PHASE LEAD OF THE BED SHEAR STRESS TO THE FREE STREAM VELOCITY OF ASYMMETRIC WAVES AT HIGH REYNOLDS NUMBER

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ABSTRACT: Turbulence behavior of asymmetric wave boundary layers is modelled by large eddy simulation approach. Particular attention is drawn to the phase lead of the bed shear stress to the free stream velocity in cnoidal and forth-leaning wave boundary layers at high Reynolds number. The results show that the bed shear stress processes under cnoidal and forth-leaning waves are very much different from that of linear waves. The peak phase lead and the valley phase lead of bed shear stress to the free-stream velocity are different. The dependences of the peak and valley phase leads on the asymmetric degree of coidal waves and the velocity-leaning index of forth-leaning waves are revealed.

INTRODUCTION

The phase lead of the bed shear stress to the free stream velocity of water waves is of significant importance for unsteady sediment transport in coastal areas [1-3]. In case of linear or sinusoidal water waves, the phase lead is exactly 45 °in laminar flow regime, and approximately 10 °or so in fully turbulent flow regime [4]. However, water waves in coastal areas are generally nonlinear or asymmetric, exhibiting asymmetric velocity processes with different amplitudes of crest and trough, or/and different acceleration and deceleration periods in half a wave cycle [5, 6]. The phase lead of the bed shear stress to the free stream velocity of these nonlinear or asymmetric water waves is not well understood.

To reveal the phase lead, we have proposed to use an infinite immersed horizontal plate oscillating non-harmonically in its own plane in a quiescent water to simulate asymmetric wave boundary layers, and have established a large eddy simulation model to manifest the flow characteristics of asymmetric wave boundary layers [7]. We further investigate the behavior of the bed shear stress under cnoidal waves and forth-leaning waves. Particular attention is drawn to the phase lead of the bed shear stress to the free stream velocity at high Reynolds number. The results show that the behavior of the bed shear stress is very much different from that of linear waves. The peak phase lead (the phase lead of the maximum shear stress to the maximum free-stream velocity) and the valley phase lead (the phase lead of the minimum shear stress to the minimum free-stream velocity) are different. We have revealed the dependences of the peak and valley phase leads on the asymmetric degree of coidal waves and the velocity-leaning index of forth-leaning waves.

PHYSICAL MODEL DESCRIPTION

We use an infinite immersed plate oscillating asymmetrically in its own plane in a quiescent water to simulate asymmetric wave boundary layers [7]. Two typical asymmetric waves have been studied. One is cnoidal waves, and the other is forth-leaning waves.

For cnoidal waves, the infinite plate oscillates with velocity expressed by

$$u(t)=\frac{U_c}{1-\overline{\mathrm{cn}^2}}\left\{\mathrm{cn}^2\left(\frac{2Kt}{T}\right)-\overline{\mathrm{cn}^2}\right\}. \tag{1}$$

Here, *cn* denotes Jacobi's elliptic function; $K=\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{1-m^2\sin^2\theta}}$ is the complete integral of the first kind elliptic function and *m* is the elliptic modulus; U_c is the maximum velocity; *t* denotes time and *T* the period of oscillations; The overbar $\overline{\ }$ denotes time average over a wave cycle. The elliptic modulus *m* solely determines the asymmetric degree of the free stream velocity, which is defined as

$$A_s=\frac{U_c}{U_c+U_t}, \tag{2}$$

where U_t is the velocity magnitudes at its valley.

For forth-leaning waves, the infinite plate oscillates with velocity expressed by

$$u(t)=\frac{U_w\sqrt{1-r^2}\sin(\omega t)}{1-r\cos(\omega t)}. \tag{3}$$

Here, U_w is the velocity amplitude; *t* denotes time; ω is the angular frequency; *r* is a variable, ranging from 0 to 1, and solely determines the forth-leaning index

$$\beta=1-\frac{2T_{cu}}{T}, \tag{4}$$

where T_{cu} is the time for velocity varying from 0 to its maximum value U_w. Hence, β=0.5 for sinusoidal waves.

MATHEMATICAL MODEL DESCRIPTION

The Navier-Stokes equations are used to model the boundary layer flow. Large eddy simulation is applied to solve the turbulence.

Uniform grids are adopted in the streamwise and spanwise directions, and non-uniform staggered grids are used in the vertical direction. The intensive degree of non-uniform staggered grids in the vertical direction can be adjusted by an intensive parameter [8]. The simulations use a mixed spectral and finite difference algorithm. Derivatives in the streamwise and spanwise directions are treated with a pseudo-spectral method, while derivatives in the vertical direction are computed with second-order center difference in the vertical staggered grids. The second-order Adams-Bashforth method is adopted for time-marching. We have verified the model by using analytical velocity profiles and bed shear stresses of laminar flow under cnoidal waves, experimental results of intermittent turbulent flow under asymmetric waves, and experimental results of fully turbulent flow under sinusoidal waves [7].

RESULTS AND DISCUSSIONS

Since the peak and valley phase differences are different between the crests and troughs of the bed shear stress and the free-stream velocity, they are defined respectively as ϕ_c and ϕ_t, which are visually shown in Fig. 1 [6].

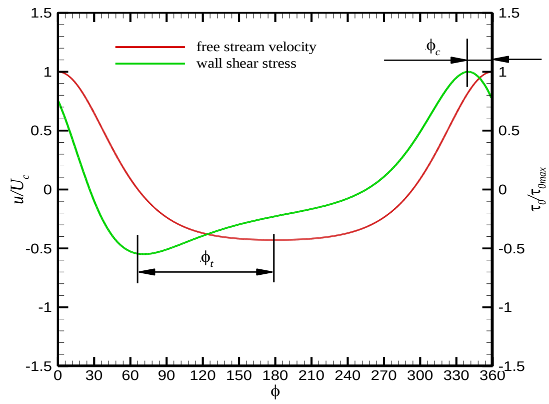

Figure 1-Definition of the peak phase lead and the valley phase lead between the crests and troughs of the bed shear stress and the free-stream velocity. τ_0 and τ_{0max} respectively stand for the bed shear stress and its maximum in a wave cycle.

By the proposed physical and mathematical models, we have obtained the processes of the bed shear stress under cnoidal and forth-leaning waves at very large Reynolds number at which the wave boundary layer flow is in the fully turbulent regime. Then, we can easily get the phase differences between the bed shear stress and the free stream velocity when Reynolds number is high.

Fig. 2 depicts the peak phase lead and the valley phase lead of the bed shear stress to the free stream velocity of cnoidal waves with different asymmetric degrees. By curve fitting, we obtained the expressions of the phase leads as follows

$$\begin{cases} \phi_c = 6.8 \\ \phi_t = \exp(9.83A_s - 3.14) \end{cases} \tag{5}$$

which are plotted in the figure. It is seen that the peak phase lead changes little between different asymmetric degrees, while the valley phase lead increases with the asymmetric degree. When the asymmetric degree is 0.5, implying that the wave approaches to the sinusoidal wave, the peak phase lead and the valley phase lead tend to equal to each other, being 7 or so, which very well reproduces the results at high Reynolds number from Jensen, et al (1989) in laboratory experiments for sinusoidal waves [4]. When the asymmetric degree is large, the wave gradually approaches to a solitary wave, where the trough of the wave becomes flatter and flatter and experiences longer and longer time. This is maybe the main reason accounting for the apparent deviation of the valley phase lead at large asymmetric degrees from that of sinusoidal waves.

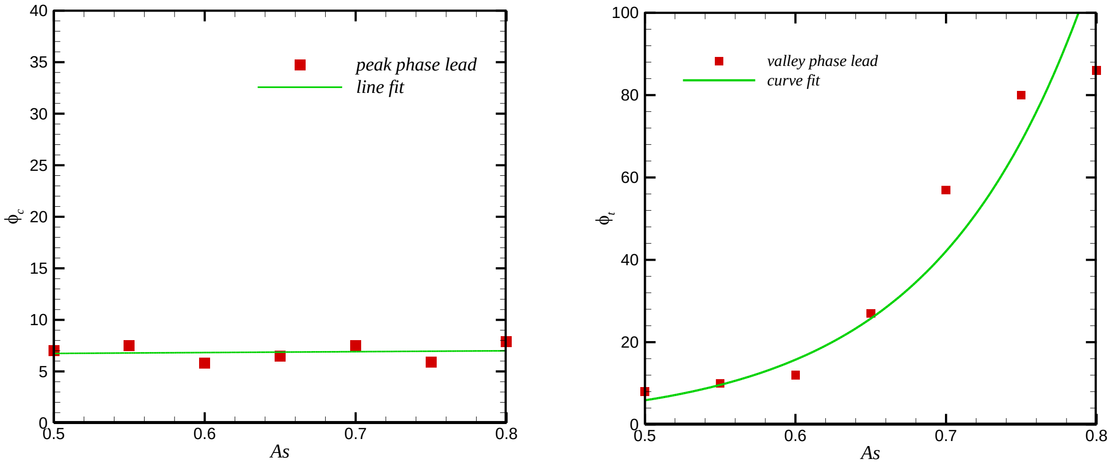

Figure 2- Dependences of the peak and valley phase lead of the bed shear stress to the free stream velocity on the asymmetric degree of cnoidal waves.

Fig. 3 depicts the peak phase lead and the valley phase lead of the bed shear stress to the free stream velocity of forth-leaning waves with different forth-leaning indexes. By curve fitting, we obtained the expressions of the phase leads as follows

$$\begin{cases} \phi_c = 7.27 \\ \phi_t = 24.92 - 29.58\beta \end{cases} \tag{6}$$

which are plotted in the figure. It is seen that the peak phase lead changes little between different forth-leaning indexes, while the valley phase lead approximately linearly decreases with the forth-leaning index. When the forth-leaning index is 0.5, the wave is exactly sinusoidal. In this case, the peak and valley phase leads are expected to be equal. The numerical results exhibit a small difference between them. This is attributed to the numerical error from finding the trough of the turbulent bed shear stress by our Matlab code. Unlike in the case of cnoidal waves, the valley phase lead is not very much different from the peak phase lead, though it depends on the forth-leaning index.

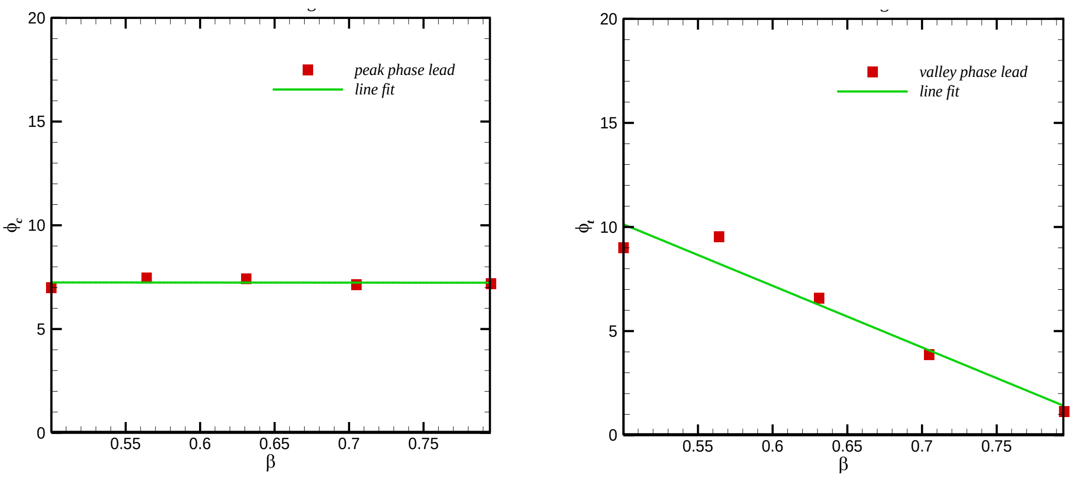

Figure 3- Dependences of the peak and valley phase lead of the bed shear stress to the free stream velocity on the forth-leaning index of forth-leaning waves.

Comparing the results of the two waves with each other, we can infer that the peak phase lead of the bed shear stress to the free stream velocity does not depend on the asymmetric degree and the forth-leaning index of asymmetric waves, whereas the valley phase lead depends. In addition, the effect of asymmetric degree is more obvious than the forth-leaning index.

CONCLUSIONS

By large eddy simulation, we have investigated the phase difference between the bed shear stress and the free stream velocity of cnoidal and forth-leaning waves at high Reynolds numbers. We find that the peak phase lead does not depend on the asymmetric degree and the forth-leaning index of asymmetric waves, and approximately equals to that of sinusoidal waves. However, the valley phase lead depends on the asymmetric degree and forth-leaning index of asymmetric waves. And the asymmetric degree takes much stronger effect than the forth-leaning index.

ACKNOWLEDGEMENTS

We very much appreciate the financial support to this work from the National Natural Science Foundation of China (11172307, 11232012 and 11572332).

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