



The Development of Phasemeter for Taiji Space Gravitational Wave Detection

Heshan Liu¹ · Ziren Luo¹ · Gang Jin^{1,2}

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Abstract

Taiji space gravitational wave detection utilizes the laser interferometer to convert the tiny distance change into the phase fluctuation of the beat note. As to realize the sensitivity of $1 \text{ pm}/\sqrt{\text{Hz}}$, the phasemeter needs to calculate the phase with the precision of $2\pi \mu\text{rad}/\sqrt{\text{Hz}}$ in the frequency range of 0.1 mHz and 1 Hz. In this paper, we report recent progress of the phasemeter for Taiji. Noises which possibly affect the measurement sensitivity are tested and discussed, especially the sampling noise and the frequency jitter. Finally, the accuracy of the phasemeter is calibrated. The result shows that the sensitivity has reached the requirement of Taiji in the frequencies between 0.01 Hz and 1 Hz, 0.1 mHz–1 mHz. Noises in the range of 1 mHz and 0.01 Hz, which have not yet depressed well, are dominated by the clocking jitter and the thermal fluctuation.

Keywords Taiji gravitational wave detection · Laser interferometer · Phasemeter

Introduction

Along with the announcement of successful gravitational waves detection by the LIGO (Laser Interferometer Gravitational-Wave Observatory) (Abbott et al. 2016), detecting the gravitational waves in the space becomes a hot topic around the world. Compared with the ground mission, the space gravitational wave detection is regarded as a complementary window for the lower frequency astronomic events (Gair et al. 2013; Pitkin et al. 2011; Freise 2010). Many proposals and techniques, such as LISA (Laser Interferometer Space Antenna) (Danzmann 1996), eLISA (evolved LISA) (Amaro-Seoane et al. 2012), BBO (Big Bang Observer) (Corbin and Cornish 2006), Taiji (Hu and Wu 2017), Tianqin (Luo et al. 2016), atom interferometry (Dimopoulos et al. 2009; Carraz et al. 2014;

Kulas et al. 2016), are put forward and taken into research by different organizations. Taiji mission is an original LISA-like mission, proposed by Chinese academic of sciences in 2015. Compared with LISA, the interferometer arm of Taiji is shortened to 3 million km. The target sensitivity is $1 \text{ pm}/\sqrt{\text{Hz}}$ in the frequency range of 0.1 mHz and 1 Hz, and it is scheduled to be launched in 2033. For the wave length of 1064 nm, the phase needs to be measured with the sensitivity of $2\pi \mu\text{rad}/\sqrt{\text{Hz}}$ in the target frequency range. Before Taiji mission, there have a Taiji pathfinder for all key technical validation, such as the inter-satellites interferometer, the telescope, the beam pointing, the ranging tone, the phasemeter, and so on. Taiji pathfinder, scheduled to be launched in 2025, is totally different with LISA pathfinder (Danzmann 2015; Armano et al. 2016) and GRACE Follow-on (Dehne et al. 2009; Schütze 2016). The arm length of Taiji pathfinder is set to 10000–100000 km level, and the sensitivity will reach $100 \text{ pm}/\sqrt{\text{Hz}}$ in the frequency range of 0.1 mHz and 1 Hz.

As the phase readout equipment, the phasemeter not only extracts the phase change of the interferometer, but also has many other functions (Esteban et al. 2009), such as the phase modulation for the data communication and the ranging tone, the phase locking, the beam pointing, and so on. In this paper, the recent progress of Taiji phasemeter is reported. The principle and implementation of the phasemeter is shown in the “[Principle](#)” and “[Implementation](#)”.

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✉ Ziren Luo
luoziren@imech.ac.cn

¹ Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China

² School of Engineering Science, University of Chinese Academy of Science, Beijing 100049, China

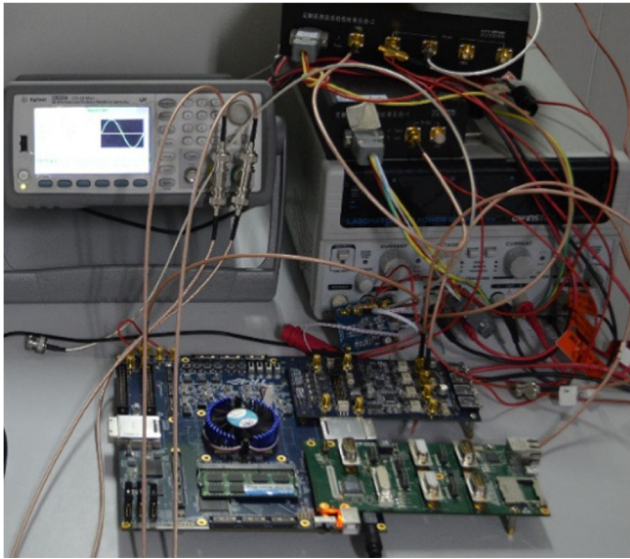


Fig. 3 The physical picture of the phasemeter prototype

Noises Analysis

In our previous research (Liu et al. 2014), outputs of the phase remaining error of the phasemeter can be written as,

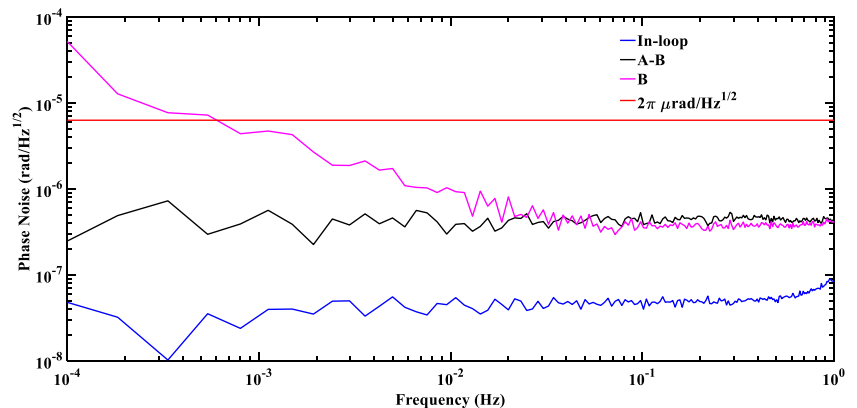
$$\phi_e = \phi_0 + \delta\Phi - \Delta\omega t + \delta\varphi + \varphi(T, \omega) + \sigma_q + \sigma_\varphi + \delta\omega t, \quad (1)$$

Where ϕ_0 , $\delta\Phi$ as the initial phase and the frequency jitter of the detected signal, $\delta\varphi$ as the sampling noise, $\varphi(T, \omega)$ as the analog frontend noise, σ_q as the ADC quantization noise, $\delta\omega t$ is the DPLL loop noise, σ_φ is the sum of other remaining noises. And, the obtained phase by the phasemeter can be written as,

$$\phi = \Delta\omega t = \phi_0 + \delta\Phi + \delta\varphi + \varphi(T, \omega) + \sigma_q + \sigma_\varphi + \delta\omega t - \phi_e. \quad (2)$$

In the following, the above noises of the phasemeter prototype of Taiji are discussed and tested. Noises spectrum are tested in the condition of the zero measurement in which the signal from a source are divided into two and then respectively delivered into different channels of the phasemeter (Two channels are labeled as “A” and “B”).

Fig. 4 Results of the phasemeter loop noise test, where the test signal is produced by another NCO. The result has been smoothed by the method of the linear amplitude spectrum density (LASD)



Although the obtained phase by the single channel of the phasemeter possible fluctuation, the phase between the channels (Common mode noise rejection) theoretically remains 0 rad in the zero measurement.

A. Initial phase ϕ_0

The obtained initial phase by the single channel A or B of the phasemeter is a constant value, which is only related with the initial time when the detecting begin.

B. Loop noise $\delta\omega t$

The loop noise, limited by the bit width of the DPLL and the finite hardware resource of the FPGA, is the sum of the loop subparts noise in the FPGA internal. The results are shown in the Fig. 4, where the test signal is produced by another NCO (1 MHz). In this situation, the value of the loop noise determines the best performance of the phasemeter.

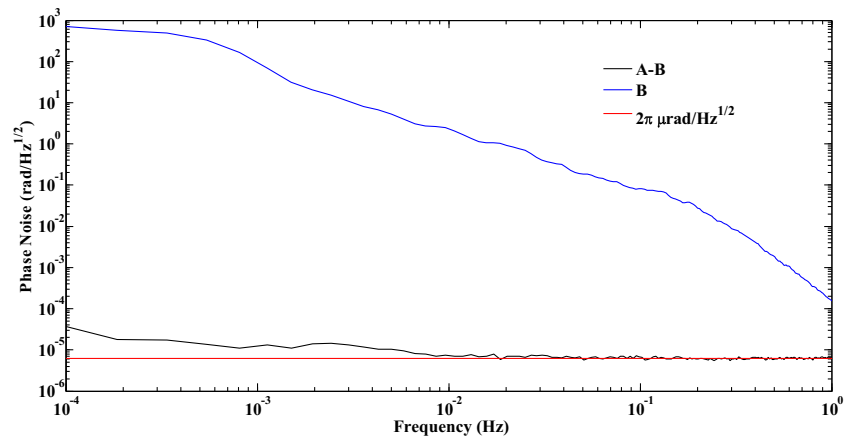
In the Fig. 4, the blue line is the in-loop noise of the channel B, which represents the loop is well locked. The pink line is the performance of the channel B of the phasemeter. This line is obviously increased in the frequency band of 0.01 Hz–0.1 mHz. It is because that the tiny quantitated frequency error in the loop always lead the integrated phase to be increased in the lower frequencies. However, this noise between channels can be well rejected, shown by the black line. It can be concluded that the loop noise of the DPLL is much lower than $2\pi \mu\text{rad}/\sqrt{\text{Hz}}$, and the loop is good enough for the accurate phase measurement of Taiji mission.

C. Quantization noise σ_q

The quantization noise will be introduced, when the analog signal is converted to the digital signal by an ADC. The value of the noise, which is determined by the sampling frequency f_s and the number of the ADC bits N , can be written as (Gerberding et al. 2013),

$$\sigma_q = \frac{\sqrt{3}}{2^N \sqrt{6} f_s}. \quad (3)$$

Fig. 5 The sampling jitter noise test, where the phasemeter is driven by an ordinary crystal oscillator and the functional generator is driven by the USO



In this paper, the sampling frequency is set to 100 MHz and the bits number of the ADC are 14. So, the value is $4.32 \times 10^{-9} \text{ rad}/\sqrt{\text{Hz}}$, much less than $2\pi \mu\text{rad}/\sqrt{\text{Hz}}$.

D. Sampling noise δ_ϕ

The sampling noise is produced by the sampling jitter of the ADC. Its value not only relates to the timing jitter of the ADC driven clock, but also the response time jitter of the ADC. The value can be written as (Heinzel et al. 2011; Barke et al. 2014)

$$\delta_\phi = 2\pi \times \delta t \times f_c, \quad (4)$$

Where f_c is the frequency of sampled signal, δt is the timing error of sampling. In Taiji mission, due to orbit drift, the highest frequency can up to 25 MHz. Therefore, for making sure the sampling noise is less than $1 \text{ pm}/\sqrt{\text{Hz}}$, the sampled timing error must be less than

$$\begin{aligned} \delta t &\leq \frac{1 \text{ pm}/\sqrt{\text{Hz}}}{1064 \text{ nm}} \times \frac{1}{25 \text{ MHz}} \\ &\approx 4 \times 10^{-14} \text{ s}. \end{aligned} \quad (5)$$

Unfortunately, there is no such stable clock, which is suited for the space gravitational wave detection mission, in

the time range 1–10000 s until now (Tinto and Yu 2015). So, the sampling noise will directly exist in the result of the single channel. However, by using the method of the common mode noise rejection between channels, this value can be largely reduced as

$$\phi_a - \phi_b = \omega (\delta t_a - \delta t_b) + \phi_{a0} - \phi_{b0}, \quad (6)$$

Where the lower-case letters of a, b represents the different channels of the phasemeter. From the Eq. 6, the value after the common mode rejection is proportional to the timing difference between channels. If two channels are synchronized by the same clock, the value can be largely decreased. For shown this, two comparison experiments are implemented, and the results are shown in the Figs. 5 and 6. In the experiment, the detecting signals are produced by a functional generator (Agilent, 33522A). One signal from the 33522A is split into two, and then respectively connect with the channels A and B of the phasemeter. The frequency of the detecting signal is set to 1 MHz. The results also have been smoothed by the method of the LASD.

In the Fig. 5, the blue line, which is the phase noise of the channel B, are dominated by the sampling noise and all useful information are submerged. However, through the common mode noise rejection, the sampling noise

Fig. 6 The sampling jitter noise test, where both the phasemeter and the functional generator are driven by the USO

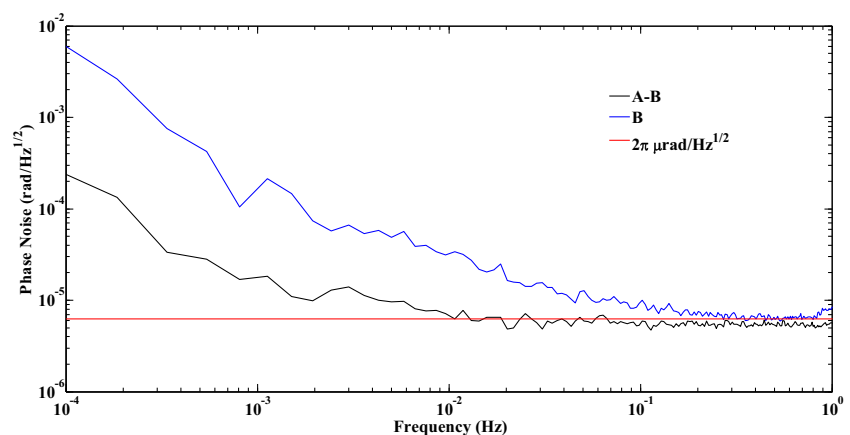
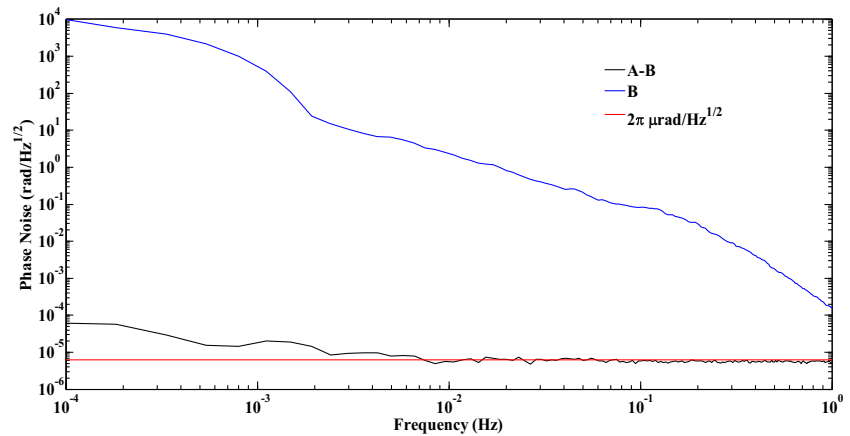


Fig. 7 The frequency jitter noise test, where the generator is driven by an ordinary clock and the phasemeter is driven by the USO



between channels, shown by the black line, can be distinctly suppressed. Compared with the Fig. 5, the blue line in the Fig. 6 is located in a lower level. It is because that the functional generator is synchronized with the phasemeter by the USO, so that the sampling noise is subtracted by the frequency jitter of the detected signal. It will be detailedly discussed in the following part of “E”. The black line also represent the common mode rejection is an effective way to suppress the sampling noise.

E. Frequency jitter $\delta\Phi$

Because of the large arm unequal, the laser frequency jitter noise is the largest one in the noises budget of Taiji interferometer. The value of the jitter can be written as (Gerberding et al. 2017; Sheard et al. 2010)

$$\delta\Phi = 4\pi \frac{\delta\mu \times \Delta L}{c}, \quad (7)$$

Where $\delta\mu$ is the laser frequency jitter, ΔL is the arm length difference, and c is the speed of light. If the signal is produced by a functional generator, frequency jitter will be determined by the driven oscillator and also can be calculated by the Eq. 4. For better understand the noise, two

tests are done, which the results are shown in the Figs. 7 and 8. In the experiments, test signals are also produced by the external 33522A. One signal from the 33522A is split into two, and then respectively be connected with the channels A and B of the phasemeter. The frequency of the detected signal is also set to 1 MHz.

Comparing with the Fig. 5, the data in the Fig. 7 presents a similar results. It is because that the frequency jitter and the sampling noise are the same in the condition of our experiment. Both of them are introduced by the jitter of the driven oscillator, and only the locations of the USO and the ordinary clock are exchanged in the two experiments. However, the results in the Fig. 8 are different with the Fig. 6. Although both of them have been used an oscillator to be synchronized, the noises present different levels. Obviously, through the Eqs. 4 and 6, using the ultra-stable oscillator to synchronize the functional generator and the phasemeter can maximally decrease the noise of the clock sampling and the frequency jitter.

F. Analog frontend noise $\varphi(T, \omega)$

The analog frontend refers to the analog circuits between the ADC and the photo detector. This circuits always

Fig. 8 The frequency jitter noise test, where both the phasemeter and the functional generator are synchronized by an ordinary clock

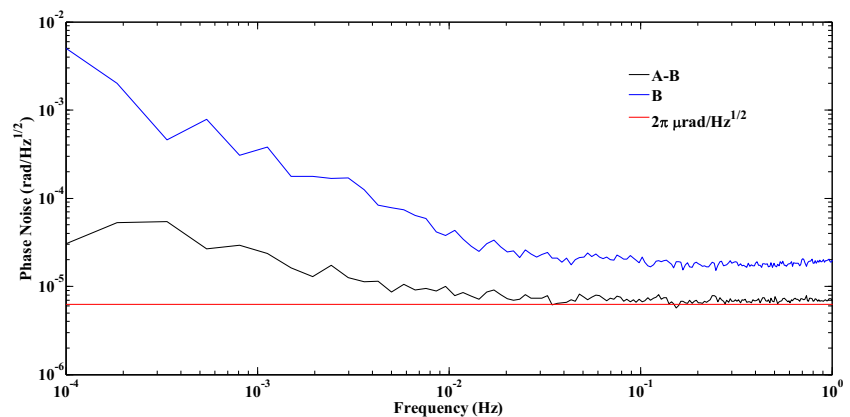
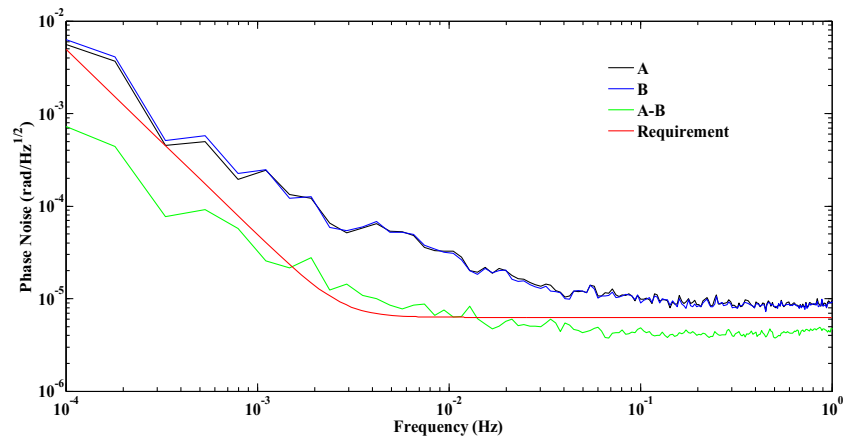


Fig. 9 The typical sensitivity curve of the phasemeter



contain the anti-aliasing filter, the RF (Radio Frequency) amplifier, the RF transformer, the transmission line, etc. The transfer time or phase response (phase) of the signal in the analog circuits are easily influenced by the environmental temperature. Moreover, the circuits present different phase delay with the frequencies changing.

G. Remaining noises

The remaining noises mainly refer to the sum of the electronic component noises, which have the property of the white noise. The electronic noise is caused by the Brownian movement of the electron. Its value is mainly determined by the temperature of the electronic parts and the components. Therefore, the components selection and the circuit design are also a significant work in the phasemeter board building.

We summarize the noises which possible influence the accuracy of the phasemeter in the above section. Some noises, like the sampling noise, the frequency jitter noise, the analog frontend noise, are the dominant parts in our recent phasemeter. We should decrease the noises as largely as possible in the actual test. Some noises, such as the loop noise, the quantization noise, have been well solved in the recent situation. Remaining noises, mainly refer to the electronic noise, are not risen to the main problem in our experiment.

The discussion of the above noises are based on the condition that parameters of the signal are permanent during the experiment. However, the coupling noise which is caused by the fluctuation of the signal parameters have not yet discussed. This noise is mainly produced by the SNR (Signal-to-Noise Ratio) variation, induced by the signal parameters change, such as the frequency, the power (Liu et al. 2014; Cervantes 2007). Although it can be ignored in this paper, the noise possibly becomes one of the main noises in Taiji condition, where the frequency of signal is fluctuated between 2 and 25 MHz and the power of the signal is also fluctuated (due to the arm length changing and the laser power drift).

Results

From the above analysis, for precisely calibrating the sensitivity of our phasemeter, we should largely depress the above noises and make sure to test the phasemeter in an idea conditions. In the calibration experiment, the phasemeter and the functional generator are synchronized by the USO. The experiment is also carried out in the condition of the zero measurement. Results are shown in the Fig. 9.

From the Fig. 9, the sensitivity (green line) after the common mode rejection can satisfy the requirement of Taiji in the frequency ranges between 0.01 Hz and 1 Hz, 0.1 mHz–1 mHz. However, noises in the bands of 1 mHz and 0.01 Hz have still not reached the requirement. From the “Noises Analysis”, the noises mainly come from two parts: (1) The analog frontend noise caused by the thermal drift. (2) The sampling noise and the frequency jitter noise. In our future research, the pilot tone will be imported to reject it (Gerberding et al. 2013, 2015; Liang 2018).

Concluding Remarks

In this paper, the phasemeter prototype have been constructed in our laboratory for Taiji and its pathfinder mission, and the sensitivity have reached the requirement in the frequencies between 0.01 Hz–1 Hz, 0.1 mHz–1 mHz. However, the noise can't be suppressed below than the requirement in the band of 1 mHz–0.01 Hz. The noise in this band are dominated by the sampling noise and the thermal noise. The pilot tone will pave the way to reject it.

The phasemeter presented here has been tested in the laboratory condition, which the SNR of tested signal is much higher and the signal parameter (frequency, power) is fixed. However, the SNR is always more lower and the parameter is also always in the dynamic condition for Taiji. More anti-aliasing filter must be imported to handle it and optimize the loop. Moreover, the phasemeter here is based

on a commercial FPGA platform, which has many useless designs for the actual experiment. A customized board for Taiji is now under developing, and we believe the better results will be obtained in the future.

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