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A DYNAMIC VIBRATION MODEL FOR ICE-STRUCTURE INTERACTION

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ABSTRACT

A dynamic vibration model for ice-structure interaction is developed on the basis of Sodhi's work. By introducing the broken length of the moving ice sheet, ice-structure interaction is divided into three phases: Loading, Extrusion and Separation. The dynamic equations of both structure and acting ice sheet and their solutions for each phase are presented in detail, and the various normalized dimensionless parameters related to this model are comprehensively studied. This model presents some meaningful results that are valuable for both structural designs against ice load and mitigation of ice-induced vibration.

INTRODUCTION

The hazardous vibration leading to the collapse of offshore structures was first experienced in Bohai Gulf in 1969 (Fang and Duan, 1994; Duan and Liu, 1995; Duan, 1997), and then observed (Maattanen, 1987) in the superstructures of the first slender steel aids-to-navigation constructed in 1973 and later collapsed in 1980 in the Gulf of Bothnia where a great number of sequences with fully developed resonance especially self-induced vibration were recorded in flexible structures such as the Norstromsgrund lighthouse (Engelbrektson, 1989, 1997). Field observation indicated even thin ice sheets of 0.1m thickness were inducing severe resonant vibrations in the 1973-constructed first Kemi-2 steel lighthouse. The substantial vibrations of the massive drilling caisson Molikpaq in the Beaufort Sea nearly brought the collapse of the structure (Jefferies and Wright 1988; Eranti, 1991; Enkvist and Eranti, 1991). Strong vibrations of channel markers were also recorded in the Baltic (Karna, 1992). CNOOC (China National Offshore Oil Corporation) and its subsidiary Bohai Oil Company have been seriously involved into the problem of severe vibrations of jacket production platforms in Bohai Gulf, with great effort being made to reduce the dynamic effect on the structures as well as the fatigue and fracture of the structures in ice-induced vibrations (Liu and Duan, 1996; Gao and Duan, 1998; Duan, Li and Li, 1999; Duan, Liu and Fan, et al., 2000). This paper presents a dynamic ice-structure interaction model to investigate the influential factors of ice and structure parameters on the vibration of offshore structures, which could be practically referred for the mitigation of ice-induced vibration.

Theories of static ice forces on structures were established from late sixties to late eighties, and many formula were developed (Sanderson, 1988). However, ice action on the structure when an ice sheet moves against a structure is actually a dynamic ice-structure interaction, the dynamic ice force demonstrating much higher peak and mean values than calculated or by the developed formula for static ice force calibrated by experiments (Duan, Liu, Fan, et al., 2000; Azarnejad, Mayne and Brown, 1999). Especially for flexible structures, the test results illustrated a dynamic amplification in the ice forces (Singh, Timco, Frederking, et al, 1990). The dynamic ice force and the structure responses are time dependent processes. Such dynamic ice-structure interaction was simulated by Sodhi (Sodhi, 1991; 1994) in a theoretical model. He described an ice-structure interaction event in three phases: loading, extrusion and separation. In the loading phase, the interaction force increases linearly with increase in the relative displacement between the ice and the structure until the ice fails as indicated by sudden decrease in interaction force. In the extrusion phase, the ice sheet moves against the structure pulverized in front of the structure. In the separation phase, the ice sheet separates from the structure and the interaction force is zero, the structure executing a transient motion until the gap is closed at the beginning of a new cycle. Sodhi presented the differential equations and gave their solutions for each phase of the interaction model. The dependence of the transition velocity and the crushing frequency on different parameters was investigated from the results of the model simulation.

This paper presents a dynamic vibration model for ice-structure interaction based on Sodhi's work. The pulverized length L of the moving ice sheet is included in the developed model, which plays an important part in ice-structure interaction. The parameters of the moving ice and the structure are normalized to study their effect on the ice structure interaction and their interaction between each other. The parameter studies present some meaningful results valuable for both structural designs against ice load and mitigation of ice-induced vibration.

NOMENCLATURE

m Mass of structure
k Stiffness of structure
c Damping of structure
X Displacement of structure
x Normalized displacement of structure
Y Displacement of the ice sheet
y Normalized displacement of the ice sheet
k_i Effective stiffness of ice sheet
P_f Limit strength of ice sheet
P_e Strength of pulverized ice sheet
L Pulverized length of ice sheet
l Normalized pulverized length of ice sheet
V Ice moving velocity
v Normalized ice moving velocity
α Strength ratio
A Normalized amplitude of the structural displacement
δ Static displacement of structure
Z Penetrated distance
f_n Natural frequency of the structure
f_b Breaking frequency of the ice sheet
f Ice action frequency
ξ Damping ratio
μ Stiffness ratio
V/Lf_n Reduced ice moving velocity

ASSUMPTION

For a single degree of freedom with the mass m, stiffness k and damp c, the ice sheet moves at a velocity of V. The following assumptions are made on the basis of experimental results and Sodhi's contributions:

In the loading phase, the interaction force $F(t)$ is in linear relation with the relative displacement $r(t)$ between the ice and the structure:

$$F(t) = k_i \; r(t) \tag{1}$$

where k_i is the effective stiffness of the ice sheet.

A limit pressure P_f of the ice on the structure is taken as the maximum interaction force of $F(t)$ when a maximum pulverized length L of the ice sheet is reached. The interaction force when the structure penetrates the pulverized length is assumed as a constant P_e. L is dependent on the properties of the ice sheet and the contact conditions of the structure.

In the separation phase, the ice sheet separates from the structure and the interaction force $F(t)$ is zero, the structure executing a transient motion until the gap is closed at the beginning of a new cycle.

The three phases of ice structure interaction are shown in Fig.1.

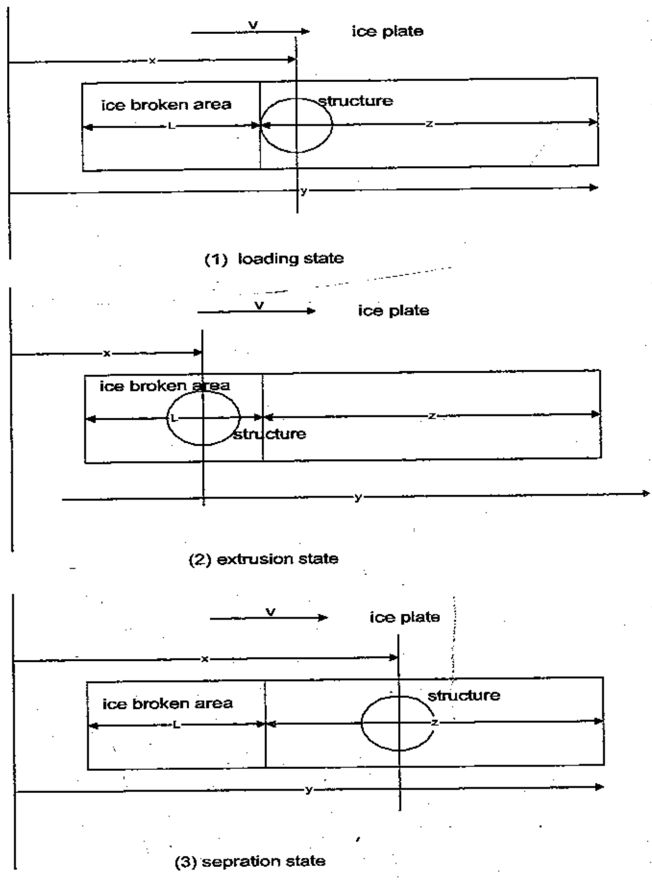

Fig.1 Three phases of ice-structure interaction

DIFFERENTIAL EQUATIONS OF MOTION FOR DYNAMIC ICE-STRUCTURE INTERACTION

Let X, $\frac{dX}{dt}$, $\frac{d^2X}{dt^2}$ stand for the displacement, velocity and acceleration of the structure, Y, $\frac{dY}{dt}$, $\frac{d^2Y}{dt^2}$ the displacement, velocity and acceleration of the ice sheet, Z the penetrated distance of the structure into the ice sheet since the beginning of the event (cycle), giving $\frac{dY}{dt}=V$, $\frac{d^2Y}{dt^2}=0$.

Let t_{j0} represent the beginning of an event/cycle, t_{jT} the end of the even/cycle, $j=l,e,s$ presenting the three phases, the equations of motion in the three phases can be written as given below:

1) Loading phase $(t_{l0}<t<t_{lT})$

$$m\frac{d^2X}{dt^2}+c\frac{dX}{dt}+kX=k_i(Y-X-Z)$$

$$\text{for} \quad k_i(Y-X-Z)\le P_f \tag{2}$$

2) Extrusion phase $(t_{e0}<t<t_{eT})$

$$m\frac{d^2X}{dt^2}+c\frac{dX}{dt}+kX=P_e$$

$$\text{for} \quad V-\frac{dX}{dt}\ge 0 \text{ and } Y-X-(Z+L)\le 0 \tag{3}$$

3) Separation phase $(t_{s0}<t<t_{sT})$

$$m\frac{d^2X}{dt^2}+c\frac{dX}{dt}+kX=0$$

$$\text{for} \quad Y-X-Z\le 0 \tag{4}$$

As stated earlier, the end of one phase is the start of the next phase, and the value of Z remains constant during all phases, except at the end of extrusion phase when it is incremented by the penetration of the structure into the ice sheet. It shall be stressed that only after the loading phase has been ended, can the interaction enter the extrusion or separation phases, and only when the accumulated distance penetrated into the ice sheet equals to the pulverized length L, can the interaction enter the next loading phase. The interaction procedure simulated is shown in Fig.2.

The ending time t_{jT} of each phase is given as below:

(1) for loading state t_{lT}: $\quad k_i(Y-X-Z)\le P_f$

$t>t_{lT}$ indicates entering the extrusion phase: $t_{e0}=t_{lT}$

(2) for extrusion state t_{eT}: $\quad Y-X-(Z+L)\le 0$

$t>t_{eT}$ indicates entering the separation phase: $t_{s0}=t_{eT}$

(3) for separation state t_{sT}: $\quad Y-X-Z\le 0$

$t>t_{sT}$ indicates entering the loading phase: $t_{l0}=t_{sT}$

Introducing the following normalized parameters:

$$x=\frac{X}{\delta} \qquad y=\frac{Y}{\delta} \qquad z=\frac{Z}{\delta}$$

$$l=\frac{L}{\delta} \qquad v=\frac{V}{\delta\omega_n} \qquad \alpha=\frac{P_e}{P_f}$$

$$\tau=\omega_n t \qquad \mu=\frac{k_i}{k} \qquad \xi=\frac{c}{2m\omega_n}$$

where $\delta=\frac{P_f}{k}$ is the static displacement, $\omega_n^2=\frac{k}{m}$ the natural frequency of the structure. Equations (2), (3) and (4) become:

(a) for loading phase $(0<\tau<\tau_{lT})$

$$\ddot{x}+2\xi_c\omega_c x+\omega_c^2 x=\mu(y_{l0}+v\tau-z_{l0})$$

$$\text{for} \quad \mu(y-x-z_{l0})\le 1 \tag{5}$$

the solution:

$$x(\tau)=k_r(y_{l0}-z_{l0}+v\tau-2\xi_c v/\omega_c)$$
$$+e^{-\xi_c\omega_c\tau}\frac{\dot{x}_{l0}+\xi_c\omega_c x_{l0}-k_r\left[(y_{l0}-z_{l0})\xi_c\omega_c+v(1-2\xi_c^2)\right]}{\omega_{dc}}\sin\omega_{dc}\tau$$
$$+e^{-\xi_c\omega_c\tau}\left[x_{l0}-k_r(y_{l0}-z_{l0}-2\xi_c v/\omega_c)\right]\cos\omega_{dc}\tau \tag{6}$$

where $\dot{x}\equiv\frac{d}{d\tau}(x)$, $\tau=\omega_n(t-t_{l0})$, $\omega_c^2=1+\mu$, $\tau_{lT}=\omega_n(t_{lT}-t_{l0})$, $\xi_c=\frac{\xi}{\sqrt{1+\mu}}$, $\omega_{dc}=\sqrt{1-\xi_c^2}\,\omega_c$, $k_r=\frac{\mu}{1+\mu}$, $y=y_{l0}+v\tau$, $x_{l0}=x|_{t=t_{l0}}$, $\dot{x}_{l0}=\dot{x}|_{t=t_{l0}}$, $y_{l0}=y|_{t=t_{l0}}$, $z_{l0}=z|_{t=t_{l0}}$ (7)

(b) for extrusion phase $(0<\tau<\tau_{eT})$

$$\ddot{x}+2\xi\dot{x}+x=\alpha$$

$$\text{for} \quad v-\dot{x}\ge 0 \text{ and } y-x-(z_{e0}+l)\le 0 \tag{8}$$

the solution:

$$x(\tau)=\alpha-e^{-\xi\tau}(x_{e0}-\alpha)\cos\omega_d\tau+e^{-\xi\tau}\frac{\dot{x}_{e0}+\xi(x_{e0}-\alpha)}{\omega_d}\sin\omega_d\tau \tag{9}$$

where $\tau=\omega_n(t-t_{e0})$, $\tau_{eT}=\omega_n(t_{eT}-t_{e0})$, $\omega_d=\sqrt{1-\xi^2}$, $x_{e0}=x|_{t=t_{e0}}$, $\dot{x}_{e0}=\dot{x}|_{t=t_{e0}}$, $z_{e0}=z|_{t=t_{e0}}$

(c) for separation phase $(0<\tau<\tau_{sT})$

$$\ddot{x}+2\xi\dot{x}+x=0$$

$$\text{for} \quad y-x-z_{s0}\le 0 \tag{10}$$

the solution:

$$x(\tau)=e^{-\xi\tau}x_{s0}\cos\omega_d\tau+e^{-\xi\tau}\frac{\dot{x}_{s0}+\xi x_{s0}}{\omega_d}\sin\omega_d\tau \tag{11}$$

where $\tau=\omega_n(t-t_{s0})$, $\tau_{sT}=\omega_n(t_{sT}-t_{s0})$, $\omega_d=\sqrt{1-\xi^2}$, $y=y_{s0}+v\tau$, $x_{s0}=x|_{t=t_{s0}}$, $\dot{x}_{s0}=\dot{x}|_{t=t_{s0}}$, $y_{s0}=y|_{t=t_{s0}}$, $z_{s0}=z|_{t=t_{s0}}$

NUMERICAL RESULTS AND DISCUSSION

As shown above, the analytical solutions of the developed model for the three ice-structure interaction phases can be obtained once given the initial conditions at the start of each phase. The numerical calculation procedures for the model are illustrated in Fig.2, and the solutions are shown in Figures 3-9. In this paper, the ice action frequency is defined as :

$f = 1/\Delta T$, $\Delta T = \sum_{n=1}^{N} \left(t_{I0}^{(n)} - t_{I0}^{(n-1)}\right)/N$. $t_{I0}^{(n-1)}$ and $t_{I0}^{(n)}$ stand for the start time of the adjacent loading phases, N the total number of the loading phases covered in the numerical solutions. The breaking frequency of the ice sheet is defined as $f_b = V/L$, the natural frequency of the structure is $f_n = \omega_n/2\pi$, the amplitude of the structural displacement $A = x_{\max} - x_{\min}$, and the ice-structure interaction force $f(\tau) = F(\tau)/P_e$.

As well known, the dynamic ice structure interaction has many influential factors, depending on ice moving velocity, ice crushing strength, ice action and breaking frequencies, ice failure modes, shape and dimension of the structure, structural stiffness and damping, etc. The influence of the factors is discussed as follows based on the numerical results.

<u>Influence of ice velocity, action frequency and structural damp on the vibration responses</u>

Figures 3-5 present the normalized relation of the dynamic displacements of the structure with the reduced ice moving velocity $V/(Lf_n)$. It can be seen that the severe vibration happens in the range of $f_b/f_n = V/(Lf_n) \le 1.5$. The displacement responses will be rapidly reduced when the frequency lies in the range of $f_b/f_n = V/(Lf_n) > 1.5$. In the range of $f_b/f_n = V/(Lf_n) \le 1.5$, there are many coupled vibration peaks each of which corresponds to an ice action frequency $f/f_n = 1/i \quad (i = 1,2,3,...)$. That is to say, the maximum vibration happens when the ice action frequency is close to the structural natural frequency and the vibration peaks occur when the action frequency is 2, 3, or more times of the structural natural frequency. The values of the peaks are mainly dependent on the damping ratio ξ and stiffness ratio of ice plate and structure $\mu = k_i/k$. The change of ξ can prominently change the values of the vibration peaks, but can not change the positions of the peaks. Differently, the stiffness ratio μ can change both the peak values and their positions. As μ increases, the range of severe dynamic vibration decreases, and the vibration responses increase with the decrease of reduced ice moving velocity.

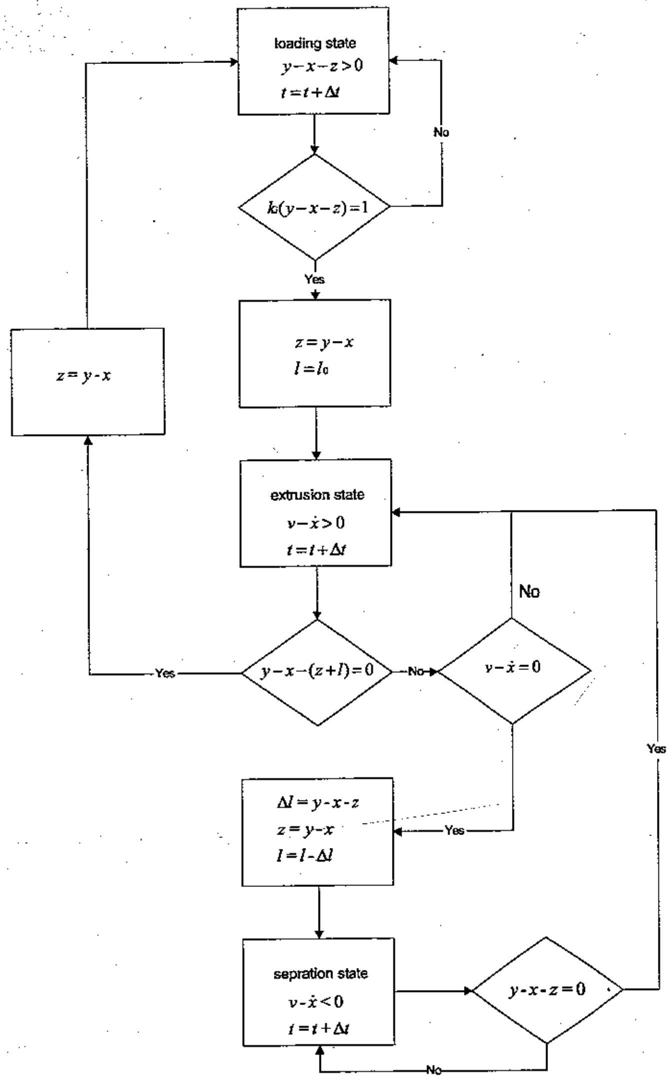

Fig.2 Numerical calculation procedures for the proposed model

<u>Relation between ice action frequency and breaking frequency</u>

Fig.6 presents the relation of the ice interaction frequency f/f_n with the breaking frequency f_b/f_n at different stiffness ratio μ. It shows that these two frequencies are in linear relation at the same μ, and f is approaching f_b when μ is increasing. The difference between f and f_b becomes larger when μ is getting smaller.

<u>Influence of structural stiffness on the vibration responses</u>

The relation of structural displacement responses with the stiffness ratio μ at the same ice moving velocity is shown in Fig.7. Apparently, the structural stiffness effectively influences the ice structure interaction frequencies, and prominently controls the structural vibration.

<u>The ice structure interaction states</u>

Figures 8-10 show the time histories of both ice forces and structural responses. It can be seen the separation phase will not occur if $\mu << 1$, i.e., for flexible structures, only loading and extrusion phases exist. The separation phase takes place only when $\mu > 1$ and the ice sheet moves at low velocities.

CONCLUDING REMARKS

This model is proposed for ice structure interaction vibrations when the ice sheet moves against the structure breaking in bending or buckling modes. It has very good application especially when some pulverized area of ice in front of the structure exists. The influence studies of the parameters such as ice moving velocity, ice action and breaking frequencies, structural stiffness and damping, etc, and their numerical results are valuable for both structural designs against ice load and mitigation of ice-induced vibration.

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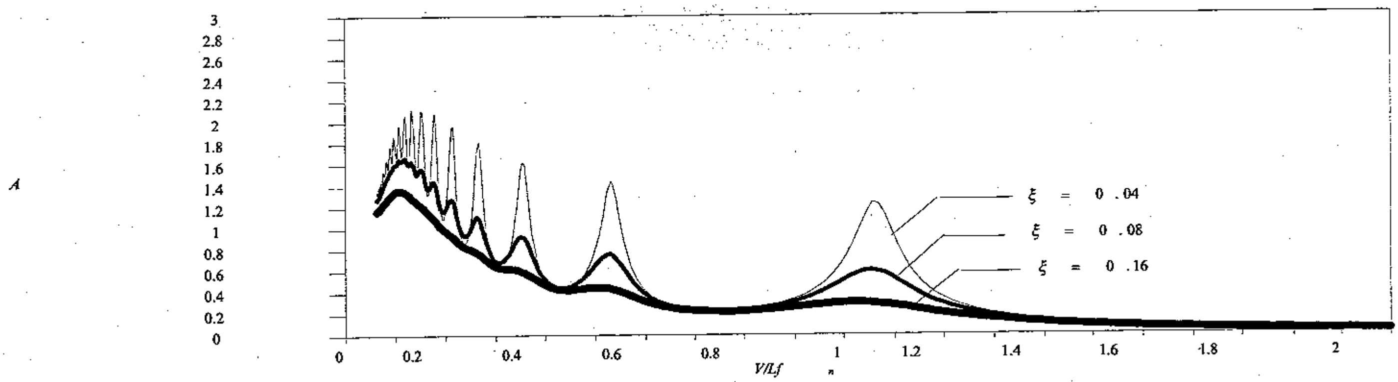

Fig.3 The displacement of structure vs. reduced ice moving velocity($l = 100, \alpha = 0.2, \mu = 0.125$)

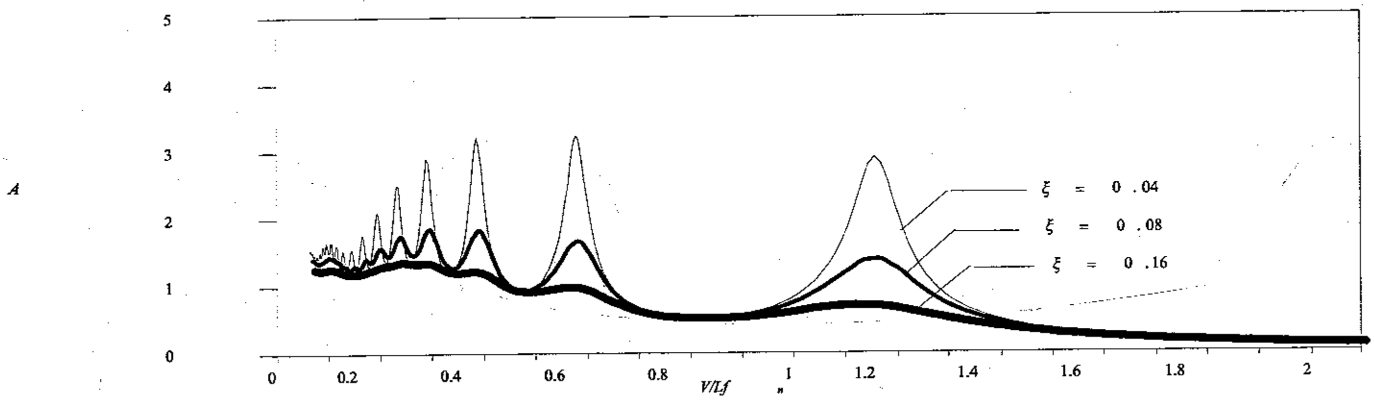

Fig.4 The displacement of structure vs. reduced ice moving velocity($l = 100, \alpha = 0.2, \mu = 0.05$)

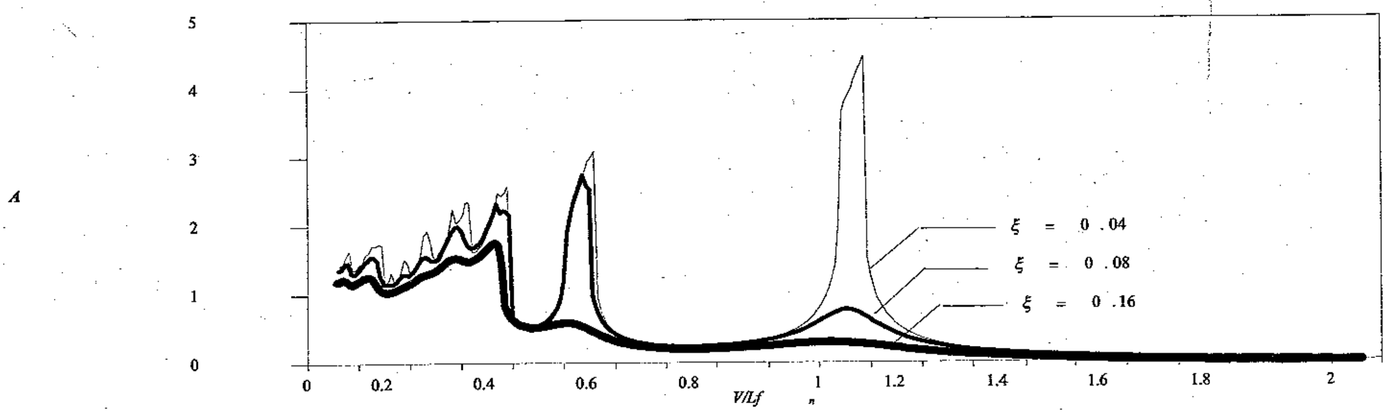

Fig.5 The displacement of structure vs. reduced ice moving velocity ($l = 10,\ \alpha = 0.2,\ \mu = 1.25$)

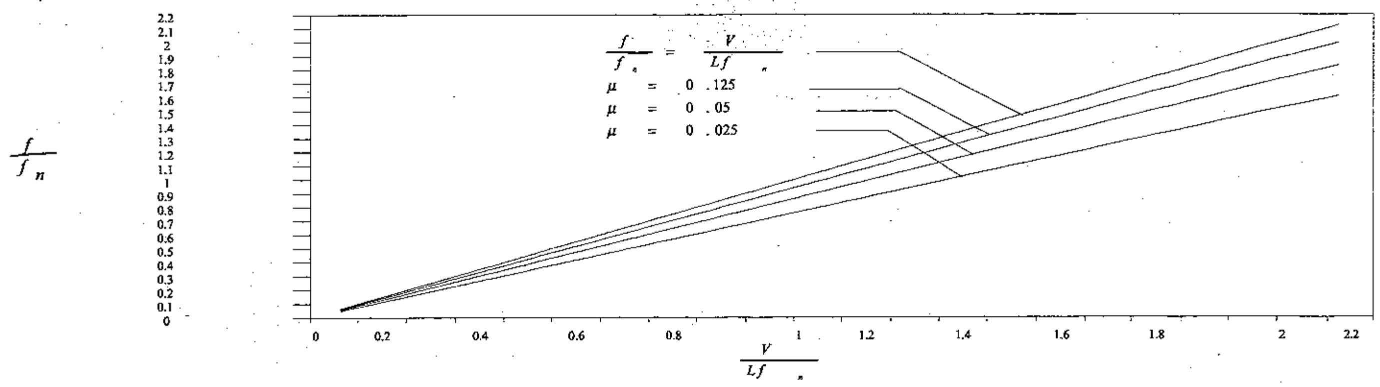

Fig.6 Ice force frequency vs. ice breaking frequency ($\xi = 0.04,\ \alpha = 0.2$)

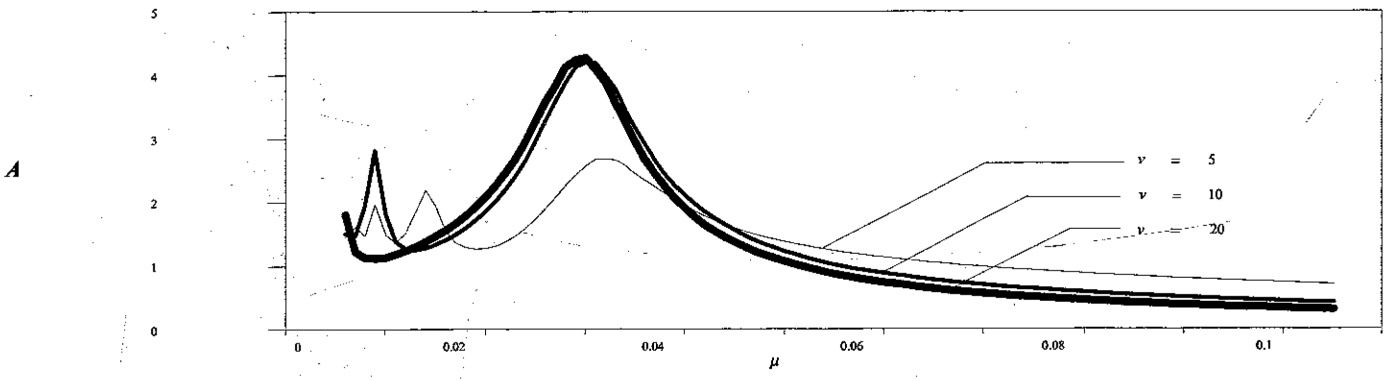

Fig7. Displacement of structure vs. stiffness ratio ($l = 100,\ \alpha = 0.2,\ \xi = 0.04$)

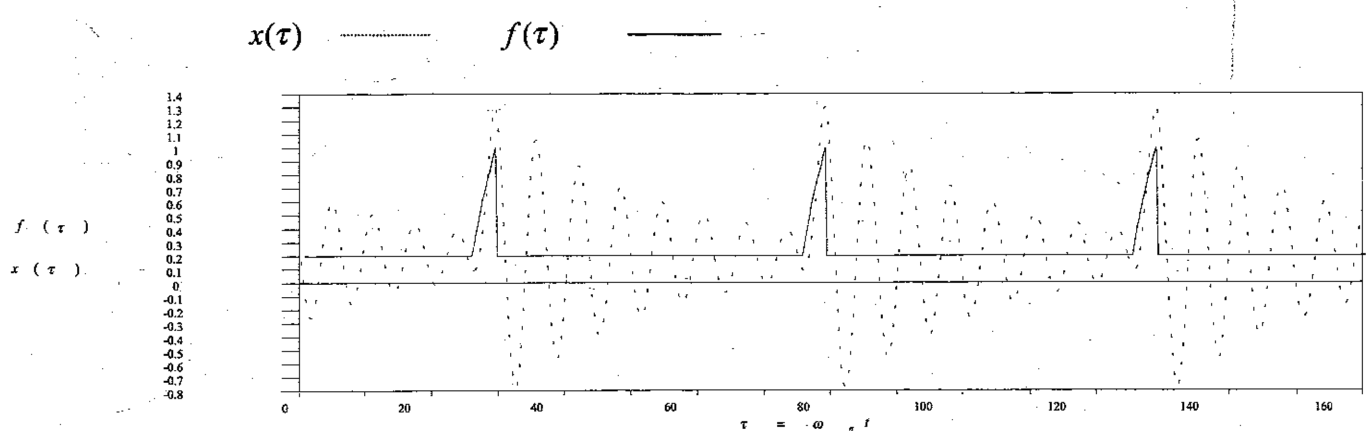

Fig.8 Time histories of ice force and structural displacement ($\nu = 2.14,\ \mu = 0.125,\ l = 100,\ \alpha = 0.2,\ \xi = 0.04$)

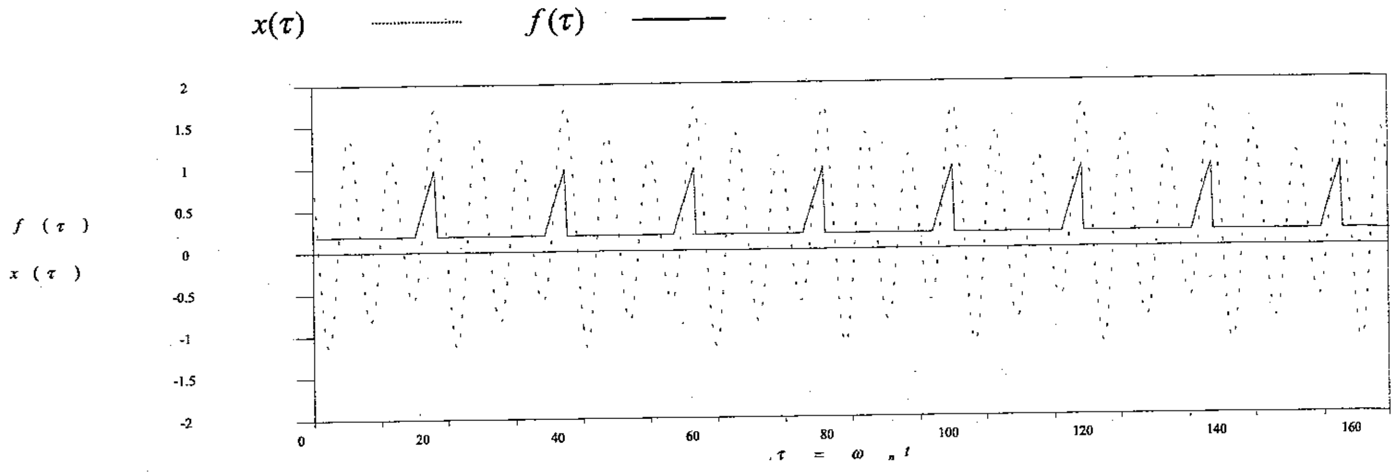

Fig.9 Time histories of ice force and structural displacement ($\nu = 6.04,\ \mu = 0.05,\ l = 100,\ \alpha = 0.2,\ \xi = 0.04$)

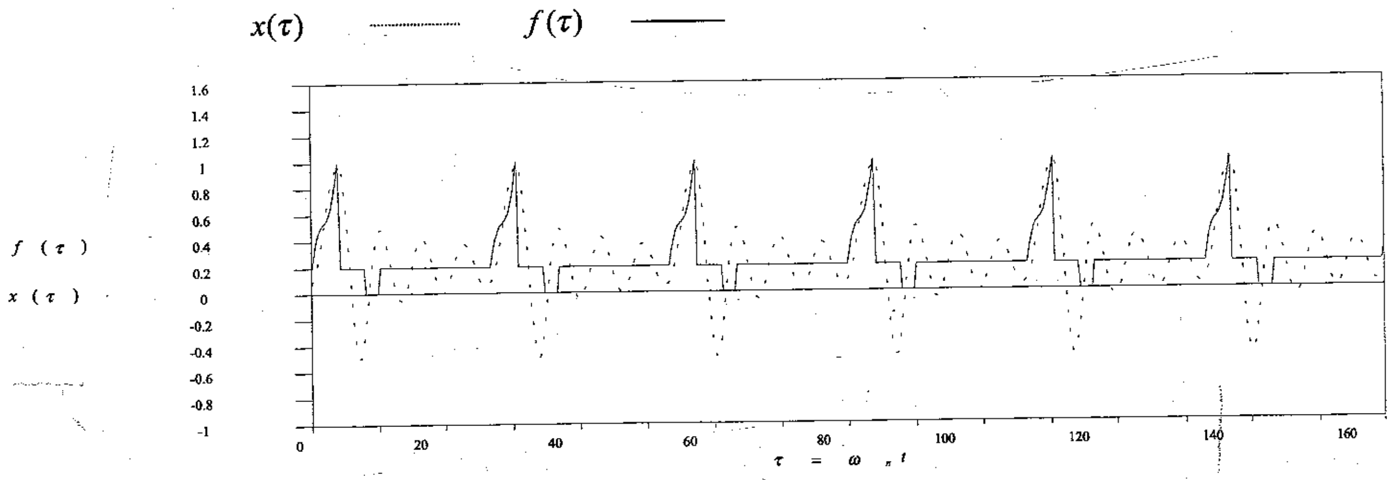

Fig10. Time histories of ice force and structural displacement ($\nu = 0.4,\ \mu = 1.25,\ l = 10,\ \alpha = 0.2,\ \xi = 0.04$)