^{2Lbm}[-(1+e^{2Lam})p\ddot{\phi}+2i\zeta e^{Lam}\delta(\zeta)]}{2p(1-e^{2(La+Lb)m})(m^2-\zeta^2-p^2\tau_2^2)} \\
 \omega_6 = \frac{(1+e^{2Lam})\ddot{\phi}p-2i\zeta e^{Lam}\delta(\zeta)}{2p(1-e^{2(La+Lb)m})(m^2-\zeta^2-p^2\tau_2^2)} \\
 \omega_7 = \frac{ie^{2Lbm}m[(1+e^{2Lam})\ddot{\phi}p-2i\zeta e^{Lam}\delta(\zeta)]}{2p\zeta(1-e^{2(La+Lb)m})(m^2-\zeta^2-p^2\tau_2^2)} \\
 \omega_8 = \frac{im[(1+e^{2Lam})\ddot{\phi}p-2i\zeta e^{Lam}\delta(\zeta)]}{2p\zeta(1-e^{2(La+Lb)m})(m^2-\zeta^2-p^2\tau_2^2)}
 \end{cases} \quad (A1)$$

where $\delta(\zeta) = \begin{cases} \infty, & \zeta = 0 \\ 0, & \zeta \neq 0 \end{cases}$.

$$\begin{cases}
 \mathbf{A}_{1,j} = \begin{cases} (i\zeta v + B_j\lambda_j)e^{-\lambda_j La} & j = 1, 2, 3, 4 \\ 0 & j = 5, 6, 7, 8 \end{cases} \\
 \mathbf{A}_{2,j} = \begin{cases} (i\zeta B_j + \lambda_j)e^{-\lambda_j La} & j = 1, 2, 3, 4 \\ 0 & j = 5, 6, 7, 8 \end{cases} \\
 \mathbf{A}_{3,j} = \begin{cases} i\zeta v + B_j\lambda_j & j = 1, 2, 3, 4 \\ -i\zeta v - B_j\lambda_j & j = 5, 6, 7, 8 \end{cases} \\
 \mathbf{A}_{4,j} = \begin{cases} i\zeta B_j + \lambda_j & j = 1, 2, 3, 4 \\ -i\zeta B_j - \lambda_j & j = 5, 6, 7, 8 \end{cases} \\
 \mathbf{A}_{5,j} = \begin{cases} 0 & j = 1, 2, 3, 4 \\ (i\zeta v + B_j\lambda_j)e^{i\lambda_j Lb} & j = 5, 6, 7, 8 \end{cases} \\
 \mathbf{A}_{6,j} = \begin{cases} 0 & j = 1, 2, 3, 4 \\ i\zeta B_j + \lambda_j & j = 5, 6, 7, 8 \end{cases} \\
 \mathbf{A}_{7,j} = \begin{cases} -1 & j = 1, 2, 3, 4 \\ 1 & j = 5, 6, 7, 8 \end{cases} \\
 \mathbf{A}_{8,j} = \begin{cases} -B_i & j = 1, 2, 3, 4 \\ B_i & j = 5, 6, 7, 8 \end{cases}
 \end{cases} \quad (A2)$$

$$\gamma_j = \begin{bmatrix} \frac{(1-v)\zeta^2+p^2\tau_2^2}{p(-m^2+\zeta^2+p^2\tau_2^2)}\delta(\zeta) \\ -\frac{2m[p(e^{Lam}+e^{(La+2Lb)m})\ddot{\phi}-i\zeta(1+e^{2(La+Lb)m})\delta(\zeta)]}{(-1+e^{2(La+Lb)m})p(-m^2+\zeta^2+p^2\tau_2^2)} \\ \frac{i\ddot{\phi}((1-v)\zeta^2+p^2\tau_2^2)}{\zeta(m^2-\zeta^2-p^2\tau_2^2)} \\ 0 \\ 0 \\ -\frac{2e^{Lbm}m[(1+e^{2Lam})\ddot{\phi}p-2ie^{Lam}\delta(\zeta)\zeta]}{(-1+e^{2(La+Lb)m})p(-m^2+\zeta^2+p^2\tau_2^2)} \\ \frac{\ddot{\phi}}{-m^2+\zeta^2+p^2\tau_2^2} \\ 0 \end{bmatrix} \quad (A3)$$

$$\left\{ \begin{aligned} K_{1,1} &= \frac{1}{2\pi} \int_0^\infty 2 \sum_{i=1}^4 \left(\frac{-ib_i \lambda_i}{\xi} - v \right) \mathbf{A}_{i,7}^{-1} \cos[\xi(X_j - \eta_k)] d\xi \\ K_{1,2} &= \frac{1}{2\pi} \left\{ \int_0^\infty \left[2 \sum_{i=1}^4 \left(\frac{-b_i \lambda_i}{\xi} + iv \right) \mathbf{A}_{i,8}^{-1} - \frac{1-v^2}{2} \right] \sin[\xi(X_j - \eta_k)] d\xi + \frac{1-v^2}{2} \frac{1}{X_j - \eta_k} \right\} \\ W_1 &= \frac{1}{2\pi} \int_0^\infty \left[\sum_{i=1}^4 \sum_{j=1}^8 (iv\xi - b_i \lambda_i) (\mathbf{A}_{ij} \cdot \boldsymbol{\gamma}_j) + \frac{i(1-e^{2Lam})(1+e^{2Lbm})((1-v)\xi^2 + p^2 \tau_2^2)}{2(-1+e^{2(La+Lb)m})\xi(-m^2 + \xi^2 + p^2 \tau_2^2)} \right] \tilde{\phi} \cos(\xi X_j) d\xi \\ &\quad + \frac{2\pi e^{Lop\tau_2}(1-e^{2Lbp\tau_2})\tau_2^2}{(1-e^{2(La+Lb)p\tau_2})(1+p\tau_1+p\tau_2^2)} + \frac{2\pi e^{Lam_0}(-1+e^{2Lbm_0})\tau_2^2}{(1-e^{2(La+Lb)m_0})(1+p\tau_1+p\tau_2^2)} \\ K_{2,1} &= \frac{1}{2\pi} \left[\int_0^\infty \left(2 \sum_{i=1}^4 \frac{i\xi b_i - \lambda_i}{\xi} \mathbf{A}_{i,7}^{-1} - (1+v) \right) \sin[\xi(x - \eta)] d\xi + \frac{1+v}{X_j - \eta_k} \right] \\ K_{2,2} &= \frac{1}{2\pi} \int_0^\infty 2 \sum_{i=1}^4 \left(\frac{-i\lambda_i}{\xi} - b_i \right) \mathbf{A}_{i,8}^{-1} \cos[\xi(X_j - \eta_k)] d\xi \\ W_2 &= \frac{1}{2\pi} \int_0^\infty \left[\sum_{i=1}^4 \sum_{j=1}^8 (i\xi b_i - \lambda_i) (\mathbf{A}_{ij}^{-1} \cdot \boldsymbol{\gamma}_j) + \frac{(1+e^{2Lam})(1+e^{2Lbm})m}{(-1+e^{2(La+Lb)m})(m^2 - \xi^2 - p^2 \tau_2^2)} \right] \tilde{\phi} \cdot (-i) \sin[\xi X_j] d\xi \end{aligned} \right. \quad (A4)$$

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