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Non-linearly restoring performance of SFT's catenary mooring-lines under consideration of its dynamic behaviors

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Abstract

Submerged floating tunnel (SFT) needs to be fixed or controlled in expected position by its supporting system which can be classified into two types: tension tendon or catenary mooring-line. The later one, catenary mooring-line, is often used for floating body due to its lower cost and easier installment. In this paper, the dynamic behaviors including inertia and hydrodynamic damping of the mooring-line are considered compared with the quasi-static method where only the static restore force is involved, and the nonlinear dynamic response model of the mooring system is developed based on finite element simulations. The influences of the amplitude and frequency of SFT motion, along with mooring-line's structural parameters, on mooring line's displacement and dynamic tension are studied. Also, the taut-slack phenomenon caused by inertial force and hydrodynamic damping is analyzed. Our results shows that when the motion of the SFT is smaller, the dynamic response of the mooring-line is a stable stand wave, and the value of dynamic tension due to dynamic characteristics is about 20% of the quasi-static method, and the tension amplitude difference is around three times larger than the quasi-static method. As SFT movement get larger, the dynamic tension corresponding is about 30% higher than the quasi-static tension. When the mooring-line becomes slack, the response is characterized as travelling wave. The magnification factor of top tension caused by mooring-line's dynamic behavior drops with increase of mooring-line mass density but rises with the increase of the initial tension ratio.

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1. Introduction

Submerged floating tunnel (SFT), or named Archimedes Bridge (AB), is a considerable alternative to connect the

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two banks of wide water area, which floats underwater at a proper depth owing to a balance of water buoyancy force, structural gravity and supporting force of the mooring system. It is a concept dating back around 150 years ago and becomes recently attractive. Compared to the traditional long-span cross-bridge or submerged underground tunnel, it is more environmentally friendly and economical[1,2]. Actually, SFT needs to be fixed or controlled in an expected position by its supporting system which can be classified into two types: tension tendon or catenary mooring-line. The later one, catenary mooring-line, is often used for floating body due to its advantages such as lower cost and easier installment. However, as a flexible body with large structural flexibility and displacement, and more, under complex actions of ambient fluid loads, the movement of mooring-line often exhibits strong nonlinear characters which introduce significant challenge to the analysis of its restoring performance. Essentially, the popular analysis is a kind of quasi-static method where only the statically restoring force is considered, or the dynamic behavior, i.e. the inertial and damping forces of the mooring-line and fluid, are not considered. Mavrakos[3] analyzed the top tension and movement of mooring-line by numerical simulations, and Papazoglou[4] presented the main factors which may influence restoring performance of a catenary. Van Den Boom[5] found that the nonlinearities coming from the geometry, elastic deformation and acting loads can significantly enlarge top tension. Gobat[6] studied the influences of floating vessel on mooring-line tension by using an empirical model.

As water depth and structural length increase, the dynamic characters of mooring-line become more profound, which may consequently change top tension and, even, may introduce transiently large snap tension due to mooring-line taut-slack. In order to examine the influence of dynamic behavior of mooring-line on restoring performance, Vassalos[7] gave the dynamic responses of a catenary mooring-line based on an simplified model where the two structural ends were assumed at same level and the initial catenary displacement is small, then he examined the mooring-line slack and snap tension using a centered-mass model[8]. Chen[9] calculated the dynamic response of a system including a SPAR and its mooring-line based on a linear coupling approach. He pointed out that if the inertial and damping forces of the mooring-line are involved during dynamic response, the top tension would get larger. Zhang[10] experimentally investigated the tension of a taut-slack case and found the snap tension was 5 times of the static one due to slack. However, the researches on the dynamic behaviors are still needed to be furthered, moreover, and it is difficult to present some extreme situations such as taut-slack based on the currently linear approach. In this paper, the dynamic behaviors including inertia and hydrodynamic damping of the mooring-line are considered. Firstly, the nonlinear dynamic response model of the mooring system is developed based on 3D dynamic theory of catenary and the numerical simulations. The influences of floating SFT's movement, e.g. its amplitude and frequency, along with mooring-line's structural properties, e.g. mass density and initial shape, on mooring line's restoring performance including its dynamic displacement and tension are studied. Additionally, the taut-slack phenomenon caused by inertial and damping forces is analyzed.

2. Dynamic governing equations of a moving catenary mooring-line

For a 3d catenary (see Fig.1), compared with the previous 2d static model, the dynamic equations includes some nonlinear terms, that makes the analysis of dynamic response more difficult[11-14].

Its governing equations of dynamics can be written as [15,16]

$$\bar{F}' + \bar{q} = \rho A \ddot{\bar{r}} \quad (1)$$

$$\bar{M}' + \bar{r}' \times \bar{F} + \bar{m} = 0 \quad (2)$$

where \bar{F} and \bar{M} are respectively the total force and moment of the catenary. \bar{q} and \bar{m} are respectively the outer force and moment acted on per unit length of the catenary. ρ and A are structural mass density and area respectively. \bar{r} represents the position vector.

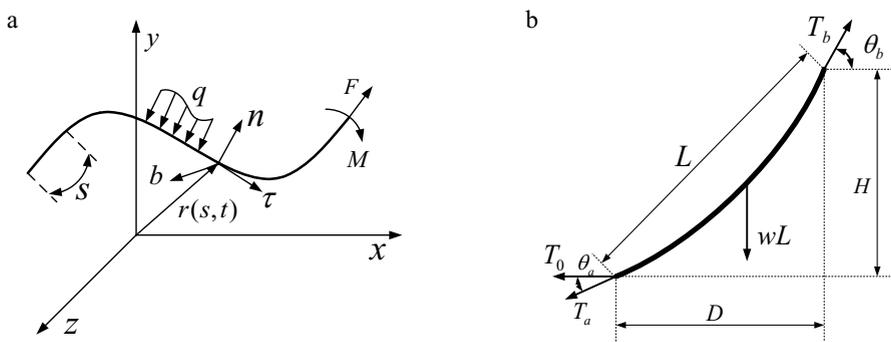


Fig.1. (a) The element of 3D flexible mooring-line; (b) Schematic diagram of the catenary

Then the expression of the bending moment and curvature is

$$\vec{M} = \vec{r}' \times (B\vec{r}'') + H\vec{r}' \tag{3}$$

where B is the structural stiffness and H is the torsion moment. Substituting Eq.(3) into (2), we have

$$\vec{r}' \times [(B\vec{r}'')' + \vec{F}] + H'\vec{r}' + H\vec{r}'' + \vec{m} = 0 \tag{4}$$

and

$$H' + \vec{m}\vec{r}' = 0 \tag{5}$$

where \vec{m} is the averaged rotation moment whose value will be zero if $\vec{m}\vec{r}' = 0$, then $H=0$, that means the rotation moment is independent on the structural arc length. Generally, the rotation moment can be neglected, or the values of both H and \vec{m} are zero. Then Eq.(4) can be rewritten as

$$\vec{r}' \times [(B\vec{r}'')' + \vec{F}] = 0 \tag{6}$$

or

$$\vec{F} = -(B\vec{r}'')' + \lambda\vec{r}' \tag{7}$$

Substituting Eq. (7) into (1) will yield

$$-(B\vec{r}'')'' + (\lambda\vec{r}')' + \vec{q} = \rho A\ddot{\vec{r}} \tag{8}$$

and the deformation equation is

$$\vec{r}' \cdot \vec{r}' = (1 + \varepsilon)^2 \tag{9}$$

where ε is the strain of the catenary. If the value of the bending moment in Eq.(7) is zero, we will have the dynamic equation of a flexible catenary of which the external loads include the gravity, buoyancy and hydrodynamic forces.

Generally, the hydrodynamic force acted on per unit structure length can be expressed by the Morison formula as

$$f = \frac{1}{2}C_D\rho D|V - \dot{y}|(V - \dot{y}) + C_A \frac{\pi D^2}{4}\rho(\dot{V} - \ddot{y}) + \frac{\pi D^2}{4}\rho\dot{V} \quad (10)$$

where D and y are the structural diameter and displacement respectively. V is the fluid velocity. Combing Eqs.(8), (9) and (10), we have a nonlinear equation groups of which direct solution could not be gotten theoretically. Here a numerical simulation is used to solve the dynamic equations. Among those direct numerical integration methods like the Newmark and the Finite Difference methods, the Newmark method is employed here so as to adjust the distribution of the structural acceleration and the nonlinearity of the catenary during the integration range by properly changing the integration parameters. The interpolation functions of the displacement and acceleration are written as:

$$\begin{aligned} \dot{u}_{t+\Delta t} &= \dot{u}_t + [(1 - \beta)\ddot{u}_t + \beta\ddot{u}_{t+\Delta t}]\Delta t \\ u_{t+\Delta t} &= u_t + \dot{u}_t\Delta t + [(\frac{1}{2} - \alpha)\ddot{u}_t + \alpha\ddot{u}_{t+\Delta t}]\Delta t^2 \end{aligned} \quad (11)$$

where the values of α and β are respectively 1/6 and 1/2 at every time step during the dynamic response. The corresponding calculations based on above mentions run into our FEM code where the structure is divided into finite bar elements, between which the rotational constraints are set to be free, and an initial shape is simulated according to the initial top tension so as to model the shape of the catenary as shown in Fig.1.

3. Dynamic response of the catenary based on the numerical simulations

The main structural and geometrical parameters of the catenary are listed in Table 1. The dynamic response of the catenary under movement of floating SFT is analyzed based on our numerical simulations. The sway of SFT is taken as an example of movement excitation, then the displacement, velocity and tension of the catenary are calculated. The bending moment and rotation constrains between the ambient element are assumed as zero during the whole dynamic responses.

Table 1 The main geometrical and material parameters of the catenary

| Geometrical | Value | Structural | Value |
|-------------------------------|-------|-----------------|-----------------------|
| Length | 800m | Young's modulus | 210GPa |
| Initial horizontal projection | 706m | Density | 2513kg/m ³ |
| Initial horizontal projection | 350m | Poisson's ratio | 0.3 |
| Diameter | 0.19m | | |

3.1. Dynamic responses under SFT's movements

The typical dynamic response of the catenary for case of a normal movement, i.e. 0.05Hz frequency and 4m amplitude, of the floating SFT is shown in Fig. 2 and 3. In Fig. 2, the top tension of the quasi-static method is also plotted as a comparison. It is seen that either the value of the peaks or the trough get extremer than, or the gap value between peak and trough gets around 3 times of the quasi-static ones owing to the inertial and damping effects, while the period of dynamic tension is consistent with that of the top SFT's sway. Observing the temporal-spatial evolution of the tension, we noted that the maximum tension always occurs at the top end when the values of tension along catenary length get their own peaks, that means tension change with time all at the same phases. Therefore, we may say the top tension is the key point if one wants to check the strength of a moving catenary.

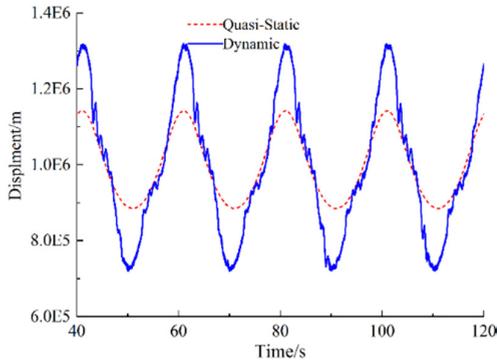


Fig.2 Top tension of the catenary mooring-line

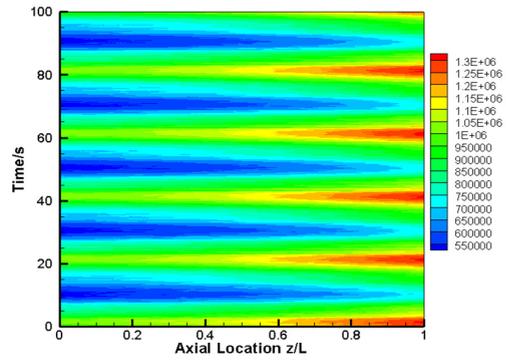


Fig.3 Temporal-spatial evolution of tension

Additionally, the spectrum plots of the top tension during the dynamic response are presented in Fig.4 where the value at 0.00Hz frequency is corresponding to the quasi-static case. At higher frequencies which are a couple of times of the excitation frequency, there are several tension peaks among which the maximum tension is at the excitation frequency, 0.05Hz. Most likely, those tension peaks in Fig.4b, at higher frequency ranging from 0.3Hz to 1.1Hz, are responsible for the small tension fluctuation with short periods in Fig.2. Moreover, if comparing the static value (at 0.00 Hz) with the additional dynamic values (e.g. at 0.05 Hz), the later one is around 20% of the former one. That means the impact caused by the dynamic behavior of the inertial and damping loads is around 20% of the total static one, and that is consistent with the time history curves in Fig. 2 where the dynamic peak is around 20% higher than the static peak.

The temporal-spatial evolutions of the displacements are presented in Fig.5. Interestingly, the horizontal displacement has two motion waves, i.e. one is around the middle part of the catenary and another one is close to the top end where the positive wave travels somewhat toward the middle part of catenary. While the vertical displacement has only one stable motion wave. The maximum displacements occur at the top end for case of the horizontal displacement and at middle point for vertical displacement respectively. Generally speaking, the movement of the catenary is characterized as a stable standing wave, or no apparent travelling wave is seen.

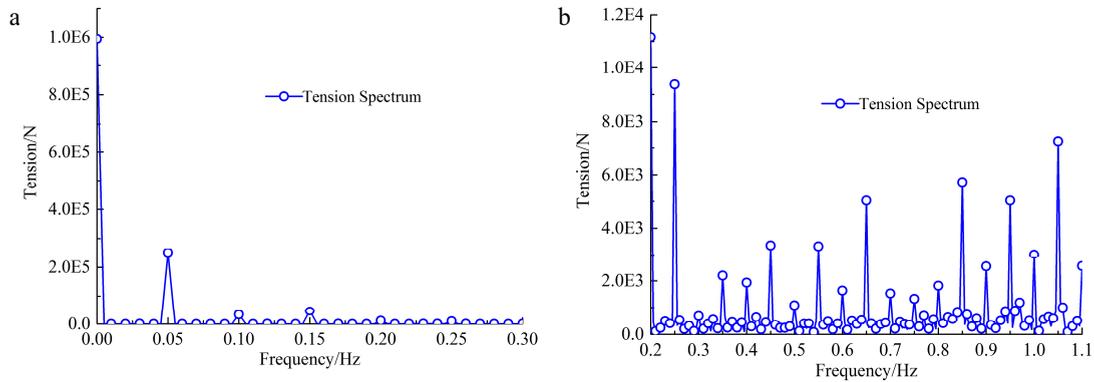


Fig.4 The spectrum plots of the top tension, (a) at frequency range 0-0.3Hz; (b) at frequency range 0.2-1.1Hz

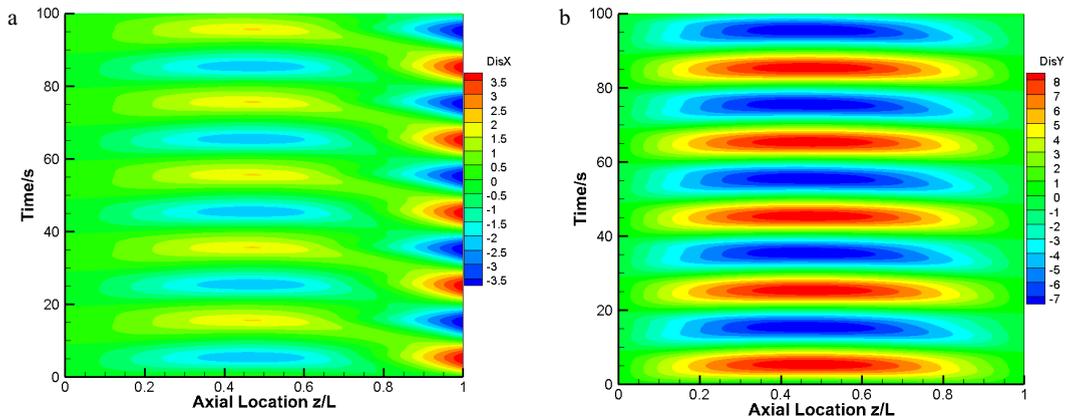


Fig.5 Temporal-spatial evolution of the catenary displacement, (a) the horizontal displacement; (b) the vertical displacement

3.2. Slack and snap tension for case of extreme situation

The tension would change a lot for case of extreme situation, like 0.1Hz frequency and 6m amplitude, as shown in Fig.6 where the taut-slack happens. It is noted that both the maximum tension and gap value between peak and trough increase, 3 times and 10 times of the quasi-static ones respectively, when the SFT movement gets larger. What’s more, the value of the minimum tension would be zero at certain time when the slack happens. The spectrum plot of the top tension during the dynamic response is presented in Fig.7 where the dynamic tension is 30% higher than the quasi-static value at the excitation frequency and up to 50% of the quasi-static value at the double- frequency of excitation. Differently from Fig.4b, the tension with higher frequency gradually decreases with the drop of frequency.

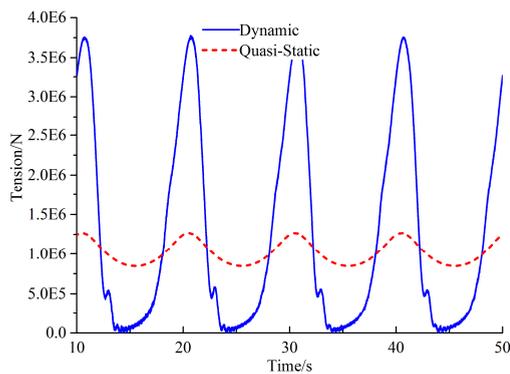


Fig.6 Top Tension of mooring line for case of the taut-slack

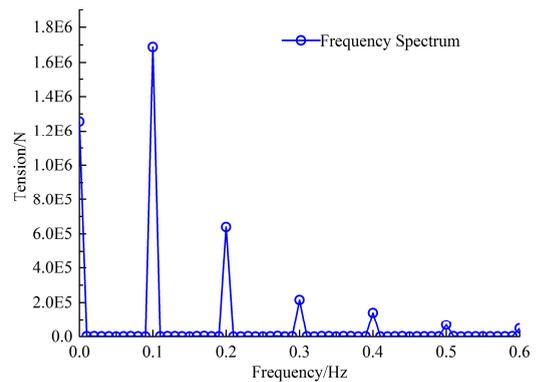


Fig.7 Spectrum plot of the top tension for case of the taut-slack

The temporal-spatial evolutions of the displacements are presented in Fig.8, and the wave forms for both displacements change a lot compared to Fig.4. More specifically, for case of the horizontal displacement, the travelling wave can be apparently seen during the positive motion but, notably, not during the negative motion. And, the vertical displacement is no longer characterized as one standing wave but, instead, two short waves close to the structure ends for case of the negative displacement, while the positive motion is still dominated by one standing wave. The maximum displacements occur at middle point for positive displacement and at two ends for negative displacement respectively. To explain all these, generally speaking, as the SFT’s motion gets larger, the gravity force of the catenary can no longer provide the restoring tension to the structure as it does for the case of small SFT’s motion due to the additional initial and damping effects compared with the quasi-static situation.

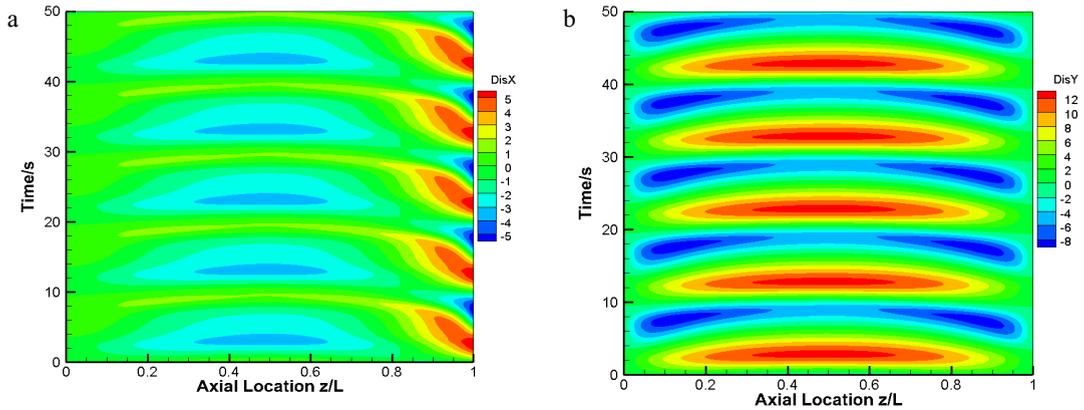


Fig.8. Temporal-spatial evolution of the displacement for case of the taut-slack, (a) the horizontal displacement; (b) the vertical displacement

4. Influences of the structural properties on the restoring performance

Here the impacts of the structural properties, such as the initial shape and mass, on the dynamic response of the catenary are examined, because the mass (the material density or that of the connected buoyancy) or shape may directly change the tension and then the dynamic response. The values of all selected density range as 8000; 10,000; 11,200; 12,000 and 14,000.

The RMS displacements along the catenary length are plotted in Fig.9 at SFT's movement of 0.1Hz frequency along with 6m and 9m amplitude respectively. As mass density gets smaller, the displacement gets larger while the slack gets more likely to happen. And, the short wave near the top end becomes more apparent of which wave length decreases but the wave amplitude increases. It is also noted that, for case of 6m SFT's amplitude, the maximum displacement (almost close to the middle point of the catenary) rises as the mass density gets larger while, for case of 9m SFT's amplitude, the maximum displacement drops. The reason for this might be the hydrodynamic drag which is proportional to the square of structural velocity. As the mass gets smaller (then the structural velocity gets larger), the damping of hydrodynamic drag to the structure movement becomes more significant than the effect of mass decrease which originally make the movement of catenary rise. Figure 10 shows the tension amplification ratio of the top tension at different mass densities, where the tension amplification rises with the increase of SFT's amplitude or the decrease of mass density and the curves gets steeper with the decrease of mass density.

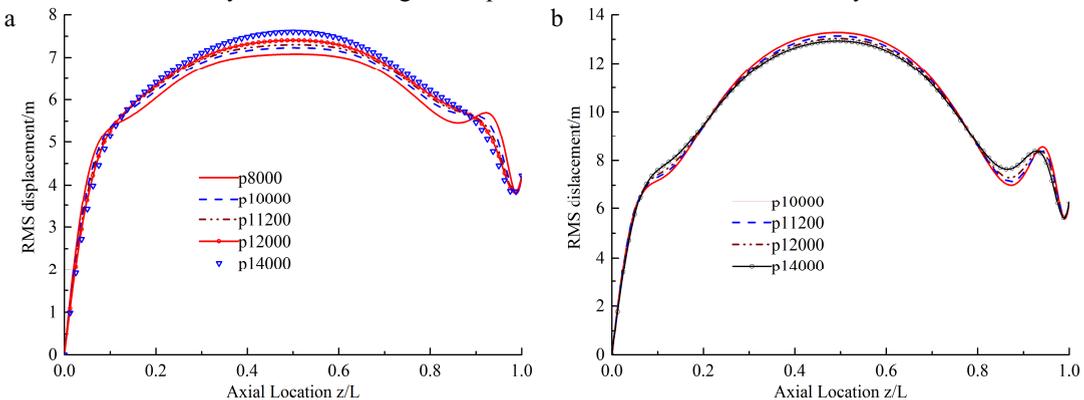


Fig.9 RMS displacement, (a) at 6m SFT amplitude; (b) at 9m SFT amplitude

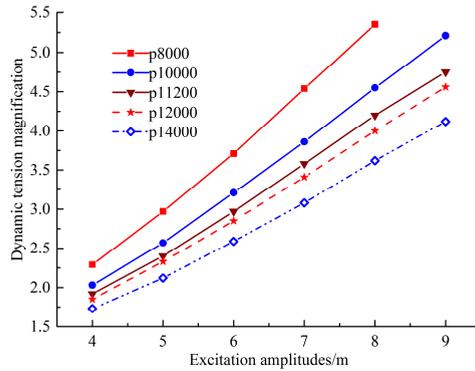


Fig.10 Tension magnification at different mass densities

Initial tension is one of important design factors by which initial angle and shape of the catenary can be determined statically, then the top tension and restoring performance would be directly influenced during dynamic response, as shown in Fig.1 and Eq.(8). The dynamic response of the catenary with different values of initial tension ratio (the top tension to the structural gravity) and the corresponding initial shape (indicated as the horizontal projection), as listed in Table 2, are analyzed so as to examine the impact of the initial tension.

Tab.2 The parameters of the mooring-line

| Case | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-------------------------|------|------|------|------|------|------|------|------|------|------|
| Horizontal projection/m | 700 | 701 | 702 | 703 | 704 | 705 | 706 | 707 | 708 | 709 |
| Initial tension ratio | 1.52 | 1.55 | 1.58 | 1.62 | 1.66 | 1.71 | 1.76 | 1.82 | 1.88 | 1.96 |

The RMS displacements along the catenary length, at SFT’s 0.1Hz frequency and 6m amplitude, are plotted in Fig.11. As the tension ratio gets larger, the RMS displacement gets larger while the slack gets more likely to happen. Principally because the traits of catenary which is initially shaped by the top tension. As a comparison, the RMS displacements of quasi-static method is also presented in Fig.12. Comparing Fig.11 and 12, we can see that, at same values of SFT’s amplitude and frequency, as the tension ratio rises the displacement gets larger, or velocity gets larger and consequently the dynamic tension is larger. Figure 13 shows the tension amplification (the ratio of the maximum dynamic tension to the static one) against the initial tension, where the tension amplification rises linearly with the increase of the initial tension. It means the impact of SFT’s amplitude on tension amplification becomes more profound as the initial tension ratio rises.

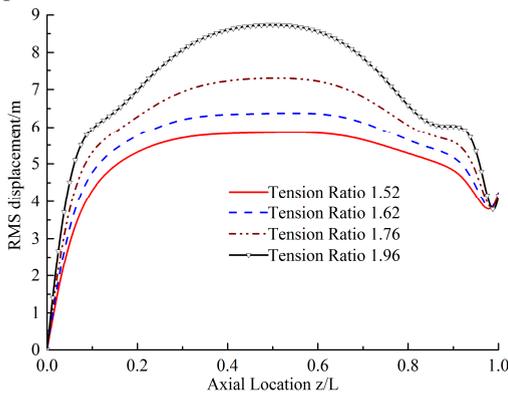


Fig.11 RMS displacement

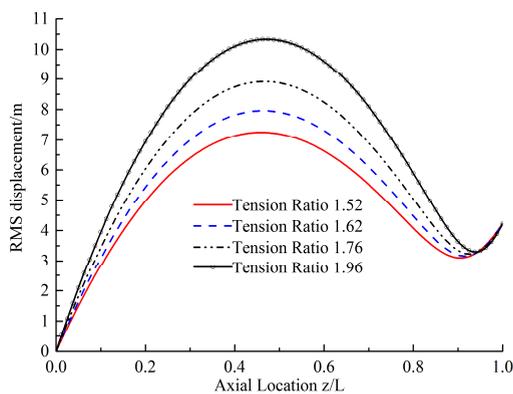


Fig.12 RMS displacement in Quasi-static condition

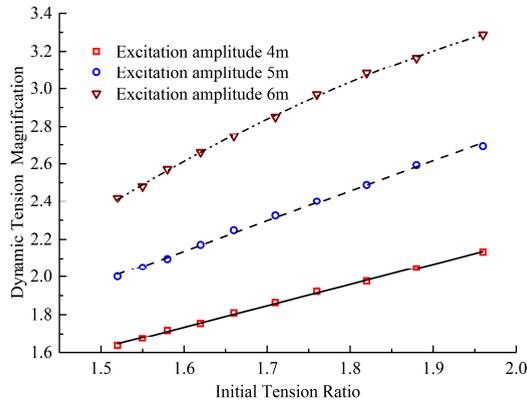


Fig.13 The tension magnification versus the initial tension ratio

5. Conclusions

The restoring performance of the catenary mooring-line is examined while the dynamic behavior of the catenary mooring-line, inertia and hydrodynamic damping, is considered, compared to the previous quasi-static method. First, the nonlinear dynamic response model of the mooring system is developed based on the 3D dynamic theory of catenary and our numerical simulations. Then the influences of the amplitude and frequency of SFT's motion, along with mooring-line's structural parameters, on the catenary's displacement and dynamic tension are studied and compared with the quasi-static results. Also, the taut-slack phenomenon caused by the inertial force and hydrodynamic damping is analyzed. Based our numerical results we draw the following conclusion:

- 1) When the motion of the SFT is smaller, the dynamic response of the mooring-line is characterized as a stable stand wave. The value of dynamic tension due to dynamic characteristics is about 20% of the quasi-static method, and the tension gap between the peak and trough is around three times larger than that of quasi-static method.
- 2) As SFT's movement gets larger, the slack is more likely to happen and the dynamic tension, at the excitation frequency, is about 30% higher than the quasi-static tension. When the mooring-line undergoes taut-slack, the response is characterized as travelling wave principally because the structural gravity can no longer provide the restoring tension, but to mainly balance with the inertial and hydrodynamic drag, to the floating top.
- 3) The influences of mooring-line's mass density and/or initial tension on the dynamic top tension, or the magnification factor of top tension caused by mooring-line's dynamic behavior, become more profound with the decrease of mass density or increase of the initial tension ratio.

Acknowledgements

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References

- [1] Ostlid H. When is SFT competitive? 1st International Symposium on Archimedes Bridge (ISAB-2010). Qiandao Lake, P R China. Oct 17-20, 2010. *Procedia Engineering*, 2010, Vol.4: 3-11.
- [2] Hong Y S, Ge F. Dynamic response and structural integrity of submerged floating tunnel due to hydrodynamic load and accidental load. 1st International Symposium on Archimedes Bridge (ISAB-2010). Qiandao Lake, P R China. Oct 17-20, 2010. *Procedia Engineering*, 2010, Vol.4: 3-11.
- [3] Mavrakos S A, Chatjigeorgiou J. Dynamic behaviour of deep water mooring lines with submerged buoys [J]. *Computers & structures*, 1997, 64(1): 819-835.
- [4] Papazoglou V J, Mavrakos S A, Triantafyllou M S. Non-linear cable response and model testing in water [J]. *Journal of sound and vibration*, 1990, 140(1): 103-115.
- [5] Van Den Boom, H.J.J. Dynamic behaviour of mooring lines. *Proceedings of Behaviour of Offshore Structures*, 1985:359-368.

- [6] Gobat J I, Grosenbaugh M A. A simple model for heave-induced dynamic tension in catenary moorings [J]. *Applied Ocean Research*. 2001, 23(3): 159-174.
- [7] Vassalos D, Huang S. Dynamics of small-sagged taut-slack marine cables [J]. *Computers & structures*, 1996, 58(3): 557-562.
- [8] Huang S, Vassalos D. A numerical method for predicting snap loading of marine cables [J]. *Applied Ocean Research*, 1993, 15(4): 235-242.
- [9] Chen X, Zhang J, Ma W. On dynamic coupling effects between a spar and its mooring lines [J]. *Ocean Engineering*. 2001, 28(7): 863-887.
- [10] Zhang S, Tang Y, Liu X. Experimental investigation of nonlinear dynamic tension in mooring lines [J]. *Journal of Marine Science and Technology*. 2012, 17(2): 181-186.
- [11] Lindahl J, Sjoberg A. Dynamic analysis of mooring cables, *Proceedings of second international symposium on ocean engineering and ship handling*, Sweden, 1983:281-319.
- [12] Paulling J R, Webster, W C. A consistent large-amplitude analysis of the coupled response of a TLP and tendon system. *Proceedings of Offshore Mechanics and Arctic Engineering, OMAE*, Tokyo, Japan, 1986, 3:126-133.
- [13] Kwan C T, Bruen F J. Mooring Line Dynamics: Comparison of Time Domain, Frequency Domain, Quasi-static Analyses. *Proceeding of the 23rd Offshore Technology Conference: Houston TX, May 1991*, 95-108.
- [14] Chen X. *Studies on dynamic interaction between deep-water floating structures and their mooring/tendon systems* [D]. Texas A&M University, 2002.
- [15] Love A E H. *A treatise on the mathematical theory of elasticity* [M]. Cambridge University Press, 2013.
- [16] Garrett D L. Dynamic analysis of slender rods [J]. *Journal of energy resources technology*. 1982, 104(4): 302-306.