# MECHANICS OF OFFSHORE PIPELINES IN SEVERAL OPERATING STATES AND ANALYZING SOFTWARE 

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#### Abstract

Offshore pipelines are indispensable structures during the marine petroleum exploitation. There are manifold operation states having to do with pipelines such as in situ operation, span, trenching, pipelaying, lifting of pipe/riser, etc. The behavior of pipelines in one state differs greatly from the others. In addition, the structure configuration, the sea severity, and the seabed soil conditions are complex. Therefore, strength analysis of offshore pipelines becomes a rather difficult and onerous task. Presently, there is a lot of technological difficulty in the course of analysis, among which are not only theoretical problems such as geometric non-linearity and moveable boundary, but also practical problems. To tackle the problems, analytical methods, numerical methods such as FEM and shooting method are adopted respectively and jointly. Based on the theoretical research, integrated software named 'Offshore Pipelines Strength Analysis' is developed. This software can carry through strength analysis of pipelines in different operating state, being up to the requirement of engineers. The theoretical background and interface of the software are presented.


Keywords: offshore; pipelines; strength; software

## INTRODUCTION

Offshore pipeline is lifeline in ocean exploitation. Due to the importance and the valuableness, it becomes a crucial problem to perform accurate and rapid strength analysis of the pipelines.

There are multiplicate operation states such as in situ operation, span, trenching, pipelaying, lifting of seabed pipe, etc. The mechanical behavior of pipelines in one state is different from the others. Furthermore, structure configuration, sea severity, and the seabed soil conditions are complex. Therefore, strength analysis of offshore pipelines becomes a very difficult and onerous task.

Presently, there are a lot of theoretical and feasible difficulties in the course of analysis. Several problems are listed as follows:
$\triangle$ Geometric non-linearity and moveable boundary in the state of pipelaying and lifting;
$\triangle$ Loads complexity because of simultaneous action of environmental loads and operation loads such as wave, current, thermal stress, operating pressure, etc.
$\triangle$ Varity of configuration of cross section, loads and boundary conditions along the zigzag axis of pipeline;
$\triangle$ Great diversities of loads and boundary conditions due to the difference between each operating state and load case.
Therefore, software that can solve the above-mentioned difficulty encountered in operating state is greatly needed.

In this paper, analytical methods, numerical methods such as FEM and shooting method are adopted respectively and jointly to tackle the technological difficulties, which are listed below:

- Strength analysis of in-situ double-wall riser. FEM method is employed to solve this problem. Several different validated FEM package can be selected as the calculating kernel up to the requirement, the preprocessor and postprocessor developed in this paper accordingly is attached to the kernel in order that the analyzing process can be executed automatically.
- The natural frequency of span pipeline taking account of the soil constraint. Part of pipeline is modeled as a beam on elastic foundation to reckon with the soil constraint.
- The non-linear problem and moveable boundary problem due to the large displacement generated in the stage of lifting and pipelaying. Singular perturbation technique and shooting method are adopted to tackle the severely non-linear difficulty.
- Strength analysis of trenching. Linear beam theory is adopted to solve the problem, the movability of boundary is considered.
Based on the theoretical research, integrated software named 'Offshore Pipelines Strength Analysis' (OPSA) is developed. This software can carry out strength analysis of pipelines in above-mentioned operating state, being up to the requirement of offshore engineers.

Hereinafter, theoretical backgrounds of every part of OPSA will be presented in turn.

## STRESS ANALYSIS OF IN-SITU RISER

Risers are pipelines transferring the products from platform to the export-lines or in field flow-lines. There are multiple kinds of loads acting on risers such as: wave, current, ice,
thermal stress, operating pressure, etc. Translations are specified in fastener where riser is tied up with the leg of platform. Because of the complexity of geometric configuration, loads and boundary conditions, the finite element method (FEM) is the best alternative.

The pipe element can be used to model single-wall riser, which is the case in special offshore structure analysis software. While for double-wall riser shown in Fig.1, this element is incapable for the reason that related formulae of general pipe element are based on single-wall pipe. To solve this problem, plate/shell element has to be adopted although obviously such measure makes the task more onerous. Unfortunately, plate/shell element is scarcely offered in special offshore software.

There are several universal and validated FEM packages offering plate/shell element. However, unlike some special offshore structure analysis software, such universal FEM packages can not execute strength analysis continuously and automatically as the special software can do.

(a)

(b)

Fig. 1 Geometric figure of double-wall riser
In this paper, we solve the problem. A preprocessor and a post-processor are designed. Together with the calculating kernel, the strength of double-wall riser can be analyzed continuously and automatically. If the essential parameters of the pipeline are inputted, the preprocessor can automatically create input data whose format is suited for FEM analysis and accords with that of some validated FEM code (the FEM code can be designated by the user). The designated FEM code acts as the calculating kernel. Plate/shell element is adopted for double-walled riser, while for single-walled riser, pipe element is adopted as usual. The post-processor can automatically generate output data, such as stress and displacement field, which can satisfy the demand of design engineers. The
preprocessor, the calculating kernel and the post-processor constitute the riser analysis module.

In this module, wave and current loads are calculated according to Morison's formula, ice and earthquake loads are calculated according to certain guideline.

As an example, a riser in both original and displaced state is shown in Fig.1. Three operation states are analyzed. According to the analyzing result, the maximum von-Mises stress is found to be in the vicinity of the elbow nearest to the platform in all states.

## SPAN ANALYSIS

Bending stresses will be introduced in the pipeline due to the irregularities of seabed. If the span length exceeds a certain value, the induced stresses of pipe will attain a dangerous level. To prevent consequently possible static damage, the allowable maximum length $l_{s}$ must be determined according to accurate stress prediction. If the span length exceeds the allowable value, measures have to be taken either to alter the pipeline route or to minimize the bottom irregularities based on technical and economic considerations.

Another indispensable problem is vortex-induced vibration (VIV). Span pipe may vibrate with large amplitude if the natural frequency of pipe and the shedding frequency of vortex satisfy some certain relation, e.g. the two frequencies coincide. Mechanism of VIV is still an active research domain for scientists. For offshore engineers, they take some practical measures to avoid VIV. One of such measures is to shorten the span length in order to keep the natural frequency of pipe away from the shedding frequency of vortex. Therefore, accurate evaluation of natural frequency of pipe becomes an important problem in the process of determining the maximum span length $l_{v}$ that can avoid VIV.

The smaller one between $l_{s}$ and $l_{v}$ is the maximum allowable span length.

## Static Analysis

The rule of determining $l_{s}$ is that the maximum von-Mises stress is less than allowable stress $[\sigma]$. Two components of stress to be considered are circumferential normal stress and axial normal stress. The other stress components are considered to be negligible because they are comparatively small.

The circumferential normal stress is given by:

$$
\begin{equation*}
\sigma_{c}=p R / t \tag{1}
\end{equation*}
$$

where $p$ is the intensity of pressure acting on pipe wall along the radial direction, $R$ is the radius of pipe, and $t$ is the thickness of pipe wall.

The axial normal stress is induced by bending, Poisson effect, and residual stress. It is calculated as follows:

$$
\begin{equation*}
\sigma_{a}=q l^{2} R / 8 I+v p R / t+\sigma_{0} \tag{2}
\end{equation*}
$$

where $q$ is load acting on pipe per unit length including selfweight, wave force, and current force. $l$ is the span length of pipe, $I$ is the moment of inertia of the cross-section about an axis through its centroid at right angles to the plane of pipe bending,
$v$ is Poisson's ratio, $\sigma_{0}$ is residual stress.
The von-Mises stress $\sigma_{v}$ of pipe is then given by:

$$
\begin{equation*}
\sigma_{v}=\sqrt{\sigma_{c}^{2}+\sigma_{a}^{2}-\sigma_{c} \sigma_{a}} \tag{3}
\end{equation*}
$$

If $\sigma_{v}$ is equal to $[\sigma]$,the correspondent span length of pipe is the maximum static allowable span length. Substituting equations (1)(2) into (3), letting $\sigma_{v}$ equal $[\sigma]$, we can obtain the maximum static allowable span length $l_{s}$.

## VIV Analysis

If the current velocity and outer diameter of pipeline are set, an effective measure to prevent the occurrence of VIV is to alter $\omega_{n}$ (the natural frequency of pipeline). Offshore engineers usually enlarge $\omega_{n}$ in order to keep it away from $\omega_{v}$ (frequency of shedding vortex). Practically, $\omega_{n}$ is often made to be some integer multiple of $\omega_{v}$ :

$$
\begin{equation*}
k \omega_{v}=\omega_{n} \tag{4}
\end{equation*}
$$

where $k$ can be taken as some integer according to engineer's experience or guideline, e.g. 10.

For simplicity, $\omega_{v}$ can be obtained by:

$$
\begin{equation*}
\omega_{v}=2 \pi S_{t} V / D \tag{5}
\end{equation*}
$$

where $S_{t}$ is Strouhal number, $V$ is the current velocity, and $D$ is the outer diameter of pipeline. For a large range of Reynolds number $R e, S_{t}$ equals to 0.2 approximately (Blevins, 1977).

The most effective method to modify $\omega_{n}$ is to shorten the span length of pipeline. Then the relations of span length and $\omega_{n}$ must be derived first.

As shown in Fig.2, pipeline is divided into three parts(Б о родавкин П.П, 1980). Part I and III are buried in soil, which are modeled as semi-infinite beam on elastic foundation. Part II is a span beam elastic supported at two ends. The origin is placed in the midpoint of part II.


Fig. 2 Span segment of seabed pipeline

The governing equation of part II is:

$$
\begin{equation*}
E I \frac{\partial^{4} y_{I I}}{\partial x^{4}}+m_{I I} \frac{\partial^{2} y_{I I}}{\partial t^{2}}=0 \tag{6}
\end{equation*}
$$

The governing equation of part I or III is:

$$
\begin{equation*}
E I \frac{\partial^{4} y_{I I I}}{\partial x^{4}}+m_{I I I} \frac{\partial^{2} y_{I I I}}{\partial t^{2}}+k_{0} D y_{I I I}=0 \tag{7}
\end{equation*}
$$

$k_{0}$ is elasticity coefficient of soil, $m_{I I}$ and $m_{I I I}$ are effective mass of part II and part I, III, respectively.

Let $y_{I I}=\bar{y}_{I I} \sin \omega_{n} t, \quad y_{I I I}=\bar{y}_{I I I} \sin \omega_{n} t, \quad$ then substitute these two equations into (6) and (7), we can obtain the solutions of (6) and (7):

$$
\begin{equation*}
\bar{y}_{I I}=A_{1} \cosh \beta x+A_{2} \sinh \beta x+A_{3} \cos \beta x+A_{4} \sin \beta x \tag{8a}
\end{equation*}
$$

$$
\begin{equation*}
\bar{y}_{I I I}=B_{1} e^{\lambda} \cos \lambda+B_{2} e^{\lambda} \sin \lambda+B_{3} e^{-\lambda} \cos \lambda+B_{4} e^{-\lambda} \sin \lambda \tag{8b}
\end{equation*}
$$

where,

$$
\begin{gathered}
\alpha=\sqrt[4]{\frac{k_{0} D-m_{I I} \omega_{n}^{2}}{4 E I}} \\
\beta=\sqrt[4]{\frac{m_{I I} \omega_{n}{ }^{2}}{E I}} \\
\lambda=\alpha \cdot\left(x-\frac{l}{2}\right)
\end{gathered}
$$

the boundary conditions are:

$$
\begin{align*}
& \text { when } x=0 ; \quad \frac{d y_{\mathrm{II}}}{d x}=0, \quad \frac{d^{3} y_{\mathrm{II}}}{d x^{3}}=0  \tag{9a}\\
& \text { when } x=\frac{l}{2} ; \quad y_{\mathrm{I}}=y_{\mathrm{II}}, \quad \frac{d y_{\mathrm{I}}}{d x}=\frac{d y_{\mathrm{II}}}{d x} \\
& \frac{d^{2} y_{\mathrm{I}}}{d x^{2}}=\frac{d^{2} y_{\mathrm{II}}}{d x^{2}}, \quad \frac{d^{3} y_{\mathrm{I}}}{d x^{3}}=\frac{d^{3} y_{\mathrm{II}}}{d x^{3}} \tag{9b}
\end{align*}
$$

$$
\begin{equation*}
\text { when } x \rightarrow \infty ; \quad y_{\mathrm{I}} \rightarrow 0, \quad y_{\mathrm{III}} \rightarrow 0 \tag{9c}
\end{equation*}
$$

Substituting (8) into (9a) and (9c), we can find $A_{2}=A_{4}=0$, $B_{1}=B_{2}=0$. Substituting (8) into (9b), we can get:

$$
\left(\begin{array}{cccc}
\cosh \frac{\beta \cdot l}{2} & \cos \frac{\beta \cdot l}{2} & -1 & 0  \tag{10}\\
\beta \sinh \frac{\beta \cdot l}{2} & -\beta \sin \frac{\beta \cdot l}{2} & \alpha & -\alpha \\
\beta^{2} \cosh \frac{\beta \cdot l}{2} & -\beta^{2} \cos \frac{\beta \cdot l}{2} & 0 & 2 \alpha^{2} \\
\beta^{3} \sinh \frac{\beta \cdot l}{2} & \beta^{3} \sin \frac{\beta \cdot l}{2} & -2 \alpha^{3} & -2 \alpha^{3}
\end{array}\right) \cdot\left(\begin{array}{l}
A_{1} \\
A_{3} \\
B_{3} \\
B_{4}
\end{array}\right)=\left(\begin{array}{l}
0 \\
0 \\
0 \\
0
\end{array}\right)
$$

The determinant of coefficient matrix of (10) should equal to 0 , then after deriving accordingly, we can obtain the following equation:

$$
\begin{equation*}
C_{1} \gamma^{4}+2 C_{2} \gamma^{3}+4 C_{3} \gamma^{2}-4 C_{4} \gamma-4 C_{1}=0 \tag{11}
\end{equation*}
$$

where,

$$
\gamma=\frac{\beta}{\alpha}
$$

$$
\begin{gathered}
C_{1}=\cosh \frac{\beta \cdot l}{2} \sin \frac{\beta \cdot l}{2}+\sinh \frac{\beta \cdot l}{2} \cos \frac{\beta \cdot l}{2}, \\
C_{2}=2 \sinh \frac{\beta \cdot l}{2} \sin \frac{\beta \cdot l}{2}
\end{gathered}
$$

$$
\begin{gathered}
C_{3}=\cosh \frac{\beta \cdot l}{2} \sin \frac{\beta \cdot l}{2}-\sinh \frac{\beta \cdot l}{2} \cos \frac{\beta \cdot l}{2} \\
C_{4}=2 \cosh \frac{\beta \cdot l}{2} \cos \frac{\beta \cdot l}{2}
\end{gathered}
$$

The maximum allowable span length avoiding VIV can be determined by solving equation (11).

With equations (4) and (5), $\omega_{n}$ can be obtained. $\gamma$ is calculated accordingly. Then solving nonlinear algebraic equation (11) with iteration method, the maximum allowable span length avoiding VIV $l_{v}$ can be gotten.

After $l_{s}$ and $l_{v}$ are obtained, the maximum allowable span length $l_{\max }$ avoiding both static damage and VIV can then be determined:

$$
\begin{equation*}
l_{\max }=\min \left(l_{s}, l_{v}\right) \tag{12}
\end{equation*}
$$

## TRENCHING ANALYSIS

Trenching operation is a procedure to lower the pipeline under mudline to protect it from adverse environmental conditions at the seabed. During this process, bending stresses will be induced in pipeline.


Fig. 3 Seabed pipeline during trenching operation

As shown in Fig.3, $\delta$ denotes the elevation of step, $\mathrm{L}_{1}$ and $\mathrm{L}_{2}$ denote suspended pipe span. In this problem, boundary is moveable because $L_{1}$ and $L_{2}$ are not known a priori.

Based on linear beam theory, the governing equations are established. Then these equations are non-dimensionalized.

The non-dimensional governing equation is:

$$
\begin{equation*}
\frac{d^{4} y_{i}^{*}}{d x_{i}^{* 4}}=-1 \quad(\mathrm{i}=1,2) \tag{13}
\end{equation*}
$$

where $x_{i}^{*}$ and $y_{i}^{*}$ are non-dimensional variables (Mousselli, 1977):

$$
x_{1}^{*}=\frac{x_{1}}{L_{c}}, \quad y_{1}^{*}=\frac{y_{1}}{L_{c}}, \quad x_{2}^{*}=\frac{x_{2}}{L_{c}}, \quad y_{2}^{*}=\frac{y_{2}}{L_{c}}, \quad L_{c}^{3}=\frac{E I}{w}
$$

The solutions of equation (13) are:

$$
\begin{equation*}
y_{1}^{*}=-\frac{x_{1}^{* 4}}{24}+k_{3} x_{1}^{* 3}+k_{2} x_{1}^{* 2}+k_{1} x_{1}^{*}+k_{0} \tag{14a}
\end{equation*}
$$

$$
\begin{equation*}
y_{2}^{*}=-\frac{x_{2}^{* 4}}{24}+d_{3} x_{2}^{* 3}+d_{2} x_{2}^{* 2}+d_{1} x_{2}^{*}+d_{0} \tag{14b}
\end{equation*}
$$

The boundary conditions and continuity requirements are:

$$
\begin{align*}
& \text { when } x_{1}^{*}=0 ; \quad y_{1}^{*}=0, \frac{d y_{1}^{*}}{d x_{1}^{*}}=0, \frac{d^{2} y_{1}^{*}}{d x_{1}^{* 2}}=0  \tag{15a}\\
& \text { when } x_{2}^{*}=0 ; y_{2}^{*}=0, \frac{d y_{2}^{*}}{d x_{2}^{*}}=0, \frac{d^{2} y_{2}^{*}}{d x_{2}^{* 2}}=0  \tag{15b}\\
& \text { when } x_{1}^{*}=-\frac{L_{1}}{L_{c}} ; y_{1}^{*}=\frac{\delta}{L_{c}}, \frac{d y_{1}^{*}}{d x_{1}^{*}}=\theta, \frac{d^{2} y_{1}^{*}}{d x_{1}^{* 2}}=\kappa  \tag{15c}\\
& \text { when } x_{2}^{*}=\frac{L_{2}}{L_{c}} ; y_{2}^{*}=0, \frac{d y_{2}^{*}}{d x_{2}^{*}}=\theta, \frac{d^{2} y_{2}^{*}}{d x_{2}^{* 2}}=\kappa \tag{15~d}
\end{align*}
$$

With satisfying equation (15), the 10 unknowns in equation (14) and (15) can be obtained by employing iterative procedures. Further, the maximum bending stress of cross-section can be derived:

$$
\sigma_{\max }=\frac{E R}{L_{c}} \frac{d^{2} y_{1}^{*}}{d x_{1}^{* 2}}
$$

$R$ is the radius of pipe.
An example is shown in Fig.4. It can be found that the maximum stress occurs at the step.



Fig. 4 Displacement and bending stress of pipeline during trenching operation

## PIPELAYING ANALYSIS

In this section, nonlinear bending analysis of pipeline during installation by laybarge is performed. Fig. 5 shows the sketch map of pipelaying. For this problem, linear beam theory
and catenary theory are not adequate. The former theory is suitable for very shallow water, the latter one suitable for very deep water. However, pipelaying procedure always carries through in moderate water. There is some special pipeline installation program that can analyze pipelaying with nonlinear finite element method. Unfortunately, such software can not handle double-wall pipeline. In addition, it is time-consuming and awkward.


Fig. 5 Sketch map of pipelaying

To win through the drawback, we model the pipeline as a nonlinear beam with large deflection and small strain. Such nonlinear beam can correctly reflect the deformation behavior of pipeline during installation. Then singular perturbation technique is adopted to solve the governing equations. Both double-wall pipeline and single-wall pipeline can be analyzed by this method. Moreover, such method can tackle the problem handily and rapidly. This advantage facilitates the procedure of preliminary design during which the plan is often modified.

The governing equation in dimensionless form is given below:

$$
\begin{equation*}
\varepsilon \cdot \frac{d^{2} \theta}{d s^{2}}+(\omega \cdot s+a+\varepsilon \cdot b) \cos \theta-\sin \theta=0 \tag{16}
\end{equation*}
$$

where $\varepsilon$ is a small dimensionless parameter, being defined as:

$$
\varepsilon=E \cdot I / H_{0} L^{2}
$$

It should be noted that $E I$ denotes bending stiffness, $H_{0}$ is horizontal force on cross-section of pipeline, and $L$ is suspended length of pipeline. $\theta$ is the angle of the tangent to the deflected axis of pipeline. The other dimensionless parameters are defined below:

$$
s=S / L, \quad \omega=w L / H_{0}, \quad a=\left.\tan \theta\right|_{s=0}, \quad b=-\left.\left(d^{2} \theta / d s^{2}\right)\right|_{s=0} /\left.\cos \theta\right|_{s=0}
$$

where $w$ is the weight of pipeline in water per unit length. $S$ is the coordinate measured along the deformed axis.

The boundary conditions at the two ends of pipeline when circular stinger is adopted are:

$$
\begin{equation*}
\left.(d \theta / d s)\right|_{s=0}=0,\left.\quad(d \theta / d s)\right|_{s=1}=L / R=\rho \tag{17}
\end{equation*}
$$

where $R$ and $\rho$ are radius and dimensionless curvature of circular stinger respectively.

A singular perturbation technique (Nayfey, 1973) is used to solve equation (16) because the limiting counterpart ( $\varepsilon=0$ ) haven't a solution satisfying the boundary conditions. After onerous and complicated derivation, solution of equation (16) is obtained:

$$
\begin{align*}
\theta= & \arctan (\omega \cdot s+a)-\sqrt{\varepsilon} \cdot\left[\frac{1}{\alpha}\left(\rho+\frac{\omega}{\alpha^{4}}\right) \cdot e^{-\alpha \eta}\right]+\sqrt{\varepsilon} \frac{\omega}{\beta^{5}} e^{-\beta \cdot \zeta} \\
& +\varepsilon \cdot\left[\frac{b_{0}}{1+(\omega \cdot s+a)^{2}}-\frac{2 \omega^{2}(\omega \cdot s+a)}{\left[1+(\omega \cdot s+a)^{2}\right]^{5 / 2}}\right] \\
& -\varepsilon \cdot\left[\frac{\omega \cdot(\omega+a)}{4 \alpha^{4}}\left(\rho+\frac{\omega}{\alpha^{4}}\right)\left(\eta^{2}+\frac{\eta}{\alpha}+\frac{1}{\alpha^{2}}\right)\right] \cdot e^{-\alpha \eta} \\
& -\varepsilon \cdot\left[\frac{a \omega^{2}}{4 \beta^{8}}\left(\zeta^{2}+\frac{\zeta}{\beta}+\frac{1}{\beta^{2}}\right)\right] \cdot e^{-\beta \cdot \zeta}+O\left(\varepsilon^{3 / 2}\right) \tag{18}
\end{align*}
$$

where

$$
\eta=(1-s) / \sqrt{\varepsilon}, \quad \alpha=\left[1+(\omega+a)^{2}\right]^{1 / 4}, \quad \zeta=s / \sqrt{\varepsilon}, \quad \beta=\left(1+a^{2}\right)^{1 / 4}
$$

For the reason that $L$ is not known a priori, an iteration procedure is needed in solve equation (16). First, the initial value of $H_{0}$ and $L$ are selected, then $\varepsilon$ can be calculated. Substituting $\varepsilon$ into equation (18), we can get $\theta$. Secondly, substituting $\theta$ into equation (19), we can calculate $H_{0}{ }^{i}$ and $L^{i}$. If $H_{0}{ }^{i}$ and $L^{i}$ satisfy convergence requirements, the approximate solution of pipelaying is obtained. Otherwise, $\varepsilon$ is updated, then iteration procedure is executed.

$$
\begin{align*}
& L=\frac{d-h_{u}+R \cdot\left(\cos \theta_{L}-\cos \beta_{u}\right)}{\int_{0}^{1} \sin \theta \cdot d s} \\
& H_{0}=\frac{T_{u}-w h_{L}-p h_{L}-w L \sin \theta_{L}}{\cos \theta_{L}\left[(a+\varepsilon \cdot b) \tan \theta_{L}+1\right]} \tag{19}
\end{align*}
$$

Parameters in equation (19) such as $h_{u}, h_{L}$, etc, are defined in

## Fig.5.

As an example, a single-wall pipeline is calculated. The obtained deflection and bending moment is compared with the results given by Robert (1996). OFFPIPE is a program that can analyze pipeline installation with nonlinear FEM. For singlewall pipeline, it can give quite accurate results. A comparison of results between singular perturbation method and FEM (Robert C. Malahy, Jr, 1996) is given in Fig.7.



Fig. 7 Solution comparison between this paper and FEM (Robert, 1996)

## LIFTING OF PIPELINE

During lifting operation, which is shown in Fig.6, pipeline is in nonlinear state. Moreover, the boundary is moveable. Finite difference method is usually employed to solve governing equations. However, such method is time-consuming and not accurate enough.


Fig. 6 Sketch map of lifting
We utilize shooting method to transform boundary value problem to initial value problem. Then the non-dimensionalized governing equations can be integrated by some conventional numerical technique.

The non-dimensionalized governing equations are given as follows:

$$
\begin{align*}
& -\sin \theta+\frac{d N_{n}}{d \xi}+Q_{n} \frac{d \theta}{d \xi}=0  \tag{20a}\\
& -\cos \theta-\frac{d Q_{n}}{d \xi}+N_{n} \frac{d \theta}{d \xi}=0  \tag{20b}\\
& \frac{d M_{n}}{d \xi}-Q_{n}=0  \tag{20c}\\
& \frac{d \theta}{d \xi}=M_{n} k  \tag{20d}\\
& \sin \theta=\frac{d y_{n}}{d \xi}  \tag{20e}\\
& \cos \theta=\frac{d u_{n}}{d \xi}+1 \tag{20f}
\end{align*}
$$

where,

$$
\begin{gathered}
\xi=\frac{s}{a}, \quad k=\frac{q a^{3}}{E I}, \quad y_{n}(\xi)=\frac{y(s)}{a}, \quad u_{n}(\xi)=\frac{u(s)}{a}, \\
N_{n}(\xi)=\frac{N(s)}{q a}, Q_{n}(\xi)=\frac{Q(s)}{q a}, M_{n}(\xi)=\frac{M(s)}{q a^{2}}
\end{gathered}
$$

$\theta$ is the angle of the tangent to the deflected axis of pipeline, $s$ is the coordinate measured along the deformed axis, $a$ is the suspended length of pipeline. $N, Q, M, q$ are axial force, shear force, bending moment and uniformly distributed load respectively. $y$ and $u$ are displacements in vertical and horizontal directions respectively.

The boundary conditions are:
when $\xi=0$;

$$
\begin{equation*}
y_{n}(0)=0, \theta(0)=0, u_{n}(0)=0, N_{n}(0)=0, M_{n}(0)=0 \tag{21a}
\end{equation*}
$$

when $\xi=1 ; \quad M_{n}(1)=0$
Let $Q_{n}(0)=\alpha$, together with (21a), equation (20) can be integrated as an initial value problem. Then adjust $\alpha$ so that the boundary condition $M_{n}(1)=0$ is satisfied. The solution can be obtained finally.

## BRIEF INTRODUCTION OF OPSA

OPSA is a computer software based on the above mentioned contents. The solution given by OPSA is accurate enough. The user interface of OPSA is designed for ease of use according to the requirements of offshore engineers. Some of important features and capabilities of OPSA are listed below.

## Some features of user interface

- Menus are provided to select program actions and load screens.
- Data are inputted by "filling in the blanks" on input form.
- Instantaneous and detailed help for each menu and data input form.
- Input data are stored in ASCII file.
- Output results are viewed on screen and can be printed in a format satisfying the requirements of offshore engineers.


## Analyzing capabilities

- Stress analysis of in-situ riser
- Span analysis ( static and VIV )
- Trenching
- Pipelaying
- Lifting
- Both single-wall and double-wall pipelines can be analyzed.
New modules, which are now under development, will provide additional analyzing capabilities (e.g. fatigue and fracture analysis) in updated versions.


## CONCLUSION

Pipelines are absolutely necessary structures during offshore oil-gas exploitation. Presently, there are lots of difficulties that are not tackled appropriately. Moreover, to the best of authors' knowledge, there is hardly any comprehensive software that can carry out strength analysis of offshore pipelines in different operation state. Therefore, we try to solve the problem in this paper.

Singular perturbation method and shooting method are employed to tackle geometric non-linearity and moveable boundary problems in the paper. Integrated software named OPSA is introduced. OPSA can perform strength analysis of offshore pipelines in 5 different operation states, including insitu analysis of riser, span, trenching, pipelaying and lifting. Both single-wall and double-wall pipelines can be analyzed by using OPSA.

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