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## Multiscale hierarchical analysis of rock mass and prediction of its mechanical and hydraulic properties

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## ABSTRACT

Engineering geological and hydro-geological characteristics of foundation rock and surrounding rock mass are the main factors that affect the stability of underground engineering. This paper presents the concept of multiscale hierarchical digital rock mass models to describe the rock mass, including its structures in different scales and corresponding scale dependence. Four scales including regional scale, engineering scale, laboratory scale and microscale are determined, and the corresponding scale-dependent geological structures and their characterization methods are provided. Image analysis and processing method, geostatistics and Monte Carlo simulation technique are used to establish the multiscale hierarchical digital rock mass models, in which the main micro- and macro-structures of rock mass in different geological units and scales are reflected and connected. A computer code is developed for numerically analyzing the strength, fracture behavior and hydraulic conductivity of rock mass using the multiscale hierarchical digital models. Using the models and methods provided in this paper, the geological information of rock mass in different geological units and scales can be considered sufficiently, and the influence of downscale characteristics (such as meso-scale) on the upscale characteristics (such as engineering scale) can be calculated by considering the discrete geological structures in the downscale model as equivalent continuous media in the upscale model. Thus the mechanical and hydraulic properties of rock mass may be evaluated rationally and precisely. The multiscale hierarchical digital rock mass models and the corresponding methods proposed in this paper provide a unified and simple solution for determining the mechanical and hydraulic properties of rock mass in different scales.

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### 1. Introduction

The rapid urbanization has posed a series of significant global challenges such as overpopulation, environmental pollution, tight supply of energy sources and global warming. We can take advantage of the natural benefits provided by underground space to meet current and future needs of the societies (Besner, 2016; Hunt et al., 2016; Kishii, 2016; Zhou and Zhao, 2016). In fact, underground space has been utilized for foundation and structures of roads and buildings, and storage of waste products, hazardous materials, energy resources and gas storage for a long time period. It helps to save land resources, promote metro development, and expand storage

space. Geotechnical engineering safety is always one of the most important issues in all phases, including planning, design, construction and operation, of underground engineering (Aksoy and Onargan, 2006; Cauvin et al., 2009; Elmo and Stead, 2010; Dindarloo and Siami-Irdemoosa, 2015; Marcoulaki et al., 2016; Qian and Lin, 2016; Ghasemi et al., 2017). Many cases show that disasters occur due to neglecting or inadequately determining the engineering geological and hydro-geological conditions at engineering sites. For example, Fig. 1a shows the Hangzhou metro pit collapse in China on November 15, 2008, in which 21 workers were killed. Ground collapse occurred at a construction site of the Subway Line 10 in Beijing, China, on March 28, 2007, as shown in Fig. 1b. The collapsed section of the tunnel, about 11 m deep, covered an area of about 20 m<sup>2</sup>. Six workers were buried in the accident. Fig. 1c shows the serious accident of water inrush and tunnel collapse that occurred in Shanghai Subway Line 4 on July 1, 2003, which caused a six-storey building to collapse, and the people in the other several

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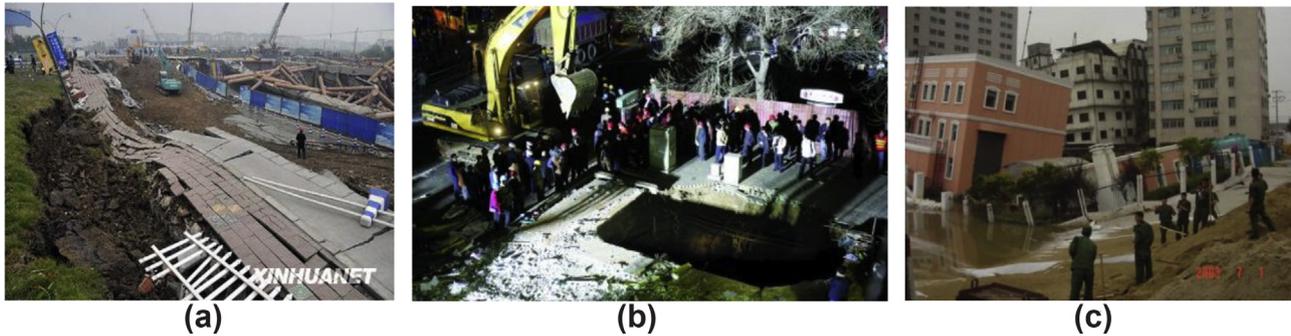


Fig. 1. Disasters occurring in underground space development: (a) Hangzhou metro pit collapse, (b) Beijing metro collapse, and (c) collapse accident in Shanghai Subway Line 4.

buildings in the neighborhood had to evacuate. Besides the poor construction and management, unclear geological conditions are the main reasons causing these engineering disasters. Engineering geological and hydro-geological characteristics of foundation rock and surrounding rock mass are the main factors that affect the stability of underground engineering.

In rock mechanics and rock engineering, related parameters are often difficult to be determined (Cai et al., 2004). Laboratory testing can be an effective method, but it has some disadvantages. By contrast, only small rock samples, regardless of fractures and joints in the rock mass, can be investigated by laboratory testing. Obviously, the results obtained cannot show the true nature of the rocks. In addition, in situ tests to determine the engineering properties of rock mass are difficult to be implemented, and are time-consuming and expensive. Some empirical or semi-empirical methods (Hoek and Brown, 1997; Singh and Goel, 1999; Smith, 2004; Ramamurthy, 2004; Singh and Rao, 2005), such as RMR (Goel et al., 1995), RQD (Singh and Goel, 1999), Q-system (Barton, 2006), and Hoek-Brown strength criterion (Hoek and Brown, 1997), were provided to evaluate the engineering properties of rock mass and to classify the rock mass. These methods have been widely adopted in rock engineering, but they often do not identify all the mechanical and hydraulic parameters of rock mass. Moreover, these rock mass quality evaluation systems do not consider local geological characteristics and use the same rating ranges for every rock type, thus they cannot sufficiently present the engineering anisotropy of rocks. Based on these, numerical modeling becomes a necessary tool for estimating the engineering properties of rock mass (Holland and Lorig, 1997; Tang et al., 2002; Jing, 2003; Liu et al., 2008).

In this paper, the concept of multiscale hierarchical digital rock mass models is presented to describe the rock mass in different scales and its scale dependence. The multiscale hierarchical digital rock mass models are some computer models, in which the main structures of rock mass in different geological units and scales are reflected. The influence of downscale characteristics (such as meso-scale) on the upscale characteristics (such as engineering scale) can be calculated by considering the discrete geological structures in the downscale model as equivalent continuous media in the upscale model. Thus the mechanical and hydraulic properties of rock mass may be evaluated rationally and precisely. Moreover, computer codes have been developed for numerically analyzing the strength, fracture behavior and hydraulic conductivity of rock mass using the multiscale hierarchical digital rock mass models. By the models and methods provided in this paper, the geological information in different geological units and scales can be considered sufficiently, and the mechanical and hydraulic parameters of rock mass can be determined quantitatively. The results show that the effect of micro-defects on the deformation behavior and the effects of fracture geometry factors, such as dip angle, trace length, fracture spacing and width, on hydraulic conductivity of rock mass in

engineering scale can be studied. The multiscale hierarchical digital rock mass models and the corresponding methods in this paper provide a unified and simple solution for determining the mechanical and hydraulic properties of rock mass in different scales.

## 2. Multiscale hierarchical digital rock mass models

Natural rock and rock mass are materials with hierarchical defect structures, such as micro-defects, micro-fracture, fractures, and faults (Liu et al., 2008, 2011). These hierarchical structures affect each other, controlling the mechanical and hydraulic behaviors of rock and rock mass (Warpinski and Teufel, 1987; Weng et al., 2011; Virgo et al., 2013; Behraftar et al., 2017; Chen et al., 2018). However, it is difficult to describe all the hierarchical structures in one model, thus the multiscale hierarchical digital rock mass models are presented to investigate the effect of these hierarchical structures on the engineering properties of rock mass. At each level of structural hierarchy, the material can be modeled as the constitution of a continuum and characteristic structures in this level for the purpose of analysis. In this paper, four scales are determined for analysis purpose, as shown in Fig. 2. In every scale, only one-level hierarchical structures exist, and the mechanical and hydraulic properties of rock mass in this scale are easily determined by the equivalent continuum method. The following sections describe the multiscale hierarchical digital rock mass models.

### 2.1. Regional scale model

Regional scale model, in which the engineering structures (e.g. dam and tunnel) and the regional geological body with topography are included, is presented for safety analysis of engineering structures. The distinct faults and rock layers, which are generally less and can be determined by geological investigation in regional scale, are classified as the first-level hierarchical structures. These structures should be considered in regional scale model, because they control the deformation characteristics and groundwater seepage in this scale. The first-level hierarchical structures divide the geological bodies in the whole region into many distinct independent bodies. These bodies are considered as equivalent continuum in this scale, as shown in Fig. 3, where an excavation process under a river is described.

### 2.2. Engineering scale model

In engineering scale, the distinct independent bodies divided by the first-level hierarchical structures are the main analysis objects. Though they have been considered as equivalent continuum in the regional scale, they are inhomogeneous and anisotropic in the engineering scale because a large number of fractures exist in them, as shown in Fig. 2b. These fractures are classified as the second-level

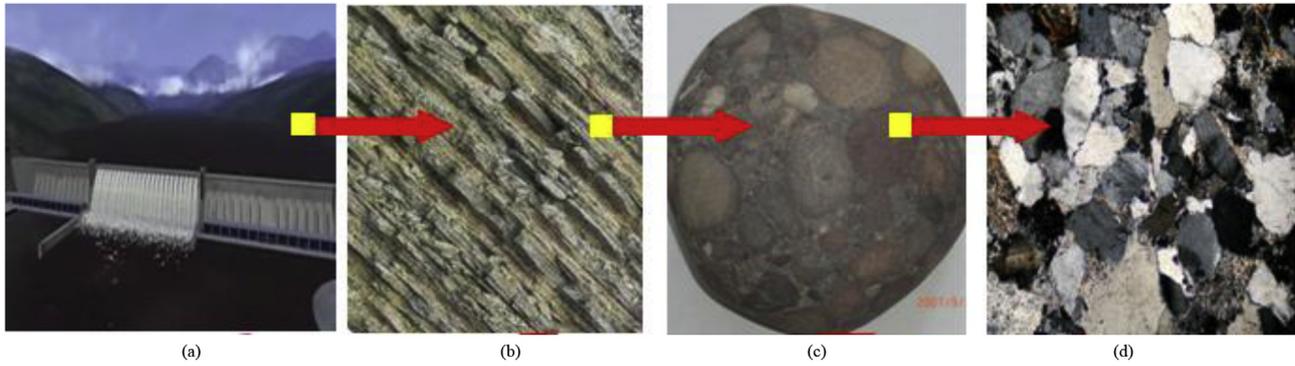


Fig. 2. Coarse-to-fine hierarchical qualitative analysis for engineering: (a) Regional scale, (b) engineering scale, (c) laboratory scale, and (d) meso-scale.

hierarchical structures. The multiplicity and randomness of the second-level hierarchical structures distribution in rock mass are the key issues in studying the engineering properties of fractured rock mass in engineering scale. The randomness of fracture geometry shapes (i.e. orientation, dip, trace length, spacing, and width) is considered on the basis of geological investigation. The geostatistics and Monte Carlo simulation technique are adopted to generate the second-level hierarchical structures. The following five types of distribution functions of the random variable are often used to describe the randomness of fracture geometric shapes:

(1) Uniform distribution, whose probability density function is

$$f(x) = \frac{1}{b-a} \quad (a \leq x \leq b) \tag{1}$$

The cumulative probability distribution function can be obtained by the following equation:

$$F(x) = \int_a^b f(t)dt = \frac{x-a}{b-a} \tag{2}$$

where  $F(x)$  is between 0 and 1.

Given  $F(x) = (x-a)/(b-a) = R$ , the random independent variable  $x_r$  is written as

$$x_r = R(b-a) + a \tag{3}$$

(2) Normal distribution, whose probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \tag{4}$$

The cumulative probability distribution function can be written as

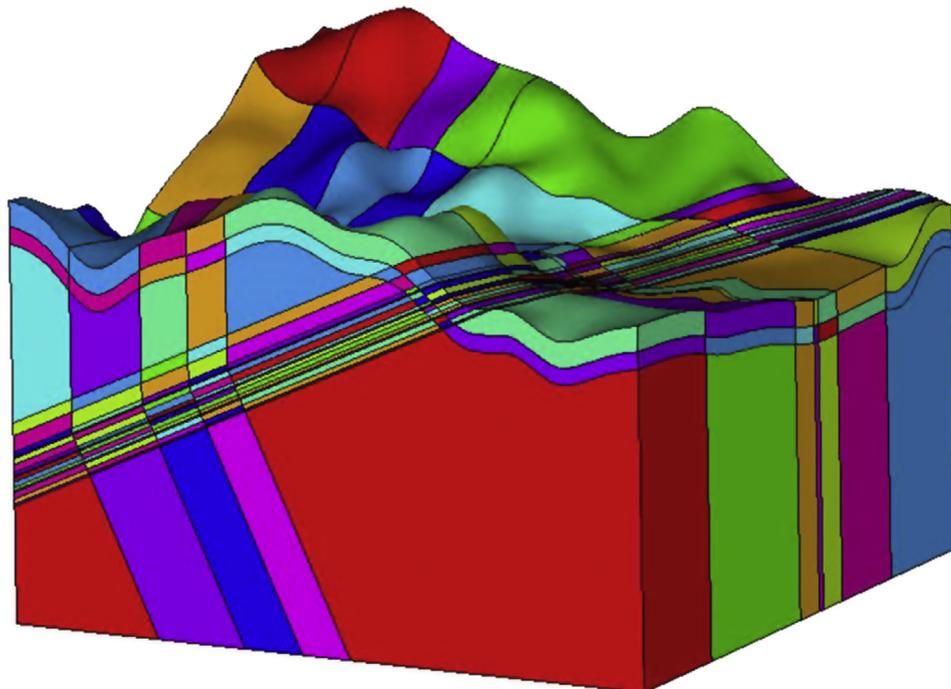


Fig. 3. Regional scale model for an excavation process.

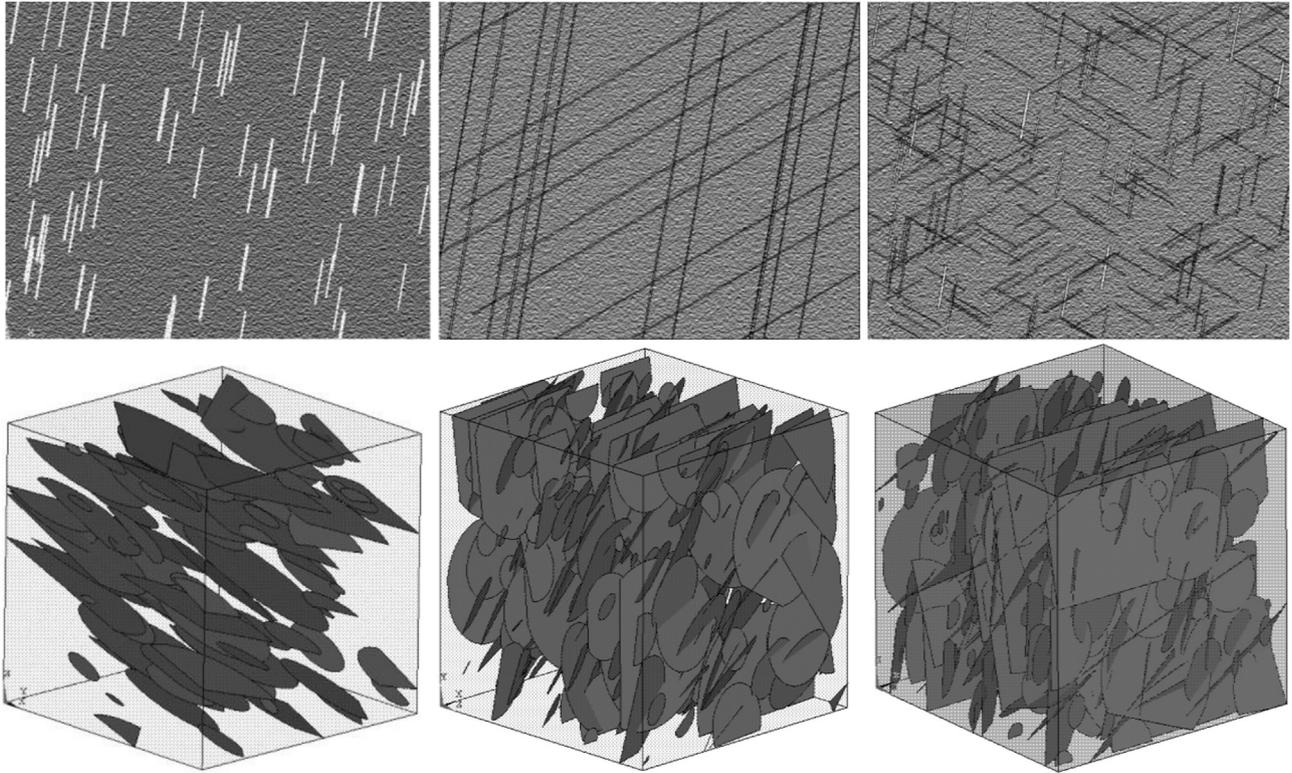


Fig. 4. Fracture network models in engineering scale.

$$F(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \quad (5)$$

$$f(x) = \frac{1}{\mu_e} e^{-x/\mu_e} \quad (6)$$

Generally, the explicit expression of  $F(x)$  cannot be obtained directly. Central limit theorem and Priest method are often used to obtain the desired random independent variable.

where  $\mu_e$  is the mean value of  $x$ .

Given the random variable  $R$  with uniform distribution between 0 and 1, the random independent variable  $x_r$  is written as

(3) Negative exponential distribution, whose probability density function is

$$x_r = -\mu_e \ln(1 - R) \quad (7)$$

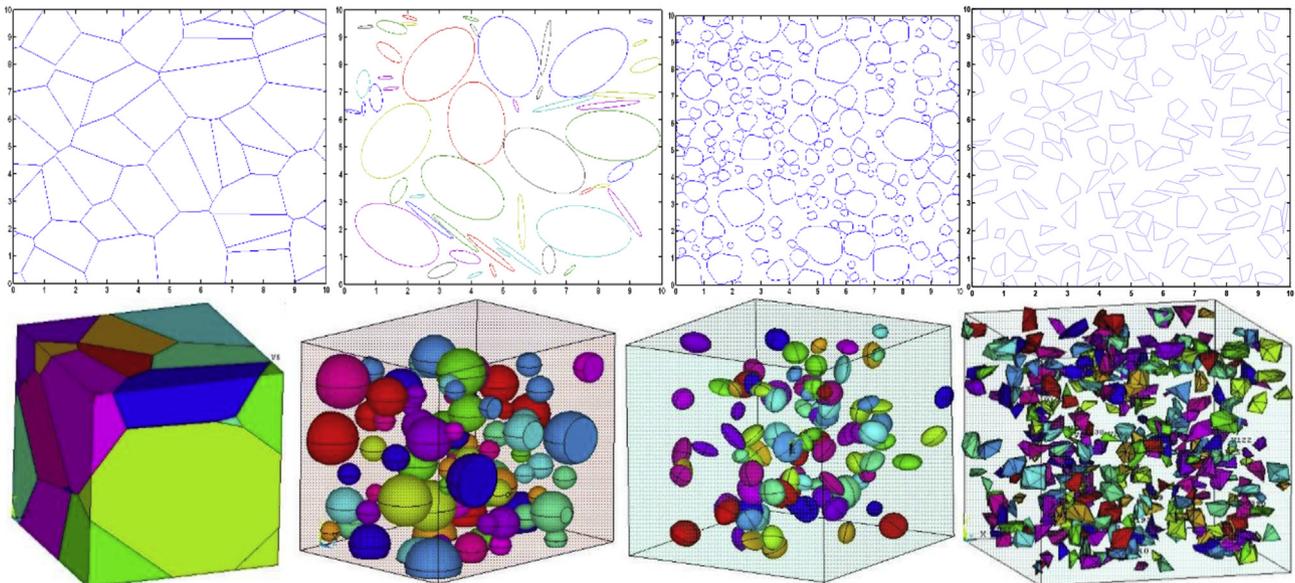


Fig. 5. Laboratory scale rock block models with sub-structures in different shapes.

(4) Logarithmic normal distribution, whose probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}(x-a)\sigma_1} \exp\left\{-\frac{[\ln(x-a)\mu_e]^2}{2\sigma_1^2}\right\} \quad (x > a) \quad (8)$$

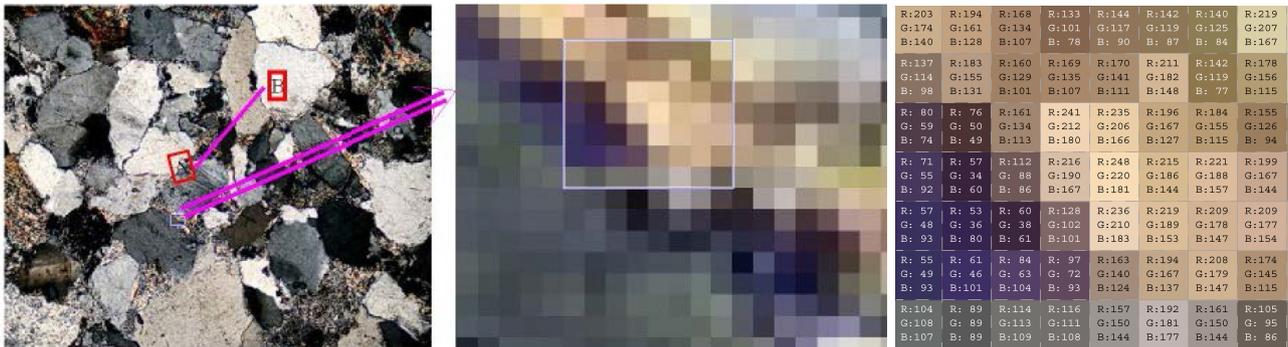
where  $\sigma_1$  is the variance of  $x$ .

The random independent variable  $x_r$  is written as

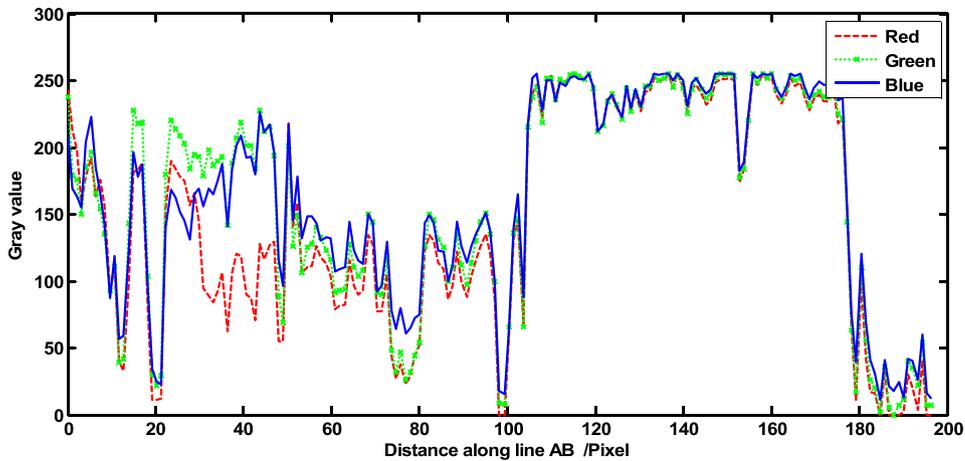
$$x_r = a + \exp(\sigma_1 r_n - \mu_e) \quad (9)$$

where  $r_n$  denotes the random number with the logarithmic normal distribution.

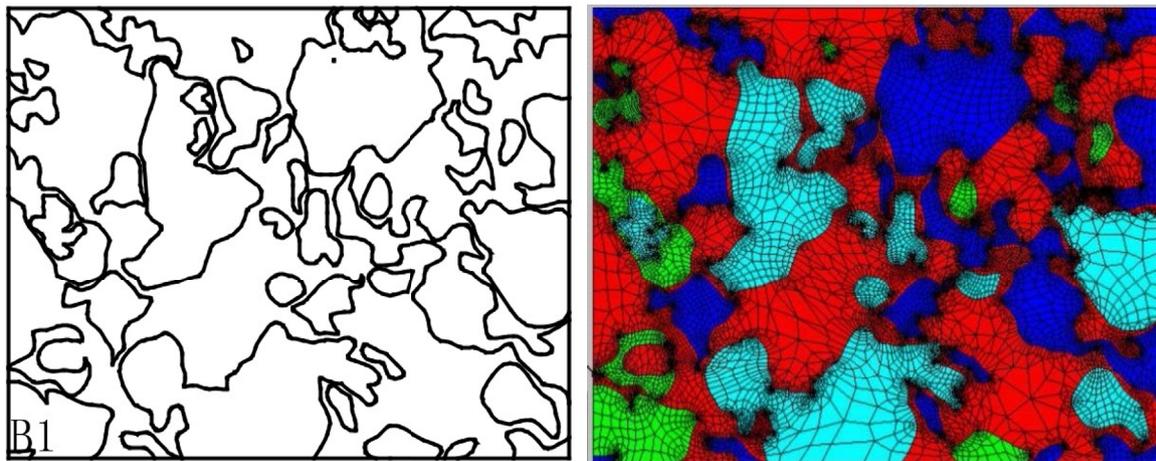
(5) Fisher distribution, which is frequently used to describe the probability distribution of normal direction of fractures in geological body. The probability density function of Fisher



(a) Meso-scale rock photo and pixel RGB values.



(b) RGB values along the line AB in (a) (segmentation threshold values can be obtained).



(c) Identified mineral borders and corresponding meso-scale rock model.

Fig. 6. Meso-scale rock model generating process by the digital image processing technique.

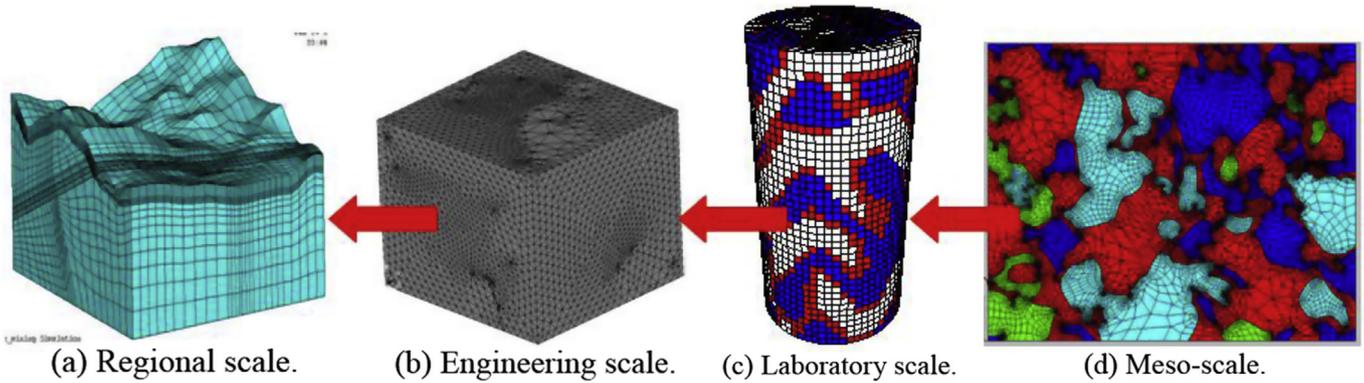


Fig. 7. Fine-to-coarse hierarchical quantitative analysis method.

distribution can be described in spherical coordinate system by the following equation:

$$f(\varphi'_n, \theta'_n) = \frac{1}{2\pi} \frac{\eta \sin \theta'_n}{e^\eta - e^{-\eta}} e^{\eta \cos \theta'_n} \quad (0 < \theta'_n < \pi, 0 < \varphi'_n < 2\pi) \quad (10)$$

where  $\varphi'_n$  and  $\theta'_n$  are the independent variables with uniform distribution between 0 and  $2\pi$ , and  $\eta$  denotes the concentration parameter that determines the degree of clustering around the mean direction. When  $\eta = 0$ , the Fisher distribution becomes the uniform distribution.

Given  $\eta$  with a value large enough, the cumulative probability distribution function can be written as

$$P(\theta'_n) \approx 1 - e^{\eta(\cos \theta'_n - 1)} \quad (11)$$

Given the random variable  $R$  with uniform distribution between 0 and 1, the random independent variable  $\theta'_{nr}$  is written as

$$\theta'_{nr} = \arccos \left[ \frac{\ln(1 - R)}{\eta} + 1 \right] \quad (12)$$

The above probability distributions are used to simulate the real fracture geometry shapes (orientation, dip, trace length, spacing, and width), and to determine which kind of probability distribution should be used to describe the fracture geometric shapes according to the engineering geology investigation. In the models shown in Fig. 4, the structure information of fractures is described and the real engineering rock mass is represented, including interpenetrated joints, non-interpenetrated fractures, and filled and unfilled fractures.

### 2.3. Laboratory scale model

Laboratory scale rock blocks often exhibit sub-structures and micro-defects, which cause nonhomogeneity and anisotropy of rocks. Based on the statistical analysis of these sub-structures and micro-defects distribution in rocks, Monte Carlo simulation technique can be adopted to simulate them. Laboratory scale rock models with sub-structures in different shapes are described in Fig. 5.

### 2.4. Meso-scale model

Digital image processing (DIP) technique is utilized to analyze the meso-scale rock pictures obtained by digital cameras or X-ray computed tomography (CT) (Yue et al., 2003). Using the DIP, the geometric distribution of all kinds of minerals in rock can be obtained, and the mineral borders can be identified, as illustrated in Fig. 6.

### 2.5. Fine-to-coarse multiscale hierarchical quantitative analysis method

Compared with the coarse-to-fine hierarchical qualitative analysis shown in Fig. 2, evaluation of rock mass quality and prediction of engineering stability are always carried out in reverse order, i.e. using fine-to-coarse hierarchical quantitative analysis method, as shown in Fig. 7. In this method, rock properties in fine scale can be determined exactly by combining experimental data and numerical results. When it is upscaled to a coarser scale, the fine scale rock is homogenized to be continuum and the structures in the fine scale are not described ever. Thus in every scale, the models are established with the continuum, whose properties are obtained in a finer scale, and the corresponding structures are obtained in the present scale. Using this method, the geological information can be transferred from one scale to another scale, and is incorporated together to analyze the engineering behavior of rock mass. It reveals the essential relation between these scales, and also connects the incomplete data from geology system, mechanical model and numerical simulation based on rigorous theories.

### 3. Mechanical and hydraulic properties of rock block or rock mass

A computer code GeoCAAS (geo-engineering computer aided analysis system) has been developed to construct the multiscale hierarchical digital rock mass models and to simulate the

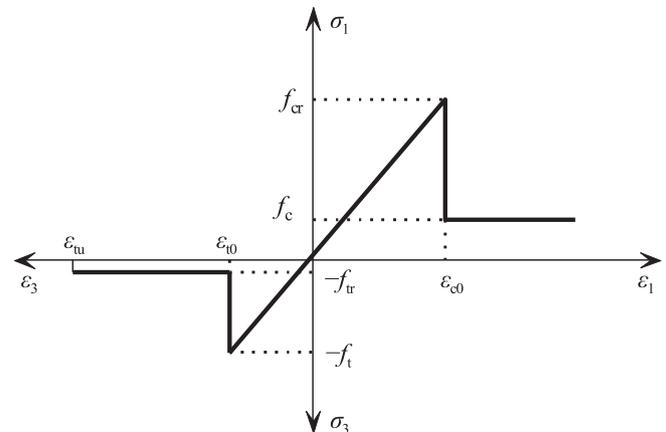


Fig. 8. Elasto-brittle constitutive relation.

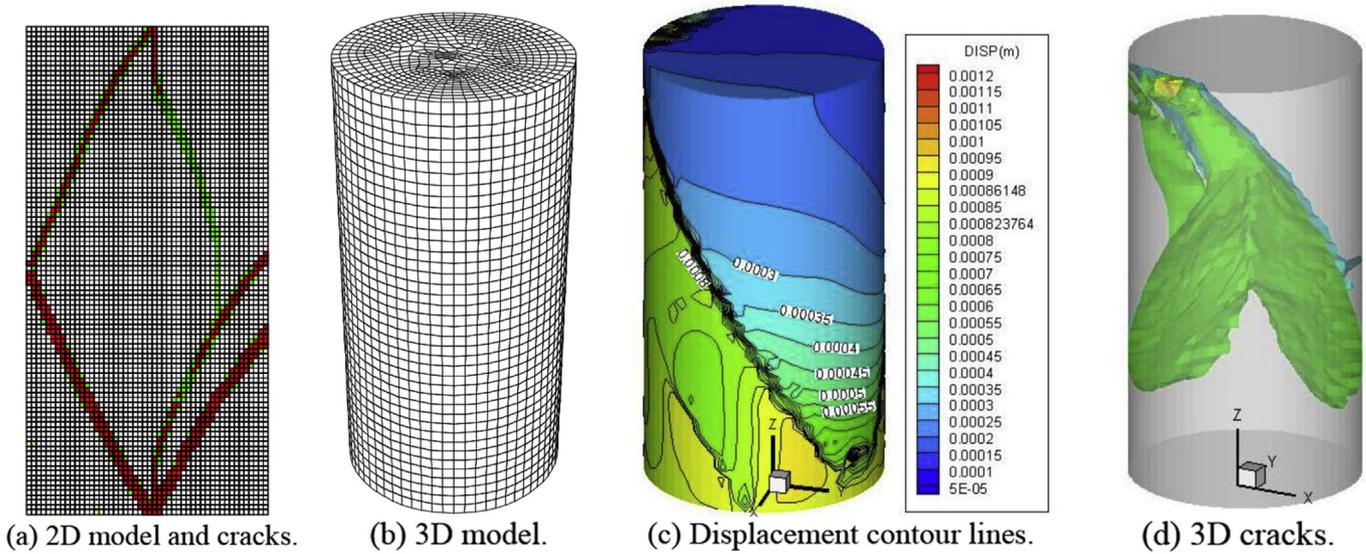


Fig. 9. Models and fractures of two-dimensional (2D) and three-dimensional (3D) numerical uniaxial compression tests for laboratory scale rock blocks.

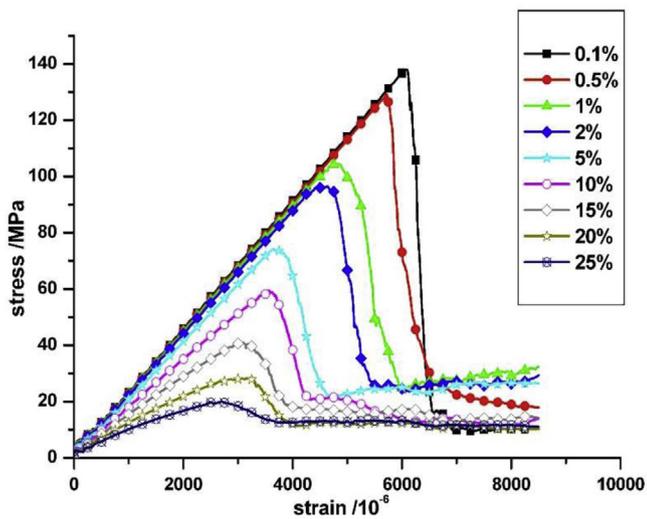


Fig. 10. Stress–strain curves of rock specimens with different contents of micro-defects.

mechanical and hydraulic properties of rock mass. Moreover, the multiscale hierarchical digital rock mass models can be discretized and then combined with numerical methods (such as finite element method (FEM), discrete element method (DEM)) to solve practical engineering problems. The following section presents the applications of the code by analyzing the strength property, fracture behavior and hydraulic conductivity of rock block or rock mass using the numerical method.

### 3.1. Mechanical properties of rock block

Elasto-brittle constitutive relation, as shown in Fig. 8, is utilized to simulate the mechanical properties of rock block:

$$\sigma = (1 - D)E_0 : \epsilon \tag{13}$$

$$D = \begin{cases} 0 & (\epsilon < \epsilon_{c0}) \\ 1 - \frac{f_{cr}}{E_0 \epsilon} & (\epsilon_{c0} \leq \epsilon) \end{cases} \text{ (compression)} \tag{14}$$

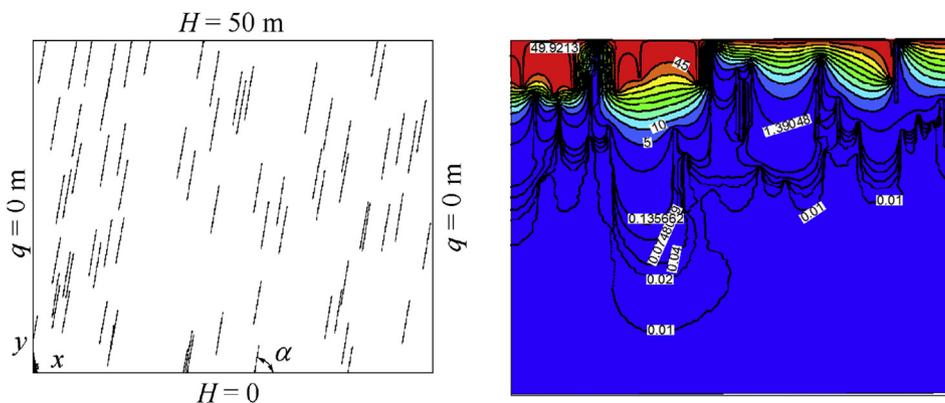


Fig. 11. Boundary conditions of numerical test for fractured rock mass model and water head distribution (unit: m).

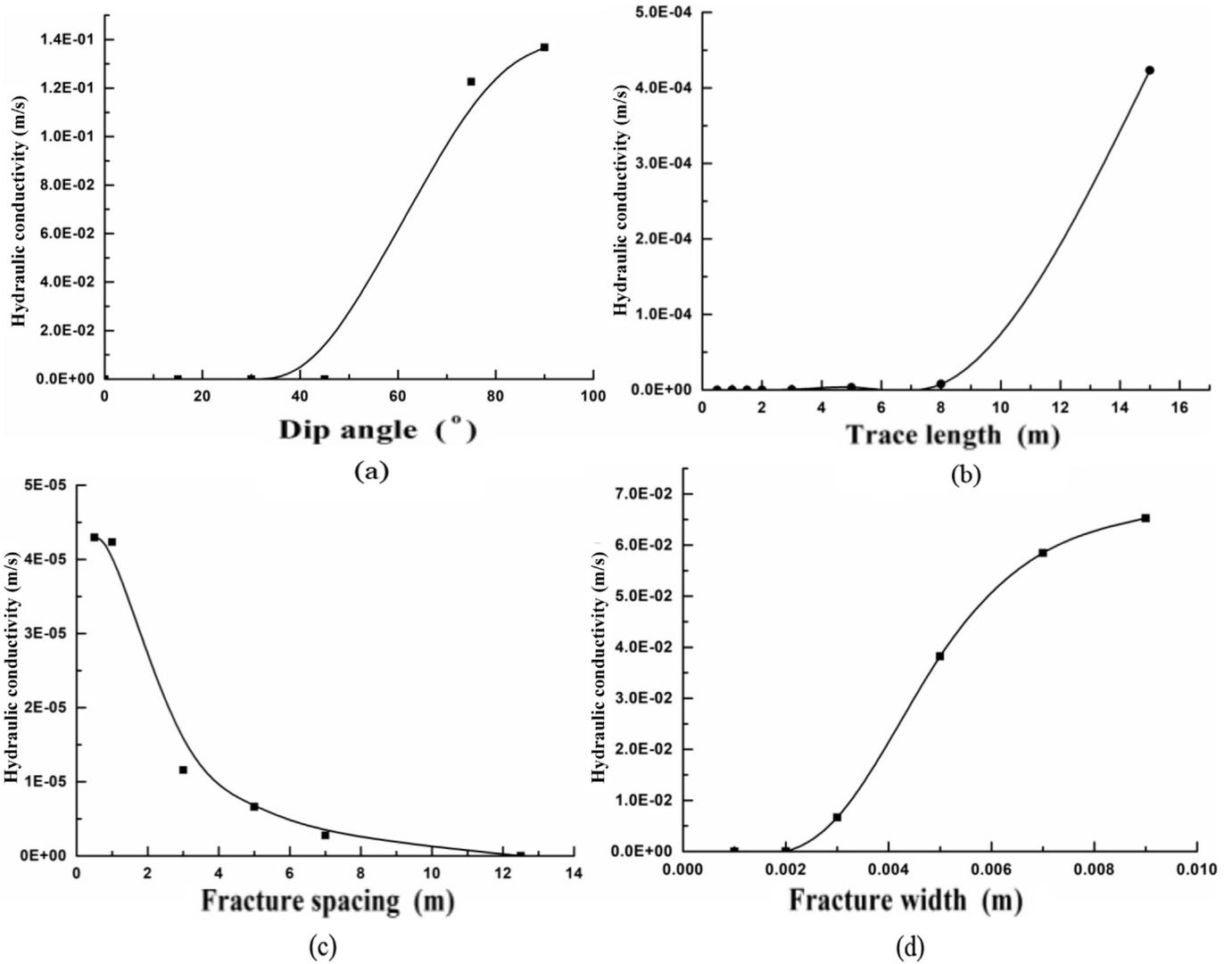


Fig. 12. Relation curves of hydraulic conductivity of fractured rock mass in vertical direction with fracture dip angle, trace length, fracture spacing and fracture width.

$$D = \begin{cases} 0 & (\epsilon_{t0} < \epsilon) \\ 1 - \frac{f_{cr}}{E_0 \epsilon} & (\epsilon_{tu} \leq \epsilon < \epsilon_{t0}) \text{ (tension)} \\ 0 & (\epsilon \leq \epsilon_{tu}) \end{cases} \quad (15)$$

Simulation results are partially shown in Fig. 9. The effects of micro-defects on the stress–strain curves are described in Fig. 10.

### 3.2. Hydraulic properties of rock mass

The hydraulic properties of rock mass, especially the fractured rock mass, are difficult to be determined in engineering practice. For example, many equations have been presented to evaluate the hydraulic conductivity tensor of rock mass, such as the cubic law for flow transportation in single fracture, but they are not always effective in determining the hydraulic conductivity tensor of intact rock mass. Numerical testing based on the above-mentioned digital rock mass models is a promising method. This method uses the seepage simulation to back analyze the hydraulic conductivity in different directions, as shown in Fig. 11:

$$K = -\frac{Q}{A} \frac{dl}{dh} \quad (16)$$

where  $K$  denotes the hydraulic conductivity,  $Q$  is the flow rate,  $A$  is the bulk cross-sectional area of flow, and  $dl/dh$  is the reciprocal of hydraulic head gradient.

Using numerical tests, the effects of fracture geometry factors, such as dip angle, trace length, fracture spacing and fracture width, on the hydraulic conductivity of intact rock mass are investigated, as shown in Fig. 12.

### 4. Conclusions

Evaluation of the mechanical and hydraulic properties is important in utilization and development of underground space. In this paper, the multiscale hierarchical digital rock mass models are presented to describe the rock mass in different scales and its scale dependence. A computer code is developed to determine the engineering properties of rock block and rock mass based on the multiscale hierarchical digital rock mass models. Because the nonhomogeneity and anisotropy of natural rock mass are reflected by the hierarchical structures in the multiscale digital rock mass models, the simpler constitutive relations of mechanical and

hydraulic behaviors are able to simulate the real conditions of rock mass. Moreover, this paper discusses the fine-to-course multiscale hierarchical quantitative analysis method, and it also provides a general and universally valid framework for analysis of geo-engineering problems.

### Conflicts of interest

The authors wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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