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The Exploration of Superiority about the Non-probability Reliability-Based Design Optimization Compared to the Safety Factor Method

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Abstract. Non-probability reliability-based design optimization (NRBDO) theory has been widely acknowledged as an advanced and advantageous methodology for complex structural system design, especially in practical engineering of aerospace. Conversely, the traditional safety factor (SF) design method faded out of the horizon of the designer since it considers only the mean value of design variable without the effect of dispersion properties, which has been verified to be not reasonable. In previous studies, only the physical interpretation that NRBDO method is superior compared to the SF method was given. In this paper, we make attempts to seek a mathematical explanation through design domain in which NRBDO method and SF method search for the optimal design solution, which presents strong evidences why NRBDO theory is more advanced and reasonable method from the aspects of mathematical proof. Eventually, the advancement of the NRBDO and the weakness of the SF method are illustrated with three numerical cases.

Introduction

All With the continuous development of technology, the complexity of the engineering structural systems increases gradually so that the anticipated influence of the uncertainty on them becomes more and more profound [1]. As a deterministic design method, the SF method has consistently dominated the industry for over 70 years with limited improvement in providing efficient operational structures, which is considered inadequate for future launch vehicles and aircraft [2]. The reason of hinder development is that there are many uncertainties stemming from practical engineering, such as geometric dimensions, material properties and external loads, all these inherent uncertain factors may lead to large variations of the structural optimization models. Thus the probability RBDO approach (PRBDO), fuzzy reliability and NRBDO method are proposed to solve this problem [3-5].

The SF method assumes that the material strength and the load-induced stress are single valued, and their ratio is an arbitrarily specified ultimate SF to account for design uncertainties. The ref [6] have a good review about the origin and development of the SF in America, subsequently, the scholars made some efforts to find a fresh approach by combining SF and reliability concept for structural design, such as the literature [7-8]. The SF method is universally applicable to most structural problems and is verifiable, however, this method is not generally understood and it is perceived as arbitrarily applied and too conservative [2], which is inadequate and deficient for the practical application of structural engineer.

The PRBDO method is the most mature theory in current uncertainty analysis, which has been used in the academia and engineering field [9]. By contrast, the NRBDO method think interval set model [10] and convex model [11] can describe the uncertain parameters well in practical engineering especially the information is scarce. Therefore, the NRBDO models, such as interval set model [12]and convex model [13], are suggested to deal with the non-deterministic structural analysis and optimization problems with respect to limited uncertain information[14].

Ben-Haim and Elishakoff [15] first introduced the concept of non-probabilistic reliability through convex model theory, which plays a well alternative role for RBDO when only a limit of information is available for uncertain factors. Then, Qiu and Elishakoff [16] suggested the interval set model for truss structures optimization with uncertain-but-bounded parameters. Guo [17] quantified the uncertain structural parameters as interval variables and proposed another measure of the 'non-probabilistic reliability', which is taken as the shortest distance from the origin to the failure surface. Wang [18] proposed a new non-probabilistic set-theoretic safety measure for structures, where based on the non-probabilistic set-theoretic stress–strength interference model, the ratio of the volume of the safe region to the total volume of the region associated with the variation of the basic interval variables is suggested as the measure of the non-probabilistic safety of the structural component.

Although the RBDO method have been under development for decades, it is still hard to be accepted by engineering designers compared to SF method. Some scholars tried to find the similarities and differences between these methods [19]. But all of these researches have no good account for the conservative of SF method and why RBDO method is more superior in the current structural design. In this paper, three consensuses are given as the foundation to make mathematical proof of that why NRBDO is more advanced than the SF method and point out the original conservative of SF method, which will be the solid theoretical basis for promoting the engineering application of NRBDO method.

The Basic Concept of Non-probability Reliability-Based Design Optimization Theory

Interval and convex set-theoretic are the two primary models of NRBDO method, these two models can mutual conversion between each other [12]. The NRBDO model expounded below mainly on the basis of interval model in this article.

Consider the safety analysis of structures subject to external loads. Stress $S^I = [\underline{S}, \overline{S}]$ and strength $R^I = [\underline{R}, \overline{R}]$ are influenced by a great deal of factors. $\overline{S}$ and $\underline{S}$ are the upper and lower bounds of the structure stress separately, $\overline{R}$ and $\underline{R}$ are the upper and lower bounds of the structure strength respectively. The limit state function of structure is expressed as the function of stress S and strength R as follows:

$$g(R,S) = R - S \tag{1}$$

It is reasonable to assume that the structure is safe when stress is less than strength, namely $g(R,S) \geq 0$, but the interference between stress and strength will occurs because of the dispersion of uncertainty factors, which is referred to as stress–strength interference model, the non-probabilistic reliability index would be computed based on the non-probabilistic set-theoretic model proposed in Ref [18].

When the interference occurs between the stress and strength interval variables, the measure of the reliability index is based on the interval interference model and the thought of volume ratio, which means the structural safety is defined as the ratio of the safe region volume to the total volume. For the two dimension situation, the measure degenerates into the ratio of the safe region area S_{safe} to the total area S_{total}, as shown in Figure 1.

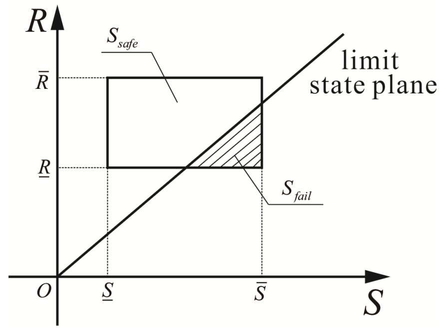

Figure 1. Non-probabilistic set-theoretic stress–strength interference model.

According to the definition of non-probabilistic reliability η, there are six different forms of formula, which is separated by the size relation between stress and strength interval boundary, the formula of solving non-probabilistic reliability is demonstrated as follows:

$$\eta=\begin{cases} 1 & ,\underline{R}>\overline{S} \\ 1-\dfrac{\left(\overline{S}-\underline{R}\right)^2}{8R^rS^r} & ,\underline{S}\le\underline{R}\le\overline{S}\le\overline{R} \\ \dfrac{S^r-S^c+R^c}{2S^r} & ,\underline{S}\le\underline{R}\le\overline{R}\le\overline{S} \\ \dfrac{R^r+R^c-S^c}{2R^r} & ,\underline{R}\le\underline{S}\le\overline{S}\le\overline{R} \\ \dfrac{\left(\overline{R}-\underline{S}\right)^2}{8R^rS^r} & ,\underline{R}\le\underline{S}\le\overline{R}\le\overline{S} \\ 0 & ,\overline{R}<\underline{S} \end{cases} \tag{2}$$

Where $S^c=(\overline{S}+\underline{S})/2$ and $S^r=(\overline{S}-\underline{S})/2$ represents the central value and radius of interval variable S^I, $R^c=(\overline{R}+\underline{R})/2$ and $R^r=(\overline{R}-\underline{R})/2$ denotes the central value and radius of interval variable R^I.

All in all, the statistical information on uncertainty may not be easily available whereas the bounds on the uncertain information can be obtained readily, under this circumstance, the NPRBDO method would be an alternative choice for designer.

The Mathematical Proof

As we all know, the disadvantage of SF method is that it is just a comprehensive representation of all empirical and uncertain information, In the meantime, the NRBDO method is one of the most advanced structural design concept, but which aspect exactly is the superiority, whether or not a theoretical support and mathematical proof can be employed to make the demonstration, which will play a decisive role in the engineering application of NRBDO method. In this section, the weakness of SF method and a mathematical proof that NRBDO is more suitable for the structure design will be presented across-the-board.

The Design Model of Two Methods

Before the proof of two approaches, the optimal design model should be introduced first. As for SF method, the model is ordinarily shown as follows:

$$
\begin{cases}
\text{find} \quad \boldsymbol{X} = [x_1, x_2, \cdots, x_n] \\
\text{min} \quad M(\boldsymbol{X}) \\
\text{s.t.} \quad G = \dfrac{h(\boldsymbol{P}^c)}{n_0} - g(\boldsymbol{X}, \boldsymbol{P}^c) \geq 0 \\
\qquad x_i \in \left[\underline{x_i}, \overline{x_i} \right], \ i = 1, 2, \cdots, n \\
\qquad p_j \in \left[p_j^l, p_j^u \right], \ j = 1, 2, \cdots, m
\end{cases} \tag{3}
$$

Where $\boldsymbol{X} = [x_1, x_2, \cdots, x_n]$ are the design variables, $\boldsymbol{P} = [p_1, p_2, \cdots, p_m]$ represents the uncertain interval parameters involved in the analysis process, $\boldsymbol{P}^c$ indicates the mean value of interval parameters; $M(\boldsymbol{X})$ denotes the objective function of design variables, and G represents the constraint function of the structure, $h(\cdot)$ is the allowable limit state function, which has nothing to do with design variables, $g(\cdot)$ denotes the calculated response function and n_0 indicates the safety coefficient in the design of structure.

The model of NRBDO is similar to SF method, while the difference between them is constraint function of NRBDO represents as:

$$
\eta(h(\boldsymbol{P}^c, \Delta \boldsymbol{P}) - g(\boldsymbol{X}, \boldsymbol{P}^c, \Delta \boldsymbol{P}) \geq 0) \geq \eta_1^*
$$

Where η_1^* indicates the structural reliability under SF method. On the basis of these two design models introduced above, the mathematical proof of why RBDO is superior to SF will be demonstrated in the following subsection.

The Mathematical Proof of two Methods

For the SF design method, due to the design process simply considered the mean value of the uncertain parameters other than take dispersion effect into account, as a result, the design result may only appear in the line of constant value h_0 / n_0. As shown in the green-dashed line in Figure 1(b), while the coordinate denotes the values of structural stress and structural strength.

In this mathematical proof, obviously, the structure mass and stress presents a relation of inverse proportion function, which means the structural mass will decrease when the stress in structure is increase. The structural stress value will be as far as possible of maximum under the constraint, while the design point continuously search from top to bottom in the green-dashed line until reach the minimum mass limit, the intersection point ② is the optimal design results of SF method (the point ① is conservative while point ③ is dissatisfactory to the constraint conditions).

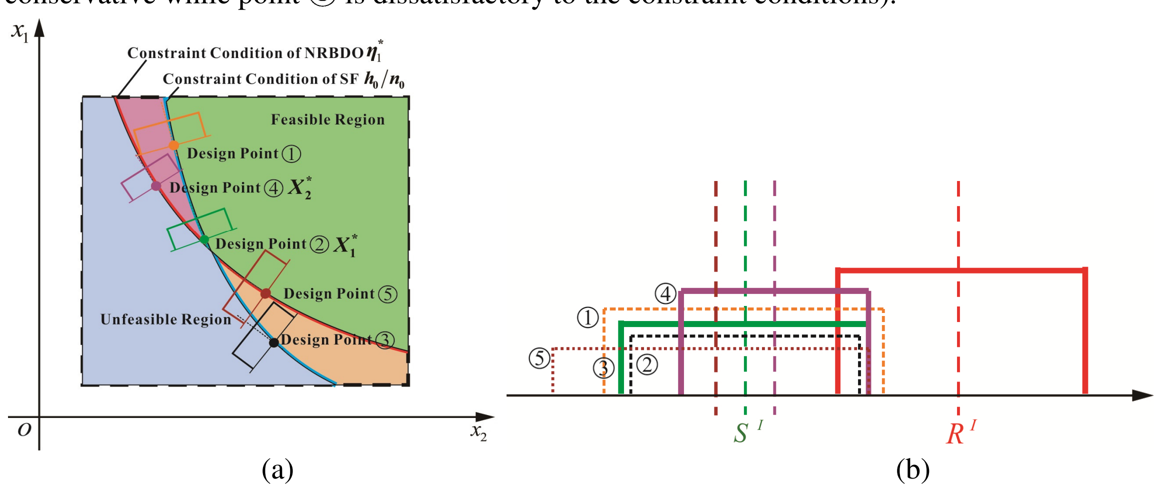

Figure 1. The comparison of design process between NRBDO and SF method: (a) the feasible region of two methods, (b) searching process of optimal design point of two methods.

As shown in Figure 1(a), the NRBDO and SF method have different design domain, it is more likely to exist other design scheme that reliability meet the requirements and the structural response is different from h_0 / n_0 born in SF method at the same time. Because of the consideration of dispersion effect, the structural response values of NRBDO method would be different from that of SF method, in other words, the design point of NRBDO can move around the green-dashed line and can get a global optimal solution in the case of satisfy the constraint conditions (the intersection point ④ is the optimal design results of NRBDO method while point ⑤ is dissatisfactory to the constraint conditions). Therefore, it is reasonable that the NRBDO method is superior to SF method.

In view of the previous section, we can know that the results of NRBDO method is no worse than the conventional SF method, but sometimes there exist this kind of situation that these two methods get the same optimal design point, so what is the dividing line between the cases? The circumstance that the NRBDO design results is better than that of SF method will be demonstrated further below.

Before the deduction and proof, we must get the following consensus:

(1) In the design of structural system, the mass of structure and structure response (such as stress and displacement) presents the relationship of inverse proportion;

(2) As for uncertain interval response parameters S_1 and S_2, when $S_1^r \leq S_2^r$, the reliability of two method can be the same reliability if only $S_1^r \leq S_2^r$;

(3) The NRBDO method considered the dispersion of parameters from the beginning of design, as a result, the value of structural response can move around the mean value results to find the global optimal design scheme.

Based on the three consensuses above, we can make the analysis that what circumstance is the result of NRBDO be superior to the result of SF method:

When the structural response of NRBDO method appeared on the left side of central value, the optimal structural mass is increased although it can meet reliability constraints, which means the occasion that the decrease of structural response value can't satisfy the conditions.

When the structural response of NRBDO method appeared on the right side of central value, only the second consensus is satisfied, structural mass can be decreased under the reliability constraints. Namely, the solution of NRBDO method is superior to SF method when the dispersion of structural response decreased, otherwise, these two methods will get the same optimal scheme.

Altogether, the NRBDO method search for the optimal design solution in the global region and can get better design result, the design result of SF method is one of the feasible solutions of NRBDO method.

The Details of Mathematical Proof about Consensus Two

The Mathematical Proof with One Dimension

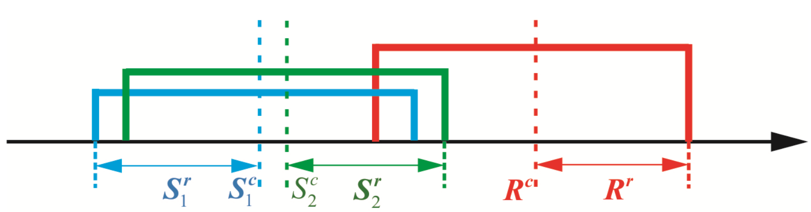

Figure 2. The model of interval analysis with one dimension.

Assume that the strength interval parameter is constant demonstrated as the red line shown in Figure 2, the blue line denotes the initial structural response of SF method, while the green line indicates the structural response of RBDO. The structural reliability of SF is shown as follows:

$$\eta_1 = \frac{S_1^c + S_1^r - R^c + R^r}{2R^r} \tag{4}$$

The structural reliability of RBDO is represented as follows:

$$\eta_2 = \frac{S_2^c + S_2^r - R^c + R^r}{2R^r} \tag{5}$$

In the reliability optimization of structure system, the constrain condition is $\eta_2 \geq \eta_1$, by substituting (4) in (5) it gives:

$$\eta_2 = \frac{S_2^c + S_2^r - R^c + R^r}{2R^r} \geq \eta_1 = \frac{S_1^c + S_1^r - R^c + R^r}{2R^r} \tag{6}$$

When $S_1^c \geq S_2^c$, the formula can be established only $S_1^r \leq S_2^r$ is the premise condition.

The Mathematical Proof with Two Dimension

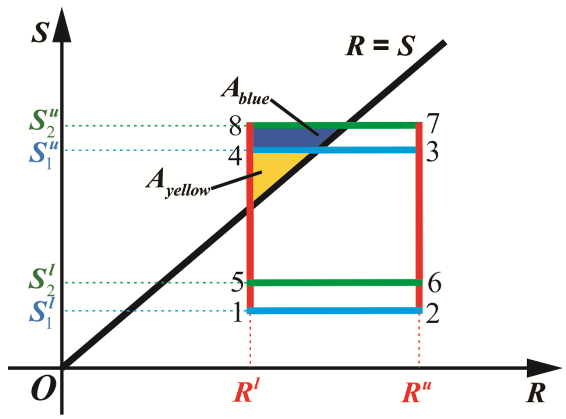

Figure 3. The model of interval analysis with two dimension.

In the analysis with two-dimensional, we marked the total area as A, the area of yellow is defined as A_{yellow}, which represented the failure probability of SF method, the area of blue is the incremental area because of the change of response value, the initial reliability of structure defined through volume ratio proposed by wang [18]. It can be expressed as follows:

$$\eta_1 = 1 - \frac{A_{yellow}}{A} \tag{7}$$

Assume that the variance of structural response and the total area remains the same when the mean value is increased, the reliability can be given:

$$\eta_2 = 1 - \frac{A_{blue} + A_{yellow}}{A} \tag{8}$$

The total area must be increased if the constrain condition $\eta_1 \leq \eta_2$ need to be satisfied, which violated the assumptions previously, thus, the assumption is failed.

When the variance of structural response increased along with the mean value, the area ratio of shadow becomes bigger, as a result, the failure probability increased and reliability decreased, which dissatisfy the requirements of reliability constraint.

Only in the situation that the mean value increased while variance of structural response decreased, the failure probability will decrease and the reliability increase, which is the desired result in structure design. Therefore, only in the case that the mean value increased while the variance of structural response decreased, the solution of NRBDO method is superior to SF method, otherwise, the optimal results of two methods with the same optimal scheme.

A Six-bar Truss Numerical Example

The analysis and optimization process of RBDO and SF method can be illustrated as Figure 4:

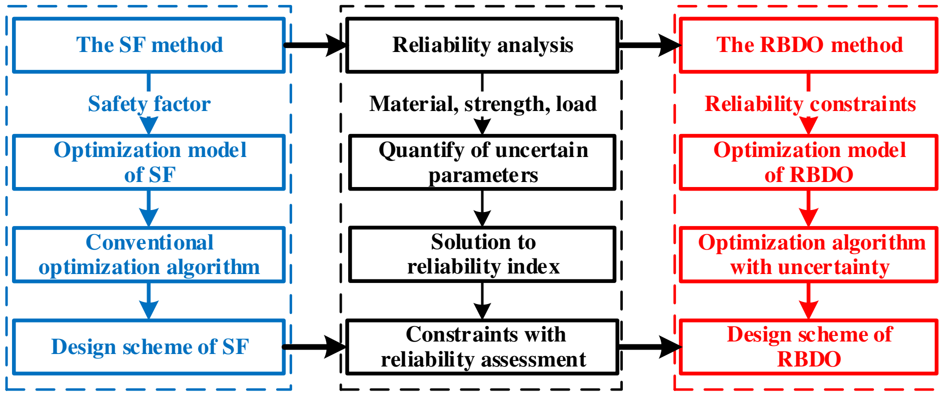

Figure 4. Flowchart of RBDO method and SF method.

A six-bar truss subjected to three horizontal uncertain forces and two uncertain vertical forces is shown in Figure 5. The allowable stress of structure and the external load are accounted as uncertain parameters with normal distribution independently. The nominal values of all external forces are P_1=50KN, P_2=30KN and P_3=20KN, while the reference coefficients of variation about their nominal values are set to 20%. As for the allowable stress, the mean and variance values are $\mu_{[\sigma]} = 25.0KN/cm^2$ and $\sigma_{[\sigma]} = 1.25KN/cm^2$ separately. All members have same Young's modulus $E = 7.06\times10^3 KN/cm^2$ and density $\rho = 2.7\times10^{-3} kg/cm^3$. The cross sectional areas of truss are considered as design variables with respect to the initial design $A_i^0 = 5cm^2 (i = 1,2,\ldots,6)$ and lower bounds $\underline{A}_i = 0.1cm^2 (i = 1,2,\ldots,6)$. The objective function is to minimize the total mass of truss system and the constraint of RBDO is the reliability is no less than that of SF method, while the constraint of SF is based on the safety factor of $n = 1.5$. The other parameters such as the length of bars are demonstrated in the figure below. All above is the initial condition for the six-truss structural system employed in this paper.

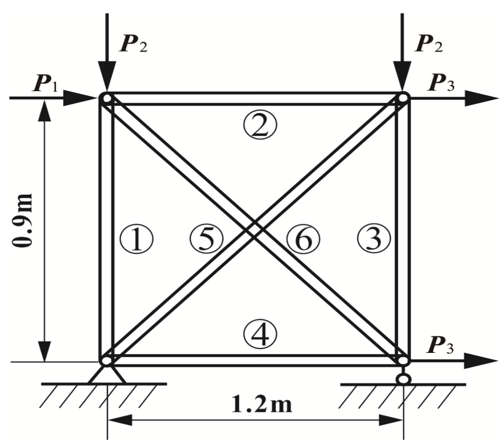

Figure 5. A six-bar truss structural system.

The optimal cross-sectional areas of SF and NRBDO models are separately shown in Table 1. It can be obtained that SF method owns a deterministic design results with total structural mass of M=4.2607kg.

Table 1. The optimal results of structure with three design methods.

	$A_1\ (cm^2)$	$A_2\ (cm^2)$	$A_3\ (cm^2)$	$A_4\ (cm^2)$	$A_5\ (cm^2)$	$A_6\ (cm^2)$	M (kg)
Initial value	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	9.7200
SF	0.3755	0.1000	2.7745	4.1255	1.6242	3.6258	4.2607
NRBDO	1.0369	0.7296	2.8052	3.3881	1.8004	2.9206	4.1798

It can be received that the reliability of six bars is 100% under the initial design. Then under the safety factor of $n = 1.5$, the bars present different levels of reliability. The first two bars have a lower reliability while the rest of four bars own a higher level of reliability, which is a threat to structure safety and waste of materials potential. By contrast, each bar has a consistent reliability relatively under the NRBDO method. It is known that the weight of each bar for the objective function is inconsistent, thus, properly decreasing the reliability of bars with higher reliability level and weight is the important reason for minimizing the mass. Namely, the redistribution of reliability due to the comprehensive consideration of design variables between different bars is the key to reduce weight.

Summary

Three conclusions can be obtained through previous theory and numerical example:

1) Normally, the result of SF method is conservative local solution because of the ignoring of dispersion of design variables; in contrast, the NRBDO method is easier to get the global optimal solution in structure design due to the comprehensive consideration of mean value and dispersion;

2) The reliability and the security level of each structural components are often inconsistent even through it is designed under the same safety coefficient; nevertheless, the structural system designed by virtue of the NRBDO method usually presented reasonable and optimal allocation of reliability, and the differences of safety level of system and irrationality can be eliminated during the design process;

3) The optimal solution of SF method is a feasible solution NRBDO method, that is to say the result of NRBDO is superior to SF method, which has been proved in the previous section.

In this paper, the physical meaning and mathematical proof of NRBDO is superior to SF method is represented, which will provide solid theoretical support and play an important role in promoting the application of reliability design methods. Although the reliability method is more reasonable for the structure design and has experienced decades of development, the variable transmission and coupling in engineering optimization still needs further study especially involved multiple disciplines.

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