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Motion error based robust topology optimization for compliant mechanisms under material dispersion and uncertain forces

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Abstract Due to the inevitable phenomena that multisource uncertainty factors in compliant mechanisms, which generally originate from material dispersion and uncertain external forces, severely affect the output motion accuracy, the robustness assessment and optimization with high confidence and efficiency is of great significance for scientists and engineers. In view of this, this study develops a novel approach of robust topology synthesis for compliant mechanisms with desired motion output by minimizing the expectation of Taguchi quantity loss function. The sensitivities of the robustness index with respect to design variables are calculated by the method of adjoint vector. Furthermore, the solution procedures of motion error based robust topology synthesis for geometrically linear and non-linear compliant mechanisms are elaborated. Two engineering examples are eventually presented to demonstrate the validity and applicability of the developed methodology.

Keywords Compliant mechanisms · Uncertainties · Motion accuracy · Robust design · Topology optimization

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1 Introduction

Compliant mechanisms are monolithic structures which can transfer the input force or motion into the output motion through their own elastic deformations (Howell 2013). Without revolute joints and assembly cost, the compliant mechanisms have numbers of advantages over traditional rigid-link mechanisms such as reserving space, reducing vibration, noise, wear and friction caused by joint clearances (Wang et al. 2015). Therefore, the compliant mechanisms are widely used in precision devices such as medical instruments and micro-electro-mechanical systems (MEMS) (Huang et al. 2014).

In recent years, topology optimization has become increasingly prevalent in the researches on the structural design since it enables designers to find the best material distribution for the required performance (Sigmund 1997). There are two main schemes in topology optimization relevant studies, the explicit geometry-based and the density-based approaches. The former approach was first proposed by Guo et al. in Ref (2014a), which was later strengthened into various problems of applying the topology optimization in practical engineering by some researchers (Zhang et al. 2017a, b; Guo et al. 2017). Despite of the explicit geometry-based approach, the other efficient and classical scheme is density-based approach, including the solid isotropic material with penalization (SIMP) method (Sigmund 1997), the level-set method (Wang et al. 2003; Allaire et al. 2004; Yaji et al. 2016), the evolutionary structural optimization (ESO) method (Xie and Steven 1993) and the sequential element rejection and admission (SERA) method (Alonso et al. 2014a, b). The brilliance of the densitybased approach is its natural ability to accommodate changes in topology and allowance for response sensitivities to be computed (Norato et al. 2015).

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As for the field of topology optimization for compliant mechanisms, both the stiffness and flexibility should be taken into account in the design procedure (Luo et al. 2005). Researchers used various functions to qualify the comprehensive influence of flexibility and stiffness and presented the algorithm to calculate the corresponding sensitivities, such as the mutual potential energy and strain energy (Ansola et al. 2007, 2010; Lin et al. 2010), geometric advantage (Luo et al. 2007) and mechanical advantage (Larsen et al. 1997; Bruns and Tortorelli 2003). Some studies focus on taking the maximum motion of output port as the objective, such as Bruggi et al. proposed the Synthesis of auxetic structures by taking the maximum output motion as objective (Bruggi et al. 2017), Leon et al. taken the maximum output motion as objective and alleviated the one-node hinges by the stress constraint (Leon et al. 2015). It has been demonstrated by the aforementioned studies that the density-based approaches are reliable and effective tools to handle the topology optimization problems with various constraints and objectives.

It should be emphasized that the material dispersion and uncertain external forces are inevitable in practical engineering (Rao and Bhatti 2001). Moreover, the small mechanisms such as MEMS are likely sensitive to the variations in material properties and applied loads (Kogiso et al. 2008). All the aforementioned researches are conducted in the deterministic assumptions, the optimums of which may fail to respond appropriately when the variations exist in material properties and external forces. Recently, the reliability or robust optimizations have become increasingly important in product design. In the field of the robust topology optimization, the compliance based robust optimizations which focus on the structural flexibility (i.e., the ability to produce the kinematic motion) are the most popular approaches. Guo et al. introduced the robust index of compliance under uncertain boundary as objective and conduct the topology optimization via level set approach (Guo et al. 2013). Jansen et al. took the variance of end-compliance calculated by Monte-Carlo simulation (MCS) as constraint in optimization formulation (Jansen et al. 2015). Richardson used the summation of mean and standard deviation of the compliance as the objective to design a monolithic structure (Richardson et al. 2015). Kogiso et al. proposed a mutual compliance based robust synthesis for structures with random applied force directions(Kogiso et al. 2008). Chen et al. developed a compliance based robust topology optimization approach for structures with interval random parameters (Chen et al. 2016). Wu et al. proposed a Level-set topology optimization method for auxetic metamaterials under hybrid uncertainties, and took the robustness of elasticity tensor index obtained by the PC expansion method (Wu et al. 2017). Liu et al. introduced the uncertain degree of mean compliance as the objective which took the correlations of the probability and fuzziness of the applied loads direction into account,

and conduct the uncertain topology optimization via BESO algorithms (Liu et al. 2015).

It is notable that in some circumstance, when considering problems of designing the positon or path generation mechanisms, the output ports of mechanisms are required to produce a given displacement or pass a set of given positions precisely (Bendsøe and Sigmund 2004). All the remarkable robust topology optimization relevant studies mentioned above was aiming at maximizing the motion ability of mechanisms robustly rather than improving the motion accuracy of output ports robustly. Therefore, the aforementioned compliance based robust topology optimization approaches may not be effective enough in the situation that the output motion accuracy is the major concern to designers. Pedersen et al. was the first to take the minimum of motion error as the objective to design a compliant mechanism with an expected output motion and conducted the topology optimization in the deterministic assumptions (Pedersen et al. 2001). Nevertheless, the studies on minimizing the motion error of compliant mechanisms with uncertain properties via robust topology optimization, namely, motion error based robust topology optimization, are still relatively rare at present. It is necessary to introduce the concept of robust design in the mechanisms synthesis which ensures the output motion realizing its target value robustly against the uncertain effects.

Aiming at finding the optimum compliant mechanism with the highest output motion accuracy and the strongest robustness under various uncertain factors by virtue of the SIMP framework, the ultimate purpose of this paper is to present a motion error based robust topology optimization approach and the corresponding methodology to calculate the sensitivities of the robust index. The expectation of Taguchi quality loss function which is widely used to qualify the robustness of motion error in rigid-link mechanisms design (Luo et al. 2012) is applied in this work as the objective function. Moreover, in order to conduct the motion error based robust topology optimization via density-based approach, the sensitivities of the expectation of Taguchi quality loss function is calculated by virtue of the adjoint method.

2 Problem statements

In this section, the definition of motion error for compliant mechanisms is discussed, and the optimal formulation is proposed.

2.1 Definition of the motion error

As shown in Fig. 1, consider the design domain of a compliant mechanism where f_{in} is the applied force at the input port, u_{out} and $\overline{u_{out}}$ are the actual output displacement and the excepted output displacement, respectively. A spring with a constant



stiffness k_s is introduced to simulate the interaction between the work-piece and the compliant mechanism. The primary purpose of a compliant mechanism is to realize an expected displacement at the output port, and where by, the objective function of mechanism synthesis is to minimize the discrepancy between the actual displacement and the required displacement. Such discrepancy can be evaluated by motion error (Mallik and Ghosh 1994), namely

$$e_{out}(X) = u_{out}(X) - \overline{u_{out}} \tag{1}$$

where X is the design variable vector filled with the relative densities of every element.

2.2 Topology synthesis of compliant mechanism

Now we carry out the motion error based topology synthesis of compliant mechanisms. A compliant mechanism is required to meet the flexibility and stiffness requirements in order to withstand the applied loads and produce the predefined displacement transmission. It worth noting that taking the minimum motion-error as objective may result in forming the *de facto* hinge regions, which will lead to the fact that the optimum function as rigid linkages and unable to be manufactured (Rahmatalla and Swan 2004). To eliminate the de facto hinge regions, Poulsen proposed a technique of imbedding the wavelet base functions (Poulsen 2001). Later, he imposed a minimum length scale constraint in the optimization formulation, namely MOLE method (Poulsen 2003). Guo et al. imposed the extreme values of the signed distance level set function as constraint to control the maximum and minimum length via the level set method (Guo et al. 2014b). To control the explicit feature of structural members via SIMP method, Zhang et al. developed an efficient local and explicit length scale control approach (Zhang et al. 2014). Besides of these morphology-based approaches, other researchers concentrated on the multi-criteria optimization method. Frecker et al. proposed an objective function which can simultaneously maximize the flexibility and minimize the stiffness (Frecker et al. 1997). Zhu et al. extended this approach to optimize hinge-free compliant mechanisms with multiple outputs (Zhu et al. 2013). Luo et al. revealed a remarkable phenomenon that the de-facto hinges is closely related to the energy dissipation from the input port to the output port. They further introduced a quadratic energy functional used in image processing applications, and conducted topology optimization of the hinge-free compliant mechanisms via the level-set method (Luo et al. 2008). Huang et al. found that the de-facto hinges is also related to the difference between the input force and output force, taken the mean compliance as the constraint and given the appropriate value of the prescribed mean compliance to effectively eliminate the *de facto* hinge regions (Huang et al. 2014).

Enlightened by Ref (Huang et al. 2014), to ensure the stiffness of compliant mechanisms and eliminate the shortage of *de facto* hinge regions, the mean compliance constraint is applied in this work. Based on (1), the optimization model of topology synthesis, especially for a Multi-Input-Multi-Output (MIMO) compliant mechanism can be formulated into

Find
$$X = (x_1, x_2, ..., x_m)^T$$

Minimize $\sum_{j=1}^k e_{out}^j (X)^2$
Subject to $\sum_{e=1}^m x_e V_e - \overline{V} \le 0$
 $C \le \overline{C}$
 $x_{\min} \le x_e \le 1$
 (2)

where k is the amount of output ports, $e_{out}^{j}(X)$ denotes the *j*th output motion error corresponding to the output port *j*, *C* and \overline{C} represent the mean compliance and the prescribed mean compliance, V_e and \overline{V} are volume of one element and the maximum allowable total volume, respectively. It is noticeable that the upper bound of mean compliance, namely the prescribed mean compliance \overline{C} may severely affects the optimal result. The optimal result will be not flexible enough to produce the expected output motion if the value of \overline{C} is too small, and the one will come with the *de facto* hinge regions if

the value of \overline{C} is too large. According to the numerical examples and discussion in Ref (Huang et al. 2014), the one-node hinge phenomena can be effectively depressed by chosen the value of \overline{C} as 3 to 4 times of C_0 , which denotes the mean compliance of the full design. In this work, the value of \overline{C} is chosen as $4C_0$. One more point should be emphasized that the traditional optimization formulation without additional mean compliance constraint can also be utilized to eliminate the *de facto* hinge regions. According to Ref (Guo et al. 2014a), the minimum and maximum length scale constraints presented by Zhang et al. can also be used to control the maximum length and the minimum length of compliant mechanisms.

3 Robust topology optimization by using the Taguchi approach

We now investigate how to insert the robustness conditions into topology optimization for the compliant mechanisms.

3.1 Robustness estimation

Uncertainties, which widely exist in practical engineering, are commonly related to manufacturing deviations, measurement errors, etc. The experimental studies have shown that the small instrument, such as MEMS devices, may be subject to severe stochastic variations in material properties and their operating environment (Maute and Dan 2003). Assuming that the material properties (i.e., the Young's module and the Poisson's ratio) and external forces are random variables, the motion error e(X) is uncertainty as well. For convenience, a vector $\mathbf{a} = (a_1, a_2, \dots, a_n)$ is established, of which the elements are all the uncertain parameters. The mean and standard deviation of random variable a_i are μ_i and σ_i . Thus, the motion error of compliant mechanism with uncertainties can be represented by e(a, X). Since the uncertainties in compliant mechanisms is usually small compared to their nominal values μ , the motion error can then be accurately approximated in the vicinity of μ . If the output motion is relatively small, the compliant mechanism can be considered as a linear model. Then the first-order approximation is indeed accurate (Wang and Qiu 2010; Wang et al. 2014). Furthermore, the high accuracy of the first-order approximation will be demonstrated in numerical examples. The motion error of compliant mechanisms can be calculated by

$$e(\boldsymbol{a},\boldsymbol{X}) \approx e(\boldsymbol{\mu},\boldsymbol{X}) + \sum_{i=1}^{n} \frac{\partial e(\boldsymbol{a},\boldsymbol{X})}{\partial a_i} \bigg|_{\boldsymbol{a}=\boldsymbol{\mu}} (a_i - \mu_i)$$
(3)

where n is the number of uncertain variables. Substituting (1) into (3), we have

$$e(\boldsymbol{a}, \boldsymbol{X}) \approx u_{out}^{\mu} - \overline{u_{out}} + \sum_{i=1}^{n} \beta_{out}^{i}(a_{i} - \mu_{i})$$

$$\tag{4}$$

where u_{out}^{μ} represents the mean output motion, and β_{out}^{i} denotes $\frac{\partial u_{out}^{\mu}}{\partial a_{i}}$. Furthermore, assuming that all the uncertain variables are independent with each other, the mean and standard deviation of motion error e(a, X) can be calculated by

$$\mu_{e}(\boldsymbol{a},\boldsymbol{X}) = u_{out}^{\mu} - \overline{u_{out}}$$
$$\sigma_{e}(\boldsymbol{a},\boldsymbol{X}) = \sqrt{\sum_{i=1}^{n} \left(\beta_{out}^{i}\sigma_{i}\right)^{2}}$$
(5)

In the studies about the non-deterministic topology optimization, some researchers used the nominal distribution to quantify the uncertain variables (Chen et al. 2016; Maute and Dan 2003; Silva and Cardoso 2017; Zhao et al. 2016; Jung and Cho 2004). If the normal distribution is utilized to quantify a_i , μ_i and σ_i in (4) and (5) are the mean and standard deviation of a normal-distributed variable $a_i \sim N(\mu_i, \sigma_i^2)$. Nevertheless, the precise distributions of the uncertainties in material properties and external forces are obtained based on a great amount of experimental samples. In practical engineering, the normal distribution assumption may not be accurate enough. For example, the range of a normal-distributed variable is from $-\infty$ to $+\infty$, which is not in accordance with the actual situation. Therefore, some non-Gaussian random variables should be considered to quantify the various uncertainties. Hence, μ_i and σ_i are the functions of the parameters of the distribution corresponded to a_i . For example, if a_i is a Weibull random variable, μ_i and σ_i are given by

$$\mu_{i} = \beta \Gamma \left(\frac{1}{\alpha} + 1 \right)$$

$$\sigma_{i} = \sqrt{\frac{\Gamma \left(\frac{2}{\alpha} + 1 \right)}{\Gamma^{2} \left(\frac{1}{\alpha} + 1 \right)} - 1}$$
(6)

where $\Gamma(\bullet)$ is the gamma function, α and β are the shape parameter and the scale parameter, respectively.

Here, we first take the linear model as an example to elaborate the concept of motion error based robust topology optimization. The displacements of all the nodes in design domain obeys

$$KU = F \tag{7}$$

where K is the stiffness matrix, F is the load vector, and U is the displacement vector. Taking the derivatives of (7), it is obvious that

$$\mathbf{K}_{\mu}\frac{\partial \mathbf{U}_{\mu}}{\partial a_{i}} = \left[\frac{\partial \mathbf{F}_{\mu}}{\partial a_{i}} - \frac{\partial \mathbf{K}_{\mu}}{\partial a_{i}}\mathbf{U}_{\mu}\right]$$
(8)

where K_{μ} , F_{μ} and U_{μ} are the mean stiffness matrix, load vector and displacement vector, respectively. It should be noted that $\beta^{i} = \frac{\partial U_{\mu}}{\partial a_{i}}$ and $\beta^{i}_{out} = \beta^{i}(out)$. According to Ref (Matthies et al. 1997), the partial derivatives of stiffness matrix and load vector about a_{i} can be approximated as

$$\frac{\partial K_{\mu}}{\partial a_{i}} = \frac{K_{\xi i} - K_{\mu}}{\xi_{i}}$$

$$\frac{\partial F_{\mu}}{\partial a_{i}} = \frac{F_{\xi i} - F_{\mu}}{\xi_{i}}$$
(9)

where $\mathbf{K}_{\xi i} = \mathbf{K}(\mathbf{a})|_{j \neq i, a_j} = \mu_i$ is the perturbation $j = i, a_j = \mu_i + \xi_i$

stiffness matrix, and
$$F_{\xi i} = F(a)|_{j \neq i, a_j} = \mu_i$$
 is the $j = i, a_j = \mu_i + \xi_i$

perturbation load vector, and ξ_i is a small perturbation.

In the process of the robust design, a quality characteristic, namely motion error is established to describe the level that significantly affects product quality and customer satisfaction (Huang and Zhang 2010). The best quality characteristic is achieved in design procedure when the motion error is zero. The deviation of motion error will certainly cause a quality loss. Therefore, it is fundamentally necessary to deduce both the mean value and the deviation of motion error in mechanism synthesis procedure. The Taguchi quality loss function is commonly used to quantify the quality loss for the uncertain system (Chen et al. 2009, 2010). For a compliant mechanism, the quality loss function can be given by

$$Q = f(e(\boldsymbol{a}, \boldsymbol{X}) - t) \tag{10}$$

where target t is zero for the mechanisms. The quality loss function can be approximated by the second order Taylor series expansion at t, thus, (10) can be calculated by

$$Q \approx Q|_{e=t} + \frac{\partial Q}{\partial e(\boldsymbol{a}, \boldsymbol{X})} \Big|_{e=t} e(\boldsymbol{a}, \boldsymbol{X}) + \frac{\partial^2 Q}{\partial^2 e(\boldsymbol{a}, \boldsymbol{X})} \Big|_{e=t} e(\boldsymbol{a}, \boldsymbol{X})^2$$
(11)

According to Taguchi (Chen et al. 2010), the quality loss function and the first order derivative at target are both zero, moreover, the second derivative at target is a constant. Therefore, (11) is converted to

$$Q \approx he(\boldsymbol{a}, \boldsymbol{X})^2 \tag{12}$$

where h is a constant variable represents the second derivative at the target. Since the quality loss function is random, the expectation of it is usually employed in robust design. Then, the expectation of quality loss function is given by

$$E(Q) = h \Big[\mu_e(\boldsymbol{a}, \boldsymbol{X})^2 + \sigma_e(\boldsymbol{a}, \boldsymbol{X})^2 \Big]$$
(13)

Substituting (5) into (13), the expectation of quality loss function can be calculated as

$$E(Q) = h \left[\left(u_{out}^{\mu} - \overline{u_{out}} \right)^2 + \sum_{i=1}^n \left(\beta_{out}^i \sigma_i \right)^2 \right]$$
(14)

Then, the objective of the optimization model in (2) is derived by finding the minimum value of E(Q). According to (14), it is notable that once μ_i and σ_i of a_i are specified, the expectation value of quality loss function does not depend on the distribution type of a_i , but the partial derivatives about a_i , namely β_{out}^i . In other word, the robust synthesis of mechanism is essentially to find an optimum result of which the mean value of motion error and the partial derivatives of output motion with respect to uncertain variable a_i are at their minimum value, as shown in Fig. 2.

3.2 Sensitivity analysis for the expectation of quality loss function

To solve the topology optimization problems via densitybased approach, it is necessary to calculate the sensitivities of the optimal objective (Sigmund 1997). According to (1) and (14), the sensitivity of objective function is given by

$$\frac{\partial E(Q)}{\partial x_e} = C \left[2 \left(u^{\mu}_{out} - \overline{u_{out}} \right) \frac{\partial u^{\mu}_{out}}{\partial x_e} + \sum_{i=1}^n 2\beta^i_{out} \sigma_i^2 \frac{\partial \beta^i_{out}}{\partial x_e} \right] \quad (15)$$

By using the method of SIMP (Bendsøe and Sigmund 2003). The Young's module E of the artificial material for each element E_e is parameterized by taking the elemental relative density as

$$E_e = x_e^p E_* \tag{16}$$

where E_* is the Young's module of the fully solid material, x_e is the element density, and p is penalization factor. In accordance with the SIMP approach, the original discrete 0-1 problem is converted to a continuous one. As shown in (15), the key object to obtain the sensitivity is to calculate $\frac{\partial u_{out}}{\partial x_e}$ and $\frac{\partial \beta_{out}^i}{\partial x_e}$. Enlightened by reference (Sigmund 1997), assuming that $K_{\mu}\lambda = \frac{\partial u_{out}^{\mu}}{\partial U_{\mu}}, \frac{\partial u_{out}^{\mu}}{\partial x_e}$ can be gained by

$$\frac{\partial u_{out}^{\mu}}{\partial x_e} = -p x_e^{p-1} \boldsymbol{\lambda}_e^T \boldsymbol{K}_{\mu}^{e*} \boldsymbol{U}_{\mu}^e \tag{17}$$

where λ_e^T is the element value of adjoint vector λ^T , U_{μ}^e is the element displacement vector, $K_{\mu}^{e^*}$ is the mean element stuffiness matrix of *e*th finite element when the density of it is 1. Furthermore, $\frac{\partial \beta_{uu}^i}{\partial x_e}$ is calculated as follows





$$\frac{\partial \beta_{out}^{i}}{\partial x_{e}} = \left[\frac{\partial \beta_{out}^{i}}{\partial \beta^{i}}\right]^{T} \frac{\partial \beta^{i}}{\partial x_{e}} \tag{18}$$

Taking the derivative of (8), one obtains

$$\begin{cases} \frac{\partial \mathbf{K}_{\mu}}{\partial x_{e}} \beta^{i} + \mathbf{K}_{\mu} \frac{\partial \beta^{i}}{\partial x_{e}} + \left(\frac{1}{\xi_{i}} \frac{\partial \mathbf{K}_{\xi}}{\partial x_{e}} - \frac{1}{\xi_{i}} \frac{\partial \mathbf{K}_{\mu}}{\partial x_{e}}\right) \mathbf{U}_{\mu} + \frac{\mathbf{K}_{\xi i} - \mathbf{K}_{\mu}}{\xi_{i}} \frac{\partial \mathbf{U}_{\mu}}{\partial x_{e}} = 0, a_{i} \in \{E, \nu\} \\ \frac{\partial \mathbf{K}_{\mu}}{\partial x_{e}} \beta^{i} + \mathbf{K}_{\mu} \frac{\partial \beta^{i}}{\partial x_{e}} = 0, a_{i} \in \{f_{1}, f_{2}, \dots, f_{m}\} \end{cases}$$

$$(19)$$

When uncertain variable a_i is a material related parameter, $\frac{\partial \beta}{\partial x_e}$ can be obtained by

$$\boldsymbol{K}_{\mu}\frac{\partial\boldsymbol{\beta}^{i}}{\partial\boldsymbol{x}_{e}} = -\left[\frac{\partial\boldsymbol{K}_{\mu}}{\partial\boldsymbol{x}_{e}}\boldsymbol{\beta}^{i} + \left(\frac{1}{\xi_{i}}\frac{\partial\boldsymbol{K}_{\xi i}}{\partial\boldsymbol{x}_{e}} - \frac{1}{\xi}\frac{\partial\boldsymbol{K}_{\mu}}{\partial\boldsymbol{x}_{e}}\right)\boldsymbol{U}_{\mu} + \frac{\boldsymbol{K}_{\xi i} - \boldsymbol{K}_{\mu}}{\xi_{i}}\frac{\partial\boldsymbol{U}_{\mu}}{\partial\boldsymbol{x}_{e}}\right]$$
(20)

It should be noted that $K_{\mu}U_{\mu} = F_{\mu}$. By taking derivative of this equation, it is obvious that

$$\boldsymbol{K}_{\mu}\frac{\partial \boldsymbol{U}_{\mu}}{\partial \boldsymbol{x}_{e}} = -\frac{\partial \boldsymbol{K}_{\mu}}{\partial \boldsymbol{x}_{e}}\boldsymbol{U}_{\mu}$$
(21)

Substituting (21) into (20), it yields

$$\boldsymbol{K}_{\mu}\frac{\partial\boldsymbol{\beta}^{i}}{\partial\boldsymbol{x}_{e}} = -\left[\frac{\partial\boldsymbol{K}_{\mu}}{\partial\boldsymbol{x}_{e}}\boldsymbol{\beta}^{i} + \frac{1}{\xi_{i}}\frac{\partial\boldsymbol{K}_{\xi i}}{\partial\boldsymbol{x}_{e}}\boldsymbol{U}_{\mu} - \frac{1}{\xi_{i}}\boldsymbol{K}_{\xi i}\boldsymbol{K}_{\mu}^{-1}\frac{\partial\boldsymbol{K}_{\mu}}{\partial\boldsymbol{x}_{e}}\boldsymbol{U}_{\mu}\right](22)$$

In accordance with (18), we arrive at

$$\frac{\partial \beta_{out}^{i}}{\partial x_{e}} = -\left[\frac{\partial \beta_{out}^{i}}{\partial \beta^{i}}\right]^{T} \boldsymbol{K}_{\mu}^{-1} \left[\frac{\partial \boldsymbol{K}_{\mu}}{\partial x_{e}} \beta^{i} + \frac{1}{\xi_{i}} \frac{\partial \boldsymbol{K}_{\xi i}}{\partial x_{e}} \boldsymbol{U}_{\mu}\right] \\ + \frac{1}{\xi_{i}} \left[\frac{\partial \beta_{out}^{i}}{\partial \beta^{i}}\right]^{T} \boldsymbol{K}_{\mu}^{-1} \boldsymbol{K}_{\xi i} \boldsymbol{K}_{\mu}^{-1} \frac{\partial \boldsymbol{K}_{\mu}}{\partial x_{e}} \boldsymbol{U}_{\mu}$$
(23)

$$\mathbf{K}_{\mu}\boldsymbol{\lambda}_{1i} = \frac{\partial\beta_{out}^{i}}{\partial\beta^{i}} \tag{24}$$

Introducing two adjoint vectors λ_{1i} and λ_{2i} into (23),

and

we have

$$\mathbf{K}_{\mu}\mathbf{K}_{\xi i}^{-1}\mathbf{K}_{\mu}\boldsymbol{\lambda}_{2i} = \frac{\partial\beta_{out}^{i}}{\partial\beta^{i}}$$
(25)

Moreover, since $\frac{\partial \beta_{out}^i}{\partial \beta^i(j)}\Big|_{j\neq out} = 0$ and $\frac{\partial \beta_{out}^i}{\partial \beta^i(j)}\Big|_{j=out} = 1$, the adjoint vector λ_{1i} can be easily obtained by the method of matrix operation. By comparisons of (24) and (25), it is significant that

$$\mathbf{K}_{\mu}\boldsymbol{\lambda}_{2i} = \mathbf{K}_{\xi i}\boldsymbol{\lambda}_{1i} \tag{26}$$

Thus, the adjoint vector λ_{2i} can be obtained by (26), and $\frac{\partial \beta'_{out}}{\partial x_e}$ can be efficiently calculated by

$$\frac{\partial \beta_{out}^{i}}{\partial x_{e}} = -\lambda_{1i}^{T} \left[\frac{\partial \mathbf{K}_{\mu}}{\partial x_{e}} \beta^{i} + \frac{1}{\xi_{i}} \frac{\partial \mathbf{K}_{\xi i}}{\partial x_{e}} \mathbf{U}_{\mu} \right] + \frac{1}{\xi_{i}} \lambda_{2i}^{T} \frac{\partial \mathbf{K}_{\mu}}{\partial x_{e}} \mathbf{U}_{\mu} \quad (27)$$

As each density variable corresponds to a unique mesh element, the value of $\frac{\partial \beta'_{out}}{\partial x_e}$ can be calculated by using (28)

$$\frac{\partial \beta_{out}^{i}}{\partial x_{e}} = -\boldsymbol{\lambda}_{1ie}^{T} \left[p x_{e}^{p-1} \boldsymbol{K}_{\mu}^{e*} \boldsymbol{\beta}^{ie} + \frac{1}{\xi_{i}} p x_{e}^{p-1} \boldsymbol{K}_{\xi i}^{e*} \boldsymbol{U}_{\mu}^{e} \right] \\ + \frac{1}{\xi_{i}} \boldsymbol{\lambda}_{2ie}^{T} p x_{e}^{p-1} \boldsymbol{K}_{\mu}^{e*} \boldsymbol{U}_{\mu}^{e}$$
(28)

where λ_{1ie}^{T} and λ_{2ie}^{T} are the elemental adjoint vectors, β^{ie} is elemental value of $\frac{\partial U}{\partial a_{i}}$, $K_{\xi i}^{e*}$ is the element stuffiness matrix when the density of *e*th finite element is unity and the value of uncertain parameter a_i is the summation of its nominal value and perturbation value.

If the uncertain variable a_i stands for load condition parameter, $\frac{\partial \beta^i}{\partial x_i}$ can be obtained by

$$\frac{\partial \beta_{out}^{i}}{\partial x_{e}} = -\left[\frac{\partial \beta_{out}^{i}}{\partial \beta^{i}}\right]^{T} \boldsymbol{K}_{\mu}^{-1} \frac{\partial \boldsymbol{K}_{\mu}}{\partial x_{e}} \beta^{i}$$
(29)

Furthermore, by introducing adjoint vector λ_1^T into (29), the sensitivity of an element *e* can be calculated by

$$\frac{\partial \beta_{out}^i}{\partial x_e} = -p x_e^{p-1} \boldsymbol{\lambda}_{1ie}^T \boldsymbol{K}_{\mu}^{e^*} \boldsymbol{\beta}^{ie}$$
(30)

In accordance with $\frac{\partial \beta_{out}^i}{\partial \beta^i} = \frac{\partial \beta_{out}^i}{\partial \beta^i}$, it can be found that $\lambda = \lambda_{1i}$.

To sum up, by substituting (29), (30) and (17) into (15), the sensitivity of objective function is given by

$$\frac{\partial E(Q)}{\partial x_e} = C \left[-2 \left(u^{\mu}_{out} - \overline{u}_{out} \right) p x_e^{p-1} \boldsymbol{\lambda}_e^T \boldsymbol{K}_{\mu}^{e*} \boldsymbol{U}_{\mu}^e + \sum_{i=1}^n 2\beta^i_{out} \sigma_i^2 \tau_i \right]$$
(31)

where

$$\tau_{i} = \begin{cases} -px_{e}^{p-1}\boldsymbol{\lambda}_{e}^{T}\boldsymbol{K}_{\mu}^{e^{*}}\boldsymbol{\beta}^{ie} - \frac{1}{\xi_{i}}px_{e}^{p-1}\boldsymbol{\lambda}_{e}^{T}\boldsymbol{K}_{\xi i}^{e^{*}}\boldsymbol{U}_{\mu}^{e} + \frac{1}{\xi_{i}}\boldsymbol{\lambda}_{2ie}^{T}px_{e}^{p-1}\boldsymbol{K}_{\mu}^{e^{*}}\boldsymbol{U}_{\mu}^{e}, \quad a_{i} \in \{E,\nu\} \\ -px_{e}^{p-1}\boldsymbol{\lambda}_{e}^{T}\boldsymbol{K}_{\mu}^{e^{*}}\boldsymbol{\beta}^{ie}, \quad a_{i} \in \{f_{1},f_{2},...,f_{m}\} \end{cases}$$
(32)

By virtue of two adjoint vectors introduced in this section, the sensitive analysis of quality loss function with respect to design variables can be easily conducted.

The major drawback of the linear projection function applied in this work is that the fading effect (existence of grey elements) along the edge of structural members cannot be prevented via the traditional Sigmud sensitivity filter approach, which is commonly regarded as the most popular and powerful method for suppressing the numerical instability of SIMP approach. The intermediate density elements will potentially affect the robustness of the compliant mechanisms. Therefore, it is necessary to eliminate the grey elements. Based on traditional Sigmud sensitivity filter approach, Zhang et al. developed an improved filter scheme which uses three adjustable parameters to prevent the undesirable boundary diffusion effect (Zhang et al. 2014). Such improved filter scheme is considered in this paper for its effectiveness and convenience, which can be expressed as

$$\frac{\partial \hat{l}}{\partial x_e} = \frac{1}{(x_e)^{\eta} + \left(\sum_{j=1}^4 x_j\right)^{\upsilon}} \frac{\sum_{i=1}^N \hat{H}_i(x_i)^{\gamma} \frac{\partial l}{\partial x_e}}{\sum_{i=1}^N \hat{H}_i}$$
(33)

where $\frac{\partial \hat{l}}{\partial x_e}$ and $\frac{\partial l}{\partial x_e}$ are filtered and unfiltered sensitivities of objective or constraint functions; η , v and γ are three adjustable parameters; \hat{H}_i is the weight factor; a more detailed explanation of (33) can be found in Ref (Zhang et al. 2014).

It should be noted that the effectiveness of the improved sensitivity filter is highly depended on the value of three adjustable parameters. When η and v are small (far less than 1), the undesirable boundary diffusion effect can be effectively prevented. While, without a structural skeleton, the improved sensitivity filter may potentially increase the time consumption to find the optimum via SIMP framework. Therefore, in this work, two optimization steps are introduced. In the first steps ($\eta = 1$, v = 0 and $\gamma = 1$), the improved sensitivity filter is converted to traditional Sigmud sensitivity filter, and the optimum of this step can be the initial design of the second step. In second step (according to Ref (Zhang et al. 2014), $\eta = 0.1$, v = 0.05 and $\gamma = 2$), the undesirable boundary diffusion effect can be prevented with the help of the improved sensitivity filter.

3.3 Solution strategy of robust synthesis for compliant mechanism with desired motion output

As expounded above, the expectation of Taguchi quality loss function is an effective way to quantify the robustness of mechanisms with respect to the randomness of motion error. Thus, robust topology optimization problem for a compliant mechanism can be formulated as

Find
$$X = (x_1, x_2, ..., x_m)^T$$

Minimize $\sum_{j=1}^{K} E(Q_{out}^j)$
Subject to $\sum_{e=1}^{m} x_e V_e - \overline{V} \le 0$
 $C \le \overline{C}$
 $x_{\min} \le x_e \le 1$
(34)

where $E(Q_{out}^{j})$ is the expectation of quality loss function for the motion error corresponding to the output port *j*.

To conduct the robust topology synthesis of compliant mechanism, the following steps should be executed.

- Step1: Define the design formulae: The design domain must be defined and meshed with the finite elements. All boundary constraints, load conditions, the uncertain characteristics of uncertain parameters, desired output motion, prescribed mean compliance and the maximum allowance volume fraction must be specified.
- Step2: Calculate the mean value of output motion and its corresponding sensitivities, for which three substeps should be followed:

- i) Calculate the mean value of nodal displacement vector U_{μ} and mean output motion u_{out}^{μ} by solving (7).
- ii) Calculate the adjoint vector $\boldsymbol{\lambda}$.
- iii) Calculate the sensitives of mean output motion with respect to the design variables, namely, $\frac{\partial u_{out}^{\mu}}{\partial x_{r}}$ by solving (17).
- Step3: Approximate the first-order derivative of output motion with respect to the uncertain parameter a_i and its corresponding sensitives with respect to the design variables, for which four sbusteps should be followed:

- i) Calculate β^i and β^i_{out} by substituting (9) into (8).
- ii) If a_i stands for a material related parameter, skip this substep. Otherwise, calculate $\frac{\partial \beta'_{out}}{\partial x_e}$ solving (30), where, $\lambda_1 = \lambda$.
- iii) Calculate the adjoint vector λ_2 by solving (26).
- iv) Calculate $\frac{\partial \beta_{out}^i}{\partial x_e}$ by substituting λ_1 and λ_2 into (28).

Repeat from substep i) to iv) until all the uncertain variables are taken into account.



- Step4: Calculate the objective function E(Q) by substituting u^{μ}_{out} and β^{i}_{out} into (14).
- Step5: Calculate the sensitivity of objective function with respect to the design variables, namely, $\frac{\partial E(Q)}{\partial x_e}$ by substituting λ_1 , λ_1 , $\frac{\partial u_{aut}^{\mu}}{\partial x_e}$ and $\frac{\partial \beta_{aut}^{i}}{\partial x_e}$ into (31) and (32). Furthermore, calculate the value of the constraints and their corresponding sensitivities with respect to design variables
- Step6: Conduct the mesh independent filtering to the sensitivity number by (33). Moreover, re-distribute the density vector X by using the Method of Moving Asymptotes (MMA)
- Step7: Repeat from step 2 to step 6 until the optimization converges are satisfied.

Besides, the whole procedure of motion error based robust topology optimization for compliant mechanisms is exhibited in Fig. 3.

3.4 Discussion of the motion error based robust topology synthesis for geometrically non-linear compliant mechanisms

For compliant mechanisms involving large displacement or large rotation as the examples in Ref (Buhl et al. 2000), it shows that the difference in stiffness obtained by linear and non-linear model are generally small when the deformation is small, and such difference is increasing along with the growth of deformation. Therefore, it is necessary to consider the geometrically non-linear effect in the process of designing a compliant mechanism with large deformation demands. The geometrically non-linear effect is important for designing a compliant mechanism with large deformation. Many researchers did a lot of tremendous work on the solution strategies of geometrically non-linear compliant mechanism design (Pedersen et al. 2001; Buhl et al. 2000; Jung and Gea 2004; Gea and Luo 2001).

Here, we discuss how to apply the proposed motion error based robust topology synthesis in the design procedure of compliant mechanisms with large deformation. The first-order Taylor series expansion is applied to evaluate the random characteristics of geometrically non-linear structures when the variances of random inputs are small (Chen et al. 2016; Jung and Cho 2004). Unlike the linear model, the tangent stiffness matrix K_T and displacement vector U is solved iteratively by using the Newton-Raphson method (Buhl et al. 2000). It is impossible to calculate β_{out}^i directly by (8) and (9). Therefore, we have to applied the difference method to approximate the differential, namely

$$\beta_{out}^{i} = \frac{u_{out\xi i} - u_{out}^{\mu}}{\xi} \tag{35}$$



Fig. 4 Design domain for the compliant inverter

where $\bullet_{\xi i} = \bullet(\mathbf{a})|_{j \neq i, a_j} = \mu_i$, ξ is a small perturbation. $j = i, a_j = \mu_i + \xi$

In the SIMP approach of geometrically non-linear compliant mechanisms, the elasticity tensor of the artificial material for each element is parameterized by taking the elemental relative densities as

$$D_e = x_e^p D_0 \tag{36}$$

where the definition of x_e and p are same with (16), D_0 is the linearized elasticity tensor of the fully solid material. Then, the sensitivity of β_{out}^i can be computed by

$$\frac{\partial \beta_{out}^{i}}{\partial x_{e}} = \frac{1}{\xi} \left(\frac{u_{out\xi i}}{\partial x_{e}} - \frac{u_{out}^{\mu}}{\partial x_{e}} \right) = -\frac{1}{\xi} \left(\lambda_{e1}^{T} \frac{\boldsymbol{p}_{e}^{\mu}}{\partial x_{e}} - \lambda_{e2}^{T} \frac{\boldsymbol{p}_{e\xi i}}{\partial x_{e}} \right)$$
(37)

where $\mathbf{K}_T^{\mu} \mathbf{\lambda}_1 = \frac{\partial u_{out}^{\mu}}{\partial U_{\mu}}$ and $\mathbf{K}_T^{\xi i} \mathbf{\lambda}_2 = \frac{\partial u_{out\xi i}}{\partial U_{\xi i}}$, \mathbf{p}^{μ} and $\mathbf{p}_{\xi i}$ are the mean internal force vector and the internal force vector when one of the uncertain input is the summation of mean value and a small perturbation.

According to (36), (37) can be converted to

$$\frac{\partial \beta_{out}^{i}}{\partial x_{e}} = -\frac{P}{\xi x_{e}} \left(\boldsymbol{\lambda}_{e1}^{T} \boldsymbol{p}_{e}^{\mu} - \boldsymbol{\lambda}_{e2}^{T} \boldsymbol{p}_{e\xi i} \right)$$
(38)

Then, the sensitivity of robust index for geometrically nonlinear compliant mechanism can be calculated by

Table 1 Uncertainty characteristics of the compliant inverter

	F _{in}	Ε	ν
Mean	$1\mu N$	10MPa	0.3
Standard deviation	$0.1 \mu N$	1 <i>MPa</i>	0.03
Distribution	Normal	Weibull	Weibull
α	_	10.428	0.31283
β	_	12.285	12.285



Fig. 5 Topology synthesis result of the compliant inverter

$$\frac{\partial E(Q)}{\partial x_e} = h \left[-2 \left(u_{out}^{\mu} - \overline{u_{out}} \right) \frac{p}{x_e} \boldsymbol{\lambda}_{e1}^T \boldsymbol{p}_e^{\mu} - \sum_{i=1}^n 2\beta_{out}^i \sigma_i^2 \frac{P}{\xi x_e} \left(\boldsymbol{\lambda}_{e1}^T \boldsymbol{p}_e^{\mu} - \boldsymbol{\lambda}_{e2}^T \boldsymbol{p}_{e\xi i} \right) \right]$$
(39)

Hence, with the sensitivity of the objective function, the motion error based robust synthesis for the geometrically nonlinear compliant mechanisms can be effectively conducted.

For geometrically non-linear finite element analysis, the problem of one-convergence will occurs in low-density elements during the optimization process. Some researchers concentrated on solving this numerical instability problem. Sigmund proposed the method of "Convergence criterion relaxation" (Sigmund 2001). Bruns et al. developed the method of "Element removal" (Bruns and Tortorelli 2003). Luo et al. presented the method of "additive hyper-elasticity technique" (Luo et al. 2015). All these great methodologies were successfully applied in SIMP framework. However, since the removed elements and the additive hyper-elasticity elements may influence the robustness of the compliant mechanisms, therefore, the method of convergence criterion relaxation may be the best choice to be utilized to conduct the motion error based topology optimization of geometrically non-linear compliant mechanisms presented in this work.

It is obvious that the motion error based robust synthesis for linear and geometrically non-linear compliant mechanisms are essentially the same. One more point should be emphasized is that the first-order approximation for geometrically non-linear strictures is sufficiently accurate only when the deviations of uncertain parameters are relatively small. In the



Fig. 6 Estimation error rates of standard deviation with different perturbation values

situation that the deviations of uncertain parameters are large, the non-liner approximation strategies should be utilized, such as high-order Taylor series expansions, the high dimensional stochastic response surface method (SRSM), high order perturbation method and even Monte-Carlo simulation. Obviously, such non-liner approximation strategies will definitely decrease the degree of efficiency. In summary, we have to evaluate the computational cost and accuracy to decide whether the proposed method is suitable for the any particular issue.

4 Numerical examples

In this section, the accuracy of the proposed robustness estimation approach is verified by a Single-Input and Single-Output compliant mechanism. Furthermore, the technique of the robust topology synthesis is applied in two engineering problems: (1) the synthesis for Two-Input and Two-Output compliant mechanism, (2) the synthesis for Three-Input and Two-Output compliant mechanism. Traditional deterministic topology synthesis is also used for comparisons purpose. The advantage of the proposed synthesis method can be clearly demonstrated in the following statements.

Table 2 Comparison of the
uncertain propagation results
corresponding to the motion at
output port obtained by Monte-
Carlo simulation and first-order
approximation

	First-order approximation	Monte-Carlo simulation	
Mean	-13.2273µm	-13.308µm	
Error rate of mean value	0.59%		
Standard deviation	1.7112µm	$1.7348 \mu m$	
Error rate of standard deviation	1.3604%		
Computational time	8.12 seconds	5.5 hours	
Computer configuration	CPU: inter core i7-3770 RAM:1	6G	



Fig. 7 Design domain for the Two-Input and Two-Output compliant mechanism

4.1 Uncertain propagation analysis for a hinge free compliant mechanism

The displacement inverter is used as a numerical example to show the accuracy of the uncertain propagation based on the first-order approximation for compliant mechanisms with material dispersion and uncertain external forces. To demonstrate the effectiveness of the proposed method, the result will be compared to the one obtained by the Monte-Carlo simulation $(1 \times 10^5 \text{ samples})$.

As shown in Fig. 4, the design domain of displacement inverter is $120 \times 120 \mu m^2$ which is discretized with 120×120 4-node quadrilateral elements. An input force $F_{in} = 1 \mu N$ is applied at the input port which is the center point of the left edge. The output port is located at the center of right edge which is designed to produce a horizontal displacement u_{out} . An artificial spring with stiffness $k_{out} = 0.01 \mu N / \mu m$ is applied at the output port to simulate the resistance from the workpiece. Moreover, the input force is assumed to be normally distributed, the material parameters are assumed to be Weibull distributed, and the uncertain properties are listed in Table 1. The topology synthesis result is shown in Fig. 5, the statistical parameters of the output motion obtained by MCS and the proposed method are are listed in Table 2. Furthermore, the influence of estimation error by the different values of small perturbation ξ in (9) is also discussed, and the error rates of standard deviation estimation corresponding to deferent values of ξ (1 $e - 6 \le \xi \le 1e - 3$) are shown in Fig. 6.

ble 3 Uncertainty aracteristics of the	Mean Standard devia		
wo-input and two- output compliant	F_{in1}	$1\mu N$	$0.04 \mu N$
nechanism	F_{in2}	$1\mu N$	$0.1 \mu N$
	Ε	10MPa	1 <i>MPa</i>
	ν	0.3	0.03



2171

Fig. 8 Result of the Two-Input and Two-Output compliant mechanism obtained by the traditional deterministic topology synthesis

It is obvious that the proposed method is sufficiently accurate to approximate the motion error of linear compliant mechanisms. Moreover, the proposed method causes much less computational cost than MCS. However, it worth noting that the proposed method will potentially overestimate the standard deviation of result due to the ignorance of higher orders. In particular, in the situation where geometrically non-linear is taken into account and the deviations of uncertainties are very large, the accuracy may be intolerable.

It is obvious that for a linear system, the value of perturbation will not affect the estimation accuracy. However, the relationship between the output motion and the Poisson ratio *v* is weak non-linear, therefore, the value of the small perturbation ξ severely affects the estimation accuracy of standard deviation, as shown in Fig. 6. It is significant that the estimation errors in Fig. 6 can be divided in three groups. (1) In the first group ($\xi \le 3e - 5$), since the difference in the denominator of (9) is by divided by a super small value, the computational instability of solution procedure is over amplified and the estimation error is intolerable. Equation (2) In the second



Fig. 9 Result of the Two-Input and Two-Output compliant mechanism obtained by the robust topology synthesis

 Table 4
 Comparison of the motion errors corresponding to the two-input and two-output compliant mechanism obtained by two synthesis approaches

Output ports	Synthesis method	$\mu_{error} (\mu m)$	$\sigma_{error} (\mu m)$	Descending rate
Port 1	Traditional method Proposed method	0.0277 0.0226	0.6229 0.5697	9.34%
Port 2	Traditional method Proposed method	0.0192 0.0213	0.7230 0.6709	7.77%

group $(3e-5 \le \xi \le 1.3e-4)$, the estimation error is relatively small, indicating that the partial derivatives approximation is sufficiently accurate. Nevertheless, for the reason that the relationship between the output motion and the Poisson ratio ν is weakly non-linear, the estimation error of the standard deviation still exists by the virtue of the higher-order ignorance in (3) and (5). Equation (3) In the last group $(1.3e-4 \le \xi)$, the estimation error is in relatively stable and in a low level. The estimation error in this group is composed by two parts, the first part is the error of partial derivatives approximation, and the second one originates from the higher-order ignorance in the linearization treatment. Based on the aforementioned conclusions, the small perturbation ξ is chosen as 1e-4 in all numerical examples.

4.2 The robust topology synthesis of the MIMO compliant mechanisms

CASE 1: two-input and two-output compliant mechanism As shown in Fig. 7, the design domain of a Two-Input and Two-Output compliant mechanism is $120 \times 120 \mu m^2$ which is discretized with 120×120 4-node quadrilateral elements. Two input forces F_{in1} and F_{in2} are applied at two input ports which are located at the left edge of the design domain. The output ports are located at the right edge of design domain, which are designed to produce two horizontal displacements, u_{out1} and u_{out2} . Two artificial springs with stiffness $k_s = 0.01 \mu N/\mu m$ are applied at the output ports to simulate the



Fig. 10 Design domain for the Three-Input and Two-Output compliant mechanism

resistance from the work-pieces. The desired outputs are $u_{out1}^{desired} = -6\mu m$ and $u_{out2}^{desired} = -7\mu m$. The volume fraction of the final design is limited to be 30%. Moreover, the material properties and input force are random variables which are assumed to be random variables and the statistical parameters are list in Table 3. The result of the traditional deterministic topology synthesis and the robust topology synthesis are shown in Figs. 8 and 9, respectively. The mean values and standard deviations of deterministic synthesis result are listed in Table 4. To quantify the effectiveness of the proposed motion error based robust topology optimization, the descending rate is established and defined as

Descending rate =
$$\frac{\sigma_{error}^R - \sigma_{orror}^D}{\sigma_{error}^R} \times 100\%$$
 (40)

where σ_{error}^{R} and σ_{error}^{D} are the standard deviations of the output motion errors of the results obtained by the robust topology optimization and the deterministic topology optimization, respectively.

CASE 2: three-input and two-output compliant mechanism The synthesis problem of a Three-Input and Two-Output compliant is shown in Fig. 10. Three input forces F_{in1} , F_{in2} and F_{in3} are applied at three input ports. The output port is located at the right edge of design domain designed to produce a horizontal displacements u_{out1} and a vertical displacement u_{out2} . Two artificial springs with stiffness $k_{out} =$ $0.01\mu N/\mu m$ are applied at the output port. The desired outputs are $u_{out1}^{desired} = -3\mu m$ and $u_{out2}^{desired} = 7\mu m$. The volume fraction of the final design is limited to be 30%. Moreover, the material properties and input forces are random variables which are assumed to be random variables and the statistical parameters are listed in Table 5. The means and standard deviations of

Table 5Uncertaintycharacteristics of thethree-input and two-output compliantmechanism

Mean	Standard deviation
$1\mu N$	$0.04 \mu N$
$1\mu N$	$0.1 \mu N$
$1\mu N$	$0.07 \mu N$
10MPa	1MPa
0.3	0.024
1μN 1μN 1μN 10MPa 0.3	0.04µN 0.1µN 0.07µN 1MPa 0.024

 Table 6
 Comparison of the motion errors corresponding to the three-input and two-output compliant mechanism obtained by two synthesis approaches

Output ports	Synthesis method	$\mu_{error} (\mu m)$	$\sigma_{error} (\mu m)$	Descending rate	
Horizontal	Traditional method Proposed method	0.0855 0.0368	0.3318 0.2839	16.87%	
Vertical	Traditional method Proposed method	0.0377 0.0538	0.6979 0.7080	-1.43%	

deterministic synthesis result and robust synthesis result are list in Table 6. The result of the traditional deterministic topology synthesis and the robust topology synthesis are shown in Figs. 11 and 12, respectively.

According to the information listed in Tables 4 and 6, some discussions can be further summarized as follows

- (1) Compared with the traditional deterministic synthesis method, the robust synthesis method proposed in this paper can effectively reduce the deviation of output displacement. As listed in Tables 4 and 6, the standard deviations of two output motions are definitely reduced.
- (2) By the optimization results of CASE1 and CASE2, it is significant that the descending rates of the standard deviations of CASE2 are much larger than the ones in CASE1. It indicates that the proposed robust synthesis method has a greater advantage over deterministic synthesis method for the MIMO compliant mechanisms with a more complicated load condition.
- (3) For the problem of designing a MIMO compliant mechanism, the robust synthesis method has a great advantage over the deterministic synthesis method. According to (31), the sensitivity of Taguchi index consists of two parts, the first being used to reduce the mean value of motion error, and the second for reducing the partial derivatives of motion error to uncertain variables. However, for a SISO compliant mechanism, there is only one input force applied on the design domain. Based on

(8) and (9), it should be noted that $\frac{\partial F_{\mu}}{\partial f_{in}} = \frac{\partial F_{\mu}}{\partial f_{in}}$, and



Fig. 11 Result of the Three-Input and Two-Output compliant mechanism obtained by the traditional deterministic topology synthesis

obviously $\frac{\partial U}{\partial f_{in}}$ equals to $\frac{U}{f_{in}}$ which has no relationship with stiffness matrix, which is why the robust synthesis cannot reduce the deviation caused by uncertain input force for SISO compliant mechanism. As for MIMO compliant mechanism, $\frac{U}{f_{in}}$ equals to $\mathbf{K}_{\mu}^{-1} \frac{\partial F_{\mu}}{\partial f_{in}}$, and $\frac{\partial F_{\mu}}{\partial f_{in}} \neq \frac{\partial F_{\mu}}{\partial f_{in}}$. Therefore, the robust topology synthesis can effectively reduce the partial derivatives of motion error to uncertain input force.

5 Conclusions

With the rapid technological advance, the robust topology synthesis of compliant mechanisms has attracted more and more concerns and discussions. Currently, most of the approaches for the robust topology synthesis are compliance based approach. In some practical circumstance, however the output motion accuracy is of major concern. It is necessary to find an approach and effective solution strategy which can ensures that the motion error of a compliant mechanism reaches zero robustly against all the uncertainties through topology optimization. In view of this, this paper introduces the expectation of Taguchi quantity loss function as the topology optimization objective, and develops a complete solution procedure of the sensitive analysis for the robustness index. As shown by the results of numerical examples, it is obvious that the motion error based robust topology synthesis has a great advantage over the traditional deterministic approach.



Fig. 12 Result of the Three-Input and Two-Output compliant mechanism obtained by the robust topology synthesis

Furthermore, it shows that the proposed method can be more effective when the load conditions and the functional requirements are more complicated.

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