LINEAR STABILITY ANALYSES OF CONVECTION IN TWO-LAYER SYSTEM
WITH AN EVAPORATING GAS-LIQUID INTERFACE.
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ABSTRACT
Marangoni-Bénard convective instability in a system of two layers consisting of an evaporating liquid layer and its vapor-phase layer, heated from below, is studied by a linear stability theory. Both phase-layers with infinite thickness are placed between two horizontal rigid boundaries while the top boundary wall is assumed as a porous medium through which the vapor phase is passed in a certain velocity. Both the unperturbed temperature profiles and the instability of two-layer convection are presented for a large variation region of the evaporating mass flow rate and different thickness ratio of two layers. The theoretical analysis results were compared with Chai & Zhang’s experiment results.

INTRODUCTION
Evaporative convection and instability represents both scientific and technological interest. Practically, a number of the industrial applications such as thin-film evaporators, boiling technologies and heat pipes concern with the evaporation process through the gas-liquid interface with the heat and mass transfer exchange. From a physical viewpoint, one of interesting questions is on the mechanisms of convection instability in thin-liquid layers induced by the coupling of evaporation phenomenon and Marangoni effect at the mass exchanged interface. In past, most scientists considered one-sided model in the study of instability of evaporation convection in thin liquid layer such as the works of A. Prosperetti, et al (1984), J.P. Bruelbach, et al (1988) 2 and P. Colinet, et al (2001) 3. Until now, available theories, such as Rayleigh’s theory, referred to as Rayleigh-Bénard convection, and Pearson’s theory, referred to as Marangoni-Bénard convection, are unable to explain completely the convection in an evaporating liquid layer. The mechanism of evaporation convection needs to be studied both theoretically and experimentally. Some different possible mechanisms (differential vapor recoil, non-linear mass transfer effects etc.) have been suggested for trying to support experimental observation. A new mechanism for convection instabilities in evaporating liquid layer was described recently by Zhang and Chao (1999) 4. They proposed a modified Marangoni number and a modified Rayleigh number to gauge the convection stability status in both evaporating layers and liquid layers heated from below without evaporation. A recent research program CIMEX (Convection and Interfacial Mass Exchange) has been proposed by J.C. Legros et al.(2001) 5 and initiated as a Microgravity Application Project for the understanding of different regimes of flow and instabilities with evaporation. One of the experimental studies is on the evaporative convection in layers with its own vapor.

In present paper, a systematic model of two-layer system consisting of the evaporating liquid layer and vapor-phase layer, heated from below, is studied by linear instability analyses. The thickness of both layers is finite and each layer has an infinite extension in length. The lower layer is considered as an evaporating liquid of single-component, which is in contact with its own vapor of the upper layer, and the interface is a free surface with evaporation. Both the top wall and the bottom wall are considered as rigid perfectly conducting boundaries. The top wall is assumed as a porous medium to vapor, through which the gas phase is passed in a certain velocity in order to control the evaporation flux and vapor pressure in the system. In the basic state, the liquid is evaporating at a certain steady evaporating rate, and it is assumed that there is no convection in vapor-gas layer and evaporating liquid layer. A non-deformable interface in the perturbation state is considered as the first step of this study, and the influence on the convection instability of the system are analyzed comparatively. Neutral stability curves of the two-layer system are presented for different cases, such as various depth ratios of two layers, and evaporating mass flux in the system of alcohol liquid

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with its own vapor phase. A comparison with the experimental results given by Zhang & Chao (1999)\textsuperscript{1} are performed in the same system of liquid and vapor. The theoretical analysis is the first part of our experimental study on the convection instability in an evaporating liquid layer enclosed in a test cell.

**MATHEMATICAL FORMULATION**

Consider a two-layer immiscible system, in which a layer of single-component in contact with its own vapor, lying on a heated rigid plate \( z = -H_2 \), the free interface between the vapor-phase and liquid is given by \( z=0 \). At the interface, evaporation may occur. The basic state is assumed one-dimensional (only varies by \( z=0 \)). At the interface, evaporation may occur. The interface between the vapor-phase and liquid is given by \( z=0 \). At the interface, evaporation may occur. The evaporating liquid layer enclosed in a test cell.

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The situation we envisage here is that the liquid layer undergoes steady evaporation from its interface. The question to be addressed is that of the stability of this two-layer system

![Geometry and coordinate system for the two-layer system](image)

**Figure 1.** Geometry and coordinate system for the two-layer system

The situation we envisage here is that the liquid layer undergoes steady evaporation from its interface. The question to be addressed is that of the stability of this process. We take the liquid and its vapor to be incompressible viscid fluids.

For incompressible, viscid fluids the conservation equations for mass, momentum, and energy are:

\[
\mathbf{\nabla} \cdot \mathbf{u}_i = 0
\]

\[
\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \mathbf{u}_i = -\frac{1}{\rho_i} \nabla p_i + \nu_i \nabla^2 \mathbf{u}_i + \mathbf{g}
\]

\[
\frac{\partial T_i}{\partial t} + \mathbf{u}_i \cdot \nabla T_i = \kappa_i \nabla^2 T_i
\]

Across the interface, the conservation of mass, momentum\(\square\) and energy are expressed by:

\[
J = \rho_s (\mathbf{u}_2 \cdot \mathbf{n}) \cdot \mathbf{n} = \rho_i (\mathbf{u}_1 \cdot \mathbf{n}) \cdot \mathbf{n}
\]

\[
J (\mathbf{u}_2 - \mathbf{u}_1) + p_2 - p_1 = 0
\]

\[
J [L + \frac{1}{2}((\mathbf{u}_2 - \mathbf{u}_{int}) \cdot \mathbf{n})^2 - \frac{1}{2}((\mathbf{u}_1 - \mathbf{u}_{int}) \cdot \mathbf{n})^2] + (\chi_s \nabla T_2 - \chi_i \nabla T_1) \cdot \mathbf{n} = 0
\]

Here \( \mathbf{u}_i \) and \( \mathbf{u}_2 \) are the velocity vectors of the two layers, \( \mathbf{u}_{int} \) is the velocity of the interface. \( \mathbf{n} \) the unit normal directed into the vapor, \( L \) the latent heat. Since the surface is assumed to be non-deformable in present paper, the vapor recoil effect is safely negligible, except for very high mass flux.

The continuity of tangential velocity:

\[
\mathbf{n} \times (\mathbf{u}_2 - \mathbf{u}_1) = 0
\]

The tangential stress boundary conditions:

\[
\mu_s \frac{\partial u_2}{\partial z} - \mu_i \frac{\partial u_1}{\partial z} = \frac{\partial \sigma}{\partial x}
\]

\[
\mu_s \frac{\partial v_2}{\partial z} - \mu_i \frac{\partial v_1}{\partial z} = \frac{\partial \sigma}{\partial y}
\]

\( u_i \) and \( v_i \) are the velocity components in \( x, y \) directions, \( \sigma \) is the surface tension coefficient. Differentiating the first of these boundary conditions with respect to \( x \), the second with respect to \( y \), adding results and using the continuity equation of incompressible fluid, we get:

\[
\mu_s \frac{\partial^2 w_1}{\partial z^2} - \mu_i \frac{\partial^2 w_2}{\partial z^2} = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \sigma
\]

Two additional boundary conditions are required to close the equation system. The first one is the continuity of temperature at the interface.

\[
T_2 = T_1 = T_{int}
\]

The second one is to use the relation connecting the mass flux \( J \) at the interface to the local temperature of the interface \( T_{int} \) and the reference temperature \( T_s \), where \( T_i \) is taken as the saturation temperature at the system pressure.

\[
J = \left( \frac{\beta \rho_s L}{T_s^2} \right) \left( \frac{M}{2\pi R} \right)^{\frac{1}{2}} (T_{int} - T_i)
\]

where \( M \) is molecular weight, \( R \) is the universal gas constant, and \( \beta \) is the accommodation coefficient, \( \rho_s \) is the density of vapor.

This linearized constitutive equation derived from
kinetic theory was used by Burel bach et al (1988) in their stability analysis of evaporation liquid films. The linear instability analysis of the Marangoni-Bénard convection in vapor-liquid system will be carried out on the basis of the government equations with Boussinesq approximation. A spatial normal disturbances proportional to $\text{exp}(\lambda t + i(x\alpha + y\alpha_y))$ is superposed on the velocity, temperature, pressure of the basic flow, where $\lambda$ is the time growth constant and $\alpha_x, \alpha_y$ are the component of a wave vector along x and y direction, respectively. If $T_{\text{int}}$ undergo small changes $T'$ the variation $J'$ in mass flux is

$$J' = \left(\frac{\beta \rho L}{T_x^2}\right)\left(\frac{M}{2\pi R}\right)^{\frac{1}{2}} T_{\text{int}}'$$

(13)

Dimensionless perturbation equations:

$$\nu' (D^2 - \alpha^2)W_1 - \beta' \alpha' \Theta_1 = -w_{10} D(D^2 - \alpha^2)$$

(14)

$$\kappa' (D^2 - \alpha^2) \Theta_1 = -Pr \frac{\partial T_{\text{int}}}{\partial z} W_1 - w_{10} D \Theta_1 = \lambda \Theta_1$$

(15)

$$D^2 - \alpha^2)W_2 = -G \alpha^2 \Theta_2 = \lambda Pr (D^2 - \alpha^2) W_2$$

(16)

$$D^2 - \alpha^2) \Theta_1 = -Pr \frac{\partial T_{\text{int}}}{\partial z} W_2 = \lambda Pr \Theta_2$$

(17)

The boundary conditions are:

$$z = h_1 : W_1 = 0, DW_1 = 0, \Theta_1 = 0$$

(18-20)

$$z = -h_2 : W_2 = 0, DW_2 = 0, \Theta_2 = 0$$

(21-23)

$$z = 0 : DW_1 = DW_2$$

(24)

$$W_1 - W_2 = B\Theta_2$$

(25)

$$W_2 = \rho^* W_1$$

(26)

$$\Theta_1 = \Theta_2$$

(27)

$$D\Theta_1 - \chi' D\Theta_1 + E(W_1 - W_2) = 0$$

(28)

$$\mu' D^2 W_1 - D^2 W_2 = \alpha^2 \frac{Ma}{Pr} \Theta_2$$

(29)

Here, $D$ is the dimensionless differential operator $d/dz$, $\alpha$ is the dimensionless wave number. The scaling factors for length, velocity, time and temperature are $H, \nu/\nu, \nu/\nu_x$ and $\Delta T (=T_2-T_1)$ respectively. $W_{10}$ is the dimensionless evaporation velocity. These equations contain the Prandtl number, $Pr = \nu/\kappa$, and the Grashof number $Gr$, is defined as $Gr = g \beta \Delta T H^3 / \nu^2$, the Marangoni number is defined as $Ma = \sigma \Delta T H / (\nu_2 \kappa_2)$, and the dimensionless ratios $\nu = \nu_2 / \nu, \mu = \mu_2 / \mu$, $\lambda = \lambda_2 / \lambda, \kappa = \kappa_2 / \kappa, \beta = \beta_2 / \beta$.

The dimensionless parameters $B, E$ are as followings:

$$B = \frac{H \Delta T}{\nu_2} \left(\frac{\beta \rho L}{T_x^2}\right)\left(\frac{M}{2\pi R}\right)^{\frac{1}{2}}$$

(30)

$$E = \frac{\rho_1 \rho_2 L}{(\rho_1 - \rho_2) \chi_2 \Delta T}$$

In the unperturbed state, the evaporation through the vapor-liquid interface is taking place while the liquid surface level decrease small. The liquid velocity in the basic state is assumed to be zero, $w_{10}=0$, and the velocity of vapor leaving the interface is $w_{20}$. In this case, the unperturbed state satisfied the equations and boundary conditions are written in dimensional form:

$$\kappa \frac{d^2 T_1}{dz^2} - w_{10} \frac{dT_1}{dz} = 0$$

(31)

$$\frac{d^2 T_2}{dz^2} = 0$$

(32)

$$z = h_1 : T_1(h_1) = T_1$$

(33)

$$z = -h_2 : T_2(-h_2) = T_2$$

(34)

$$z = 0 : T_1(0) = T_2(0)$$

(35)

$$\chi_2 \frac{dT_1(0)}{dz} - \chi_1 \frac{dT_2(0)}{dz} + JL = 0$$

(36)

$$J = \frac{\rho_1 \rho_2}{(\rho_2 - \rho_1)} w_{10}$$

**NUMERICAL RESULT AND DISCUSSION**

In the linear stability analysis of Marangon-Bénard with evaporation, alcohol liquid and its own vapor...
are considered as a quasi-enclosed two-layer system with evaporation in present study. The physical properties of the alcohol system could be found in the handbook (see Ref. 6). Their ratios of the properties of liquid and its vapor are $\nu^*=285.4, \rho^*=2.93\times10^{-5}, \chi^*=0.084, \kappa^*=4907.7$ respectively, and the Prandtl number $Pr=14.87$.

**Evaporation Effect on the Basic Temperature Distribution**

First we study the influence of temperature gradients in both vapor and liquid layer with evaporation on the instability of Marangoni-Bénard convection in the vapor-liquid system is induced by the surface-tension gradient along free surface. Marangoni-Bénard convection of the system can be established and sustained only when the temperature gradient is negative. When evaporation occurs, the temperature distributing of the two layers may be changed with the evaporation flux. The temperature distribution of steady state in two-layers is one of major factors which affect the convective instability of the system.

Figure 2. Dimensionless temperature gradient of liquid layer $A_2$ versus dimensionless evaporation velocity in the steady state. ($Pr=14.87, Gr=0, \nu^*=285.4, \rho^*=2.93\times10^{-5}, \chi^*=0.084, \kappa^*=4907.7$)

Figure 2 shows that the dimensionless temperature gradient of the lower layer is changed with the dimensionless evaporation velocity. When the evaporation velocity increases, the temperature gradient in the liquid layer also grows.

The dimensionless temperature gradient in the lower layer, $A_2$ is an important factor that influences the Marangoni-Bénard instability of the system, even if we suppose that evaporation is not considered in the linearized perturbed equations of the system. In this case, we may consider only the influence of the temperature profile of the layers induced by evaporation on the instability of classic Marangoni-Bénard convection. That means we do not consider evaporation effect in the equation (25) and allow the parameter $B=0$. Figure 3 shows how the evaporation velocity affects the critical Marangoni number $Ma_c$. It shows that when the dimensionless evaporation velocity $W_{10}$ augments, the $Ma_c$ number will decrease. When $W_{10}>0.01$, the $Ma_c$ decreases intensely with augmentation of evaporation velocity. This indicates that the increasing of the evaporation mass flux has the destabilizing effect on the vapor-liquid system.

Figure 3. Critical Marangoni number of two layers $Ma_c$ versus dimensionless evaporation velocity ($Pr=14.87, Gr=0, \nu^*=285.4, \rho^*=2.93\times10^{-5}, \chi^*=0.084, \kappa^*=4907.7, h=1$).

**Evaporation Instability**

The neutral stability curves of Marangoni-Bénard convection in the system of alcohol liquid and its vapor are presented in Fig. 4 for the liquid layer $H_2=2\text{mm}$. Here, we use the Marangoni number $Ma_2$ corresponding to the liquid layer defined as:

$$Ma_2 = \frac{\sigma H_2 \Delta T_2}{\mu_2 \kappa_2}$$

where $H_2$ is the depth of the liquid layer, $\Delta T_2$ is the temperature between the bottom and the interface. In the perturbed state, if we introduce the variation of the evaporation velocity, the evaporation mass flux presents stabilizing effect. The parameter $B$ in the jump mass balance boundary (24) measures the degree of the non-equilibrium at the evaporating interface. When the accommodation coefficient $\beta$ increases, the parameter $B$ will augment. In the Fig. 4, the neutral stability curves move up continuously when $\beta$ changes from 0 to 0.1. It shows evidently that the critical Marangoni number $Ma_{2,C}$ increases with the augmentation of $\beta$. When the
accommodation coefficient of evaporation $\beta=0$, it corresponds to the non-volatile case in which the evaporation mass flux $J$ is zero. When $\beta$ is very large, it corresponds to the quasi-equilibrium limit, where the interfacial temperature is constant. In this case, the system is stable. In the case that evaporation does not occur, the critical wave number is 2.0 when $\beta=0$. When $\beta$ increases, the critical wave number is about 2.8.

For another experimental case of $H_2=1\text{mm}$ where $Ma_{2,c}=351$, we find the corresponding coefficient $\beta=0.0037$. The corresponding parameter $B$ of the system is $B=151.51$ for $Ma_{2,c}=1228$ and $B=19.30$ for $Ma_{2,c}=351$ and $\beta=0.0037$. By compared these two cases considered by Zhang & Chao (1998)$^7$, the accommodation coefficient $\beta$ that we found by linear instability analysis is between 0.001 to 0.01.

**Comparison with Experiment Results**

In comparison of our linear instability result in this quasi-two-layer system of liquid and vapor with the experimental results presented by Zhang & Chao (1998)$^7$, we changed the accommodation coefficient $\beta$ in our calculation in order to determine the value of the parameter $B$ where the critical Marangoni number of the liquid layer $Ma_{2,c}$ of linear instability analysis corresponds to the one obtained in Zhang & Chao’s experimental observation.

Figure 5 shows the variation of the critical Marangoni number $Ma_{2,c}$ in the function of the accommodation coefficient $\beta$ for the same evaporating liquid used in Zhang & Chao’s experimental investigation. The critical Marangoni number $Ma_{2,c}$ of the system observed in the experiment by Zhang & Chao (1998)$^7$ is 1228 for the evaporating liquid layer $H_2=2\text{mm}$ and evaporation velocity $w_{10}=1.56\times10^{-3}\text{mm/s}$, but the accommodation coefficient $\beta$ is unknown in their experiment. In figure 5, we can find the corresponding accommodation coefficient $\beta=0.0083$ when take the same Marangoni number $Ma_{2,c}=1228$ in our numerical computation. For an other experimental case of $H_2=1\text{mm}$ where $Ma_{2,c}=351$, we find the corresponding coefficient $\beta=0.0037$. The corresponding parameter $B$ of the system is $B=151.51$ for $Ma_{2,c}=1228$ and $B=19.30$ for $Ma_{2,c}=351$ and $\beta=0.0037$. By compared these two cases considered by Zhang & Chao (1998)$^7$, the accommodation coefficient $\beta$ that we found by linear instability analysis is between 0.001 to 0.01.

**CONCLUSION**

In this paper, Marangoni-Bénard convection instability with an evaporation interface in a two-layer system consisting of liquid and its own vapor are studied theoretically. On the physical model of the system, we considered both liquid layer and vapor phase layer with finite thickness. At first step we concentrate on the study of evaporation affects on the Marangoni-Bénard instability of the system. The linear instability analysis results show that the evaporation not only drive evidently the temperature difference $\Delta T_2$ between the bottom and the interface of evaporating liquid, but also play an important role in the convective instability of the two-layer system, even if the case that we suppose a non-deformable interface. Influences of the deformable interface and the vapor-recoil effect at the evaporation interface on the system of quasi-enclosed two-layers will be studied in our further works.
NOMENCLATURE

$C_p$  heat capacity at constant pressure of liquid $kJ/kgK$

$L$  enthalpy of vaporization, $kJ/kg$

$\sigma$  surface tension coefficient, $N/m$

$\sigma_T$  surface tension gradient with temperature, $N/mK$

$\rho_i$  density of fluid, $Kg/m^3$

$\chi_i$  thermal conductivity, $W/mK$

$\kappa_i$  thermal diffusivity, $m^2/s$

$t$  time, $s$

$\nu_i$  kinetic viscosity of the fluid layers, $m^2/s$

$\mu_i$  dynamic viscosity of the layer, $Ns/m^2$

$\beta_i$  cubic expansion coefficient, $1/K$

$\beta$  accommodation coefficient of the evaporation

$H_i$  thickness of two layers, $m$

$u_i$  the velocity of the layer, $m/s$

$T_i$  temperature of the two layers, $K$

$p_i$  pressure of the two layer system, $N/m^2$

$g$  the acceleration of gravity $m/s^2$

Subscripts

$int$  physical properties of the interface

$i$  physical properties of layer $i$, $i=1,2$

$c$  critical value

Superscript

$*$  physical property ratios of layer 1 to layer 2

$'$  disturbance physical variable

REFERENCES


