Hierarchical-microstructure based modeling for plastic deformation of partial recrystallized copper

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A B S T R A C T

Hierarchical microstructure in partial recrystallized materials can simultaneously improve the strength and ductility of metallic materials. Modeling the mechanical behavior of partial recrystallized materials helps to process materials with superior combination of ductility and strength. Here, using experimental characterization, cellular automation (CA) and finite element method, hierarchical-microstructure based modeling was proposed to simulate the tensile deformation of partial recrystallized copper. Firstly, partial recrystallized coppers with different volume fractions of recrystallization were produced by means of extrusion machining and subsequent heat treatment (HT). Uniaxial tensile tests and microstructural observations show that the hierarchical-microstructure of recrystallized grains (RGs) surrounded by elongated subgrains has a significant effect on the mechanical properties. Then, based on the experimental results, a hierarchical-microstructure based plasticity model was developed to describe the yield surface of partial recrystallized materials. CA was further employed to simulate the hierarchical microstructure. By embedding the plasticity model and simulated hierarchical-microstructure in finite element method, a finite element model (FEM) for mechanical behavior of partial recrystallized copper was proposed, where the elongated subgrain with forest dislocation and low angle grain boundary, the RG with few dislocations and twin boundary, and volume fraction of recrystallization were taken into consideration. Finally, the experimental data and the comparison with the conventional plasticity model validate the rationality of the proposed model.

1. Introduction

For metallic materials in many applications, there is significant interest in enhancing strength in balance with good ductility (Meyers and Chawla, 2009; Ming et al., 2018; Valiev et al., 2016). Extensive investigations have proved that superior combination of ductility and strength is able to be achieved by combining severe plastic deformation (SPD) technology with heat treatment (HT) (Ma and Zhu, 2017; Ovid’ko et al., 2018). Bimodal copper produced by cryogenic rolling and subsequent annealing possesses superior balance between strength and ductility (Wang et al., 2002). Nanostructured copper could combine high strength and good ductility by inducing static recrystallization during annealing process (Li et al., 2008b; Zhang et al., 2008). Dynamic plastic deformation and subsequent thermal annealing were employed to produce good-ductility and high-strength Mg-Zn alloy by mixing nanotwin bundles with microsized recrystallized grains (Xiao et al., 2011). Asymmetric rolling and partial recrystallization were conducted to fabricate heterogeneous lamella structural titanium with high-strength and coarse-grain-ductility (Wu et al., 2015). Rapid annealing treatment after high pressure torsion was used to achieve full recrystallization in ultrafine grains, resulting in improvement of both strength and ductility (Zheng et al., 2017). To-date, the trade-off of strength and ductility has been overcome in various metallic materials by combining SPD and subsequent HT: (i) the conventional metals, including copper (Tian et al., 2018), stainless steel (El-Tahawy et al., 2018; Sun et al., 2019) and titanium (Huang et al., 2019); (ii) the new class of materials, including medium-entropy (Laplanche et al., 2017) alloy and high-entropy alloy (Sun et al., 2018; Wu et al., 2019).

Modeling the deformation behavior of materials processed by SPD and HT is an important research area, which helps to optimize the processing parameters for enhancement of both strength and ductility. For metallic materials, the elastic deformation is linear and obeys
Hooke’s law (Hooke, 1678). Plastic deformation often stems from non-linear processes like dislocation, void, slip, twinning, necking, shear banding, crack etc.; therefore, plentiful models for plastic deformation have been proposed for different considerations. Hill (1948) put forward a quadratic yield criterion to predict the plastic deformation of isotropic hardening. Hosford (1979) introduced a yield criterion to characterize yielding behavior of materials subjected to plane stress. In the condition of finite strains, Loret (1983) established constitutive equations for anisotropic materials. Barlat et al. (1991) developed a yield function with six independent components for anisotropic materials by means of two linear transformations of the stress tensor. Karafillis and Boyce (1993) further generalized linear transformation to develop the yield function for both isotropic and anisotropic materials. Considering the dependence of strain rate and temperature, Nemat-Nasser et al. (1998) proposed a constitutive model for face-centered cubic (FCC) single crystals. Zikry and Kameda (1998) put forward a dislocation-density based constitutive framework to understand the deformation mechanisms of intermetallics at high strain rate. Because the macrostress has an effect on the macrostress in nature, Hori and Nemat-Nasser (1999) advised a hybrid micromechanics model for heterogeneous materials. In order to model deformation of heterogeneous nanograined materials, Altan and Subhash (2003) presented the representative volume element with two isotropic material properties. A gradient plasticity model was introduced to characterize the deformation patterns of slip lines and slip bands (Tsagarakis et al., 2006; Zaiser and Aifantis, 2006). A theory of dynamic recrystallization softening was suggested to model the formation of shear bands (Osovski et al., 2013; Rittel et al., 2008, 2006). Srivastava and Nemat-Nasser (2011) presented universal theorems to formulate the dynamic behavior of linearly elastic heterogeneous solid. The homogenous anisotropic hardening model was developed for the description of the strain hardening response of metallic materials under non-proportional loading (Barlat et al., 2014; Ha et al., 2013). By coupling plasticity to martensitic phase transformation, Sadjadpour et al. (2015) extended a thermo-mechanical model to investigate the dynamic shear deformation. Recently, by means of a self-consistent method, Abdul-Latif et al. (2018) developed a model to describe the mechanical response of heterogeneous ultrafine grained and nanocrystalline FCC metals. Ziaei and Zikry (2019) understood the aggregate behavior of multi-phase crystalline by combining microstructural interactions with the dislocation-density framework. Considering the effect of transient mobile dislocation density, Varma et al. (2019) provided a stress relaxation constitutive model to predict strain hardening rate. Moreover, some microstructural observations were carried out on the materials processed by SPD and HT to analyze the mechanism of plastic deformation (Ma and Zha, 2017; Ovidko et al., 2018; Tian et al., 2018; Zheng et al., 2017). The experimental results show that the hierarchical-microstructure with the heterogeneous grains separated and surrounded by nanostructures can be created in partial recrystallized materials. These results inspired us to develop a hierarchical-microstructure based plasticity model for the description of mechanical behavior in partial recrystallized materials.

In present work, a theoretical model is proposed to characterize the mechanical behavior of partial recrystallized cooper with hierarchical-microstructure. Partial recrystallized coppers were processed using a SPD method of extrusion machining and subsequent HT at different temperatures. Based on tensile tests and microstructural observations, we investigated the relationship between microstructure and mechanical property. After analyzing the experimental results, a theoretical model combining cellular automation (CA) and finite element method was proposed to describe the mechanical response of partial recrystallized cooper. The accuracy and rationality of the proposed model were further discussed.

2. Experiments

2.1. Experimental procedure

Fig. 1 illustrates the initial microstructure of cooper with the composition (wt%): Cu (99.95%), Fe (0.004%), S (0.004%), Pb (0.003%), Zn (0.003%), Sn (0.002%). The initial copper possesses coarse grain face center cubic (FCC) microstructure and the average grain size is ~100 μm. The initial copper was first processed at the shear strain of 3.1 by using the SPD method of quasi-static extrusion machining (QSEM) (Liu et al., 2019), and then the SPDed copper was annealed for 5 min at the temperatures of 293, 473, 523, 548, 573, 773 K. After QSEM and subsequent HT, cooper with a gauge dimension of 12 × 2.5 × 0.5 mm3 was tested by means of Instron E10000. All uni-axial tensile tests were conducted at the same strain rate (2.5 × 10−4 s−1). Copper annealed at each temperature was tested at least 3 times. Electron backscatter diffraction (EBSD) at the scanning step length of 0.5 μm was conducted on JOEL JSM-7800F. Transmission electron microscope (TEM) test at 200 kV operating voltage was performed on JEM 2010.

2.2. Mechanical property

The tensile test results were shown in Fig. 2. As for initial copper, the 0.2% yield strength is ~ 47 MPa, the ultimate tensile strength is ~196 MPa, and the uniform elongation is ~41.7%. SPDed copper annealed at low temperature possesses improved strength but poor uniform elongation. High ductility but low strength can be achieved if SPDed cooper is annealed at high temperature. As illustrated in Fig. 2b, 523 K is the transition temperature of HT. SPDed copper annealed
below 523 K possesses high strength but low ductility; otherwise, copper has low strength but high ductility. In particular, compared with the initial workpiece, SPDed copper annealed at 573 K has doubled yield strength but no reduction in ductility.

2.3. Microstructural observations

Fig. 3 shows the distribution of grain boundary misorientation which contains low angle grain boundary (LAGB) and high angle grain boundary (HAGB). According to Fig. 3a, a mass of LAGBs but few twin boundaries (TBs) appear in SPDed copper annealed at 293 K. LAGB resulting from forest dislocation helps to improve the strength. As for high HT temperature (Fig. 3c-d), the frequency of ∼60° misorientation angle is relatively high, which indicates the formation of Σ3 twin boundaries (TBs). Σ3 twin boundaries are coherent interfaces providing a possible slip transfer (Christian and Mahajan, 1995; Lu et al., 2009), which could obstruct the motion of dislocations and accommodate some dislocations.

Fig. 4 shows EBSD image quality map of SPDed copper annealed at different temperatures. SPDed copper annealed at 293 K (Fig. 4a) is filled with severe-plastic-deformed (SPDed) grains which contain a mass of LAGBs. If the HT temperature is greater than or equal to 523 K (Fig. 4b-d), there are two types of grains in copper sheet, i.e., SPDed grains and recrystallized grains (RGs). It is illustrated in Fig. 4 that the volume fraction of RG is larger at higher HT temperature.

Fig. 5 shows the TEM observation of SPDed copper annealed at
523 K. RGs are surrounded by SPDed grains. The average grain size of RG is ∼4 μm (Fig. 5a). SPDed grains are elongated subgrains which are full of forest dislocations (Fig. 5b). The capacity to store new dislocation is poor in SPDed grains. SPDed grains possess high strength but poor ductility. Comparing Fig. 5b and c, fewer dislocations appear within RG. New dislocations could be stored in RGs during plastic deformation, which helps to the improvement of ductility.

Orientation imaging maps (OIMs) for processed coppers are illustrated in Fig. 6. Compared with initial copper (Fig. 1a), SPDed grains tend to rotate toward the texture component {111} in SPDed copper (Fig. 6a). If subsequent HT is conducted in SPDed grains, small RGs emerge with the random orientations (Fig. 6b-d). The pole figures of processed copper at different HT temperatures are plotted in Fig. 7 to further reveal the random orientation of RGs. As shown in Fig. 7a, the processed copper without RGs possesses strong basal texture ((100)||ND). As for partial recrystallized copper in Fig. 7b, RGs with random orientation weaken the basal texture ((100)||ND). The basal texture becomes weaker with the increasing HT temperature (Fig. 7c). When HT temperature is higher than 573 K, full recrystallization happens in copper. The randomization of the texture is observed in full recrystallized copper (Fig. 7d), which indicates that RG texture is nearly random.

It is illustrated in Figs. 3-7 that partial recrystallization happens during subsequent HT of SPDed copper. A hierarchical microstructure, i.e., small RGs surrounded by SPDed grains, can be produced in partial recrystallized copper. According to Hall-Petch relationship (Hall, 1951; Petch, 1953), small RGs can improve the strength of material. There is a difference of texture component between SPDed grain and RG, which could have an inevitable effect on the mechanical behavior. SPDed grains saturated with forest dislocation improve strength. RGs with few dislocations and coherent TBs inside help to the improvement of ductility. High strength and high ductility of partial recrystallized copper attribute to the hierarchical microstructure.

3. Model for partial recrystallized copper

Partial recrystallized copper possesses hierarchical microstructure; therefore, the conventional plasticity model only for SPDed grains or RGs is not suitable for mechanical behavior of partial recrystallized copper. A new plasticity model considering the hierarchical-microstructure is needed to characterize the plastic deformation of partial recrystallized copper.

3.1. Cellular automaton simulation

Cellular automation (CA) has been widely employed to simulate microstructural evolutions, e.g., solidification (Chen et al., 2015), phase transition (Bussemaker et al., 1997), grain growth (Davies, 1997; Gandin and Rappaz, 1997), static recrystallization (Kühbach et al., 2016; Marx et al., 1999), and dynamic recrystallization (Chen et al., 2010; Popova et al., 2015). The detailed theory for CA has been described in the literatures (Chen et al., 2009; Ding and Guo, 2001). Here, the algorithm of CA is directly used to simulate the hierarchical-microstructure of partial recrystallized copper. A two-dimensional CA model was employed. The 2-D CA model is composed of 200 × 200 equal-sized square cells. Four typical state transformation rules were employed. As shown in Fig. 8, an arbitrary site is determined by its neighbouring sites.
Rule 1: the orientation in cell C5 depends on both nearest (C2, C4, C6 and C8) and next-nearest (C1, C3, C7 and C9) neighbors. The state of cell C5 could transform to the state S in the next cellular automaton step if the number of its neighboring cells with state S is not less than five.

Rule 2: the orientation in cell C5 is determined by its nearest neighbors (C2, C4, C6 and C8). The state of cell C5 could transform to the state S in the next cellular automaton step if arbitrary three of the cells C2, C4, C6 and C8 possesses the state S.

Rule 3: the orientation in cell C5 is related with its next-nearest neighbors (C1, C3, C7 and C9). The state of cell C5 could transform to the state S in the next cellular automaton step if arbitrary three of the cells C1, C3, C7 and C9 possesses the state S.

Rule 4: if none of the above rules 1–3 is satisfied, the state of cell C5 could randomly transform to an arbitrary state of its nearest (C2, C4, C6 and C8) and next-nearest (C1, C3, C7 and C9) neighbors.

The probability of cell C5 state transition relies on the change of interfacial energy between before and after the trial transition. The state transition happens if the total interfacial energy decreases after the trial transition. The lowest energy principle (Chen et al., 2010; He et al., 2006) is used to characterize the normal grain growth:

\[
P = \begin{cases} 
1, & \Delta E_{ij} < 0 \\
0, & \Delta E_{ij} \geq 0 
\end{cases}
\]  

(1)

\[
E_i = \gamma \sum_j \left(1 - \delta_{C_i C_j}\right)
\]  

(2)

\[
\Delta E_{ij} = E_j - E_i
\]  

(3)

In Eqs. (1)-(3), \(\gamma\) is grain boundary energy, \(\delta\) is Kronecker symbol, \(k\) is the \(k\)th neighbor to site \(i\), and \(c\) is the total number of site \(i\). \(C_i\) and \(C_k\) are respectively the orientations of site \(i\) and \(k\). Site \(j\) or \(k\) refers to one of the nearest (C2, C4, C6 and C8) and next-nearest (C1, C3, C7 and C9) neighbors. \(P\) is the transition probability controlling the state of cell C5.

The nucleation and growth of recrystallization in CA are modeled by the following expressions (Ding and Guo, 2001):

\[
\eta(t, T) = C_n \dot{\varepsilon} \exp\left(-\frac{Q_{\text{act}}}{RT}\right)
\]  

(4)

\[
\dot{v}_i = \frac{h D_b \dot{F}_i}{4 \pi r_i^2 K T} \exp\left(-\frac{Q_b}{RT}\right)
\]  

(5)

Here, \(\eta(t, T)\) is the nucleation rate of per unit grain boundary area, \(\dot{\varepsilon}\) is strain rate, \(Q_{\text{act}}\) is activation energy, \(R\) is gas constant, and \(T\) is absolute temperature. In Eq. (5), \(h\) is characteristic grain boundary thickness, \(D_b\) is boundary self-diffusion coefficient, \(Q_b\) is boundary diffusion activation energy, \(K\) is Boltzmann’s constant, \(b\) is Burger’s vector, \(r_i\) is the radius of RG, and \(v_i\) is the growth velocity of the \(i\)th RG. The driving force \(F_i\) is related with the dislocation density and radius of the \(i\)th RG.

In this work, the values of the parameters in Eqs. (1)-(5) can refer to the references (Chen et al., 2010; Ding and Guo, 2001; He et al., 2006). The simulated mean RG size is 4 μm obtained from TEM observations in Fig. 5a, and then \(r_i\) is equal to 2 μm. Based on the experiments in Fig. 4b, the nucleation site of recrystallization can be on grain boundary or within SPDed grain. By means of MATLAB-R2009b, the simulated hierarchical-microstructure at the RG volume fraction of 30% is

![EBSD image quality maps of processed cooper at different HT temperatures](image_url)
3.2. Multiple microstructures

In order to calculate the mechanical response of partial recrystallized copper, the hierarchical-microstructure simulated by CA is combined with finite element method. Using the commercial ABAQUS/Explicit software (ABAQUS 6.11-1), a 2-D finite element model (FEM) with 200 × 200 random elements was set up for uniaxial tension. Based on the experimental measurements (Kumar et al., 2019), the following boundary conditions are used in the proposed FEM. The displacement along x direction at the top boundary is the same as that at the bottom boundary. Nodes can be free to move along y axis at both the top and bottom boundaries. Displacement along x direction is fixed in the left side. Displacement in the right side increases linearly at a specific loading speed. Every element in FEM corresponds to one of 200 × 200

![Fig. 5. TEM micrographs of partial recrystallized copper: (a) Recrystallized grains (RGs) are surrounded by elongated subgrains; (b) local magnification of subgrain; (c) local magnification of recrystallized grain; (d)-(e) are the selected area diffraction patterns (SADPs) of (b)-(c) respectively.](image-url)
equi-sized square cells in CA, i.e., an element belongs to either SPDed grain or RG. The element size was 1 × 1 μm after mesh sensitivity tests. Two user material subroutines were needed for partial recrystallized copper. One is for SPDed grain and the other is for RG.

Volume fraction of recrystallization can be calculated by using the following equation (Avrami, 1939):

\[ X_B = 1 - \exp(-Bt^n) \]

where \( X \) is volume fraction, \( t \) is HT time, \( B \) and \( n \) are recrystallization kinetic parameters. \( B \) is related to the HT temperature. The relationship between \( B \) and HT temperature has been put forward in the literature (Liu et al., 2019). Using the results from Liu et al. (2019), the RG volume fraction at different HT temperatures can be given. The RG volume fraction at the HT temperature of 293 K, 473 K, 523 K, 548 K, 573 K and 773 K is 0%, 11%, 37%, 57%, 76% and 100% respectively. Combining RG volume fraction and CA method, the multi-microstructures at different HT temperatures were illustrated in Fig. 10.

Fig. 10a is full of SPDed grains. Fig. 10f is filled with RGs. The RG volume fraction increases from Fig. 10b to e.

### 3.3. Constitutive model

The mechanical behaviors of SPDed grains and RGs are assumed to be isotropic. The deformation of partial recrystallized copper is elastoplastic. The deformation gradient tensor \( \mathbf{F} \) can be split into two parts, i.e., an elastic part \( \mathbf{F}_e \) and a plastic part \( \mathbf{F}_p \) (Lee, 1969):

\[ \mathbf{F} = \mathbf{F}_e \mathbf{F}_p \]

For elastic deformation in the current configuration, the relationship between Cauchy stress tensor \( \mathbf{\sigma} \) and the logarithmic elastic strain is linear:

\[ \mathbf{\sigma} = \frac{1}{2} \mathbf{J}^{-1} \mathbf{C} : \ln \mathbf{b}_e \]

where \( \mathbf{b}_e \) is elastic Finger tensor and \( \mathbf{J} \) is the determinant of deformation gradient tensor \( \mathbf{F} \). The elastic stiffness \( \mathbf{C} \) is in the following form:

\[ \mathbf{C} = K \mathbf{I} \otimes \mathbf{I} + 2G \left( \mathbf{I}' - \frac{1}{3} \mathbf{I} \otimes \mathbf{I} \right) \]

where \( K, G, \mathbf{I} \), and \( \mathbf{I}' \) are respectively bulk modulus, shear modulus, second order unit tensor, and fourth order symmetric unit tensor. \( K \) and \( G \) are related with elastic modulus and Poisson’s ratio.

According to the experimental results in Fig. 2a, the elastic responses of SPDed grains and RGs are assumed to be identical. The two microstructures differ only through the yield strength and hardening property. For metal, the plastic deformation is determined by the stress deviator:

\[ \mathbf{s} = \mathbf{\sigma} - \frac{1}{3} \mathbf{tr}(\mathbf{\sigma}) \mathbf{I} \]
\[ J^2 = \sqrt{3} T^2 - \left[ a^S_0 + \sigma^S_1 V + H^S_2 (\epsilon^S_2)^n^S + H^S_3 (\epsilon^S_3)^n^S V \right] \leq 0 \]
\[ J^2 = \sqrt{3} T^2 - \left[ a^R_0 + \sigma^R_1 (1 - V) + H^R_2 (\epsilon^R_2)^n^R + H^R_3 (\epsilon^R_3)^n^R (1 - V) \right] \leq 0 \]  
(11)

with SPDed and RG labeled by superscript S and R respectively. In Eq. (11), \( V (0 \leq V \leq 1) \) is the volume fraction of recrystallization. The elastic domains of SPDed grains and RGs are bounded by Eq. (11). The equivalent stresses in SPDed and recrystallized material are defined by:

\[ \sigma^S = \sqrt{3} T^2 = \sqrt{3} \sigma^S_1 \]
\[ \sigma^R = \sqrt{3} T^2 = \sqrt{3} \sigma^R_1 \]  
(12)

\( \epsilon^S_2 \) and \( \epsilon^S_3 \) are equivalent plastic strain in SPDed and recrystallized material respectively, which can be calculated by the following form:

\[ \{ \epsilon^S_2 = \frac{1}{3} \epsilon^S, \epsilon^S_3 \} \]  
\[ \{ \epsilon^R_2 = \frac{1}{3} \epsilon^R, \epsilon^R_3 \} \]  
(13)

where \( \epsilon^S \) and \( \epsilon^R \) are the strain deviator of SPDed and recrystallized material respectively. They are defined by:

\[ \{ \epsilon^S = \epsilon - \frac{1}{3} \text{tr}(\epsilon) I \}
\{ \epsilon^R = \epsilon - \frac{1}{3} \text{tr}(\epsilon) I \} \]  
(14)

where \( \epsilon^S \) and \( \epsilon^R \) are plastic strain tensor of SPDed and recrystallized material respectively. In Eq. (11), \( a^S_0 \) and \( a^S_1 \) are the yield strength of single-microstructural SPDed material and recrystallized material respectively. They can be determined from the experimental measurements in Fig. 2a. \( H^S_2 \) and \( H^S_3 \) are respectively the strain hardening modulus and exponent of recrystallized material. Due to the mechanical incompatibility (Ovid'ko et al., 2018), the recrystallized material and SPDed material affect the strength and ductility of each other. The term \( (a^S_0 + H^S_2 (\epsilon^S_2)^n^S) V \) and \( (a^R_0 + H^R_2 (\epsilon^R_2)^n^R (1 - V) \) in Eq. (11) are the effect of hierarchical-microstructure on the mechanical property of SPDed and recrystallized material respectively. For \( V = 0 \), there is no RG in SPDed copper. Eq. (11) becomes the following form:

\[ \sqrt{3} T^2 - \left[ a^R_0 + H^R_2 (\epsilon^R_2)^n^R \right] \leq 0 \]  
(15)

which is the same as the conventional plastic model of SPDed material. As for \( V = 1 \), completed recrystallization happens in SPDed copper.

Fig. 7. The pole figures of processed copper at different HT temperatures: (a) HT at 293 K; (b) HT at 523 K; (c) HT at 573 K; (d) HT at 773 K.

Fig. 8. 2-D Moore configuration with both nearest (C2, C4, C6 and C8) and next-nearest (C1, C3, C7 and C9) neighbors.
There is no SPDed grain in copper; therefore, Eq. (11) degrades into the conventional plastic model of recrystallized material:

$$J_{R}^3 (\rho) \leq 0$$

(16)

Because of the mechanical incompatibility (Ovid'ko et al., 2018), the strength and strain hardening rate of RG core are related with the surrounding subgrains, which are suggested by:

$$\sigma_{n}^R = \frac{2Eb}{l}$$

(17)

$$H_{R}^R = -\alpha Gb \left( \frac{1}{bd} + k_{1} \sqrt{\rho_{S}^R} - k_{2} \rho_{S}^R \right)$$

(18)

In Eq. (17), $E$ is elastic modulus, $b$ is magnitude of Burger's vector and $l$ is the average spacing of cell-block boundary (CBB) (Hamelin et al., 2011). Eq. (18) is derived by combining Kocks-Mecking-Estrin (KME) (Estrin and Mecking, 1984; Mecking and Kocks, 1981) model and Taylor equation (Taylor, 1934, 1938). In Eq. (18), $\alpha$ is a dislocation interaction term, $\rho_{S}^R$ is the dislocation in...
Fig. 11. Equivalent stress distributions at different volume fractions of recrystallization: (a) 0%; (b) 11%; (c) 37%; (d) 57%; (e) 76%; (f) 100%.

Fig. 12. Equivalent plastic strain distributions at different volume fractions of recrystallization: (a) 0%; (b) 11%; (c) 37%; (d) 57%; (e) 76%; (f) 100%.

Fig. 13. Plastic strain and stress distributions of 76% recrystallized copper at the engineering strain of 30%: (a) Plastic strain distribution in SPDed material; (b) Plastic strain distribution in recrystallized material; (c) Stress distribution in SPDed material; (d) Stress distribution in recrystallized material.
SPDed grains, $d$ is the average grain size of RG, $k_1$ is dislocation storage rate, and $k_2$ is dynamic recovery rate. Eq. (18) is negative, which means that the mechanical incompatibility between SPDed grain and RG results in the decreasing strain hardening rate in RG.

The effect of RG core on the surrounding subgrains is given by the same forms:

$$\sigma_i^S = \frac{2Eb}{d}$$  \hspace{1cm} (19)

$$H_1^S = \alpha Gb \frac{1}{2\sqrt{\rho_i^R}} \left( \frac{1}{bd} + k_1 \sqrt{\rho_i^R} - k_2 \rho_i^R \right)$$  \hspace{1cm} (20)

The negative Eq. (19) indicates that the mechanical incompatibility between SPDed grains and RG leads to the decrease of strength in SPDed grains. $\rho_i^R$ in Eq. (20) is the dislocation density in RG.

Because partial recrystallized copper is composed of SPDed grains and RGs, it is required to introduce an interface into the proposed FEM. On the interface $A$ between SPDed material and recrystallized material, it is necessary to have:

$$u^S_A = u^R_A$$  \hspace{1cm} (21)

where $u^S$ and $u^R$ are respectively the displacement vector of SPDed and recrystallized material.

4. Numerical result and discussion

Based on the proposed theory, a user material subroutine was established into ABAQUS to predict the mechanical behavior of partial recrystallized copper. FEMs for different volume fraction of recrystallization have been set up in Fig. 10; however, the material parameters in Eq. (11) are needed to simulate the tensile deformation. The experimental data of SPDed copper and annealed copper
4.1. Validation of numerical results

For partial recrystallized copper, the mechanical responses of SPDed material and recrystallized material are separated to analyze their effect to the strength and ductility of partial recrystallized copper. Fig. 13 shows the plastic strain and stress distributions of 76% recrystallized copper at the engineering strain of 30%. According to Fig. 13a-b, the plastic strain level in SPDed material is far less than that in recrystallized material. As illustrated in Fig. 13c-d, the average stress in SPDed material is larger than that in recrystallized material. The combination of SPDed material and recrystallized material can simultaneously improve the strength and ductility. This explains the improvement of strength without losing the ductility in partial crystallized copper (Fig. 2a). Meanwhile, it is found from Fig. 13a-b that most of the plastic strain in partial recrystallized copper can be shared by recrystallized material. The average level of plastic strain in SPDed material is ~5 × 10⁻⁵ although the strain level in 76% recrystallized copper is 30%. The plastic strain level in SPDed material is still within the plastic strain range over which the data (Fig. A) were fitted to determine material parameter; thus, it is reasonable to use the parameters of SPDed material within a low plastic strain range for simulating the large deformation behavior of partial recrystallized copper.

The stress-strain responses were further calculated to validate the rationality of the present model. Fig. 14a shows the simulated stress-strain responses of partial recrystallized copper during tensile deformation. SPDed copper annealed at low temperature possesses high strength but poor uniform elongation. Both the yield strength and ultimate tensile strength decrease with the increasing HT temperature. The uniform elongation is larger for higher HT temperature. The predicted stress-strain curves (Fig. 14a) are in good accordance with the experimental results in Fig. 2a. The comparisons of ultimate tensile strength and uniform elongation between FEM and experiments are also presented in Fig. 14b. According to Fig. 14b, the experimental measurements validate the rationality of FEM simulation. Therefore, the proposed model is capable of predicting the mechanical behavior of partial recrystallized copper.

4.2. Comparison of two plasticity models

To understand the proposed model comprehensively, the differences between hierarchical-microstructure based model and conventional model are discussed in this section. Eq. (11) describe the flow stress in the frame of hierarchical-microstructure based plasticity model. The

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**Fig. 15.** (a) Comparison of flow stress between hierarchical-microstructure based model and conventional model; (b) Comparisons of strain hardening rate between hierarchical-microstructure based model and conventional model.

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(Mishra et al., 2007; Wang et al., 2002) were employed to determine the corresponding material parameters. The detailed derivations of the parameter values in plasticity model are demonstrated in Appendix A. The elastic deformations for all microstructures in partial recrystallized copper are assumed to be the same. The values are achieved from experimental measurements (Li et al., 2008a; Tian et al., 2018). The material parameters for hierarchical-microstructures are obtained from the measurements in Figs. 3-7. The detailed material parameters for the simulations are provided in Table 1.
terms \((a_S^p + H_V(a_S^p)^{n_S})V\) and \((a_R^p + H_V(a_R^p)^{n_R})(1 - V)\) in Eq. (11) characterize the effect of hierarchical-microstructure on the mechanical property of SPDed and recrystallized material respectively. As for conventional plasticity model, the effects of hierarchical-microstructure on the mechanical properties are not taken into consideration. Therefore, the conventional plasticity model is given by:

\[
\begin{align*}
\sqrt{3J_2^S} - \left[ a_S^0 + H_V(a_S^0)^{n_S} \right] & \leq 0 \\
\sqrt{3J_2^R} - \left[ a_R^0 + H_V(a_R^0)^{n_R} \right] & \leq 0
\end{align*}
\]

(22)

where SPDed grain and RG are respectively labeled by superscript S and R.

As shown in Eqs. (11) and (22), the material parameters in conventional plasticity model are one part of the material parameters in hierarchical-microstructure based model. Substituting the material parameters in Table 1 into Eqs. (11) and (22), the flow stresses and strain hardening rates for different plasticity models are obtained. Fig. 15 shows the comparisons between hierarchical-microstructure based model and conventional model in the case of 30% recrystallization volume fraction. For both SPDed grain and recrystallized grain, flow stress predicted by hierarchical-microstructure based model is larger than that predicted by conventional model. For recrystallized grain, the deviation of flow stress prediction between hierarchical-microstructure based model and conventional model decreases with the increasing equivalent plastic strain; however, the deviation for SPDed grain increases with the increasing equivalent plastic strain. As shown in Fig. 15b, the strain hardening rate in hierarchical-microstructure based model is larger than that in conventional model. Comparing Eqs. (11) and (22), the strengthening mechanism from hierarchical microstructure is not taken into consideration in conventional model. This explains the low flow stress and low strain hardening rate predicted by conventional model. The neglect of the strengthening mechanism from hierarchical microstructure indicates that the stress-strain response predicted by conventional model could deviate from the experimental results of Fig. 2.

Figs. 16 and 17 show respectively the equivalent stress distribution and the equivalent plastic strain distribution for the recrystallization volume fraction of 76%. Compared Fig. 16a-b with d-e, the stress distribution of hierarchical-microstructure based model is similar to that of conventional model for early and middle stage of tensile deformation. If engineering strain is less than 16%, plastic strain distributions simulated by hierarchical-microstructure based model are similar to
those simulated by conventional model (Fig. 17a-b and d-e). When engineering strain exceeds a certain value, i.e. 32% in this FEM, stress concentration and deformation localization happen in conventional plastic model (Figs. 16f and 17f). Deformation localization indicates that the material lose the load carrying and energy absorption capacity (Dodd and Bai, 2012; Meyers and Chawla, 2009). However, at the identical engineering strain, there is no stress concentration or deformation localization in hierarchical-microstructure based plastic model (Figs. 16c and 17c).

The comparison of the two models with experiments is further illustrated in Fig. 18. For only one type of grain, i.e., full severe plastic deformed (SPDed) grain (Fig. 18a) or full recrystallized grain (Fig. 18f), both the present model and the conventional model can capture well the experimental results. According to the proposed theory, the present model can degrade into the conventional model when recrystallization fraction is 0% or 100%. This explains the results in Fig. 18a and f. When volume fraction of recrystallization is less than 50%, the difference between conventional model and experiments becomes larger with the increasing volume fraction of recrystallization. However, when volume fraction of recrystallization is larger than 50%, the difference between conventional model and experiments becomes smaller with the increasing HT temperature.

(1) Uniaxial tensile tests reveal that uniform elongation of partial recrystallized copper increases but its ultimate tensile strength decreases with the increasing HT temperature.

(2) Microstructural observations show that a hierarchical-microstructure of recrystallized grains (RGs) surrounded by elongated subgrains affects the mechanical properties of partial recrystallized copper and the volume fraction of RG increases with the increasing HT temperature.

(3) A hierarchical-microstructure based plasticity model is proposed to predict the mechanical behavior of partial recrystallized copper, where elongated subgrain, RG, forest dislocation, low angle grain boundary (LAGB) and twin boundary (TB) are taken into consideration.

(4) Compared with the conventional plasticity model, the stress-strain response predicted by hierarchical-microstructure based plasticity model is in better agreement with the corresponding experimental data. Therefore, the present model is valid to predict mechanical properties of partial recrystallized copper.

5. Conclusions

A SPD method of quasi-static extrusion machining (QSEM) was used to produce SPDed copper. Then, a systematic heat treatment (HT) at six annealing temperatures was conducted on SPDed copper to obtain partial recrystallized copper with different volume fraction of recrystallization. The microstructure and mechanical property of partial recrystallized copper were investigated in this study. The following conclusions can be drawn.

(1) Uniaxial tensile tests reveal that uniform elongation of partial recrystallized copper increases but its ultimate tensile strength decreases with the increasing HT temperature.

(2) Microstructural observations show that a hierarchical-microstructure of recrystallized grains (RGs) surrounded by elongated subgrains affects the mechanical properties of partial recrystallized copper and the volume fraction of RG increases with the increasing HT temperature.

(3) A hierarchical-microstructure based plasticity model is proposed to predict the mechanical behavior of partial recrystallized copper, where elongated subgrain, RG, forest dislocation, low angle grain boundary (LAGB) and twin boundary (TB) are taken into consideration.

(4) Compared with the conventional plasticity model, the stress-strain response predicted by hierarchical-microstructure based plasticity model is in better agreement with the corresponding experimental data. Therefore, the present model is valid to predict mechanical properties of partial recrystallized copper.

Declaration of Competing Interest

None.
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Supplementary materials


Appendix A

The available experimental data (Mishra et al., 2007; Wang et al., 2002) were used here to obtain the parameter values in plasticity models (11) and (22). The uniaxial tensile data for SPDed copper and RGed copper in references (Mishra et al., 2007; Wang et al., 2002) are illustrated in Fig. A(a). The mean yield strengths for SPDed and RGed copper are directly calculated in Fig. A(a), i.e., $\tau_0 \approx 450$ MPa and $\tau_0 \approx 50$ MPa. The recrystallization volume fraction in SPDed cooper is zero. The recrystallization volume fraction in RGed cooper is 100%. There are no hierarchical microstructures in SPDed and RGed copper. Therefore, the plastic deformation can be controlled by plasticity model ((22), i.e.,

Fig. A. Experimental data from Mishra et al. (2007) and Wang et al. (2002): (a) True stress vs true strain curves for SPDed and RGed copper; (b) Relationships between $\log_{10}(\sigma - \sigma_0)$ and $\log_{10}(\varepsilon_p)$ for SPDed and RGed copper.
In Eqs. (A1), SPDed and RGed copper are respectively labeled by superscript S and R. as is equivalent stress during uniaxial tensile tests. Eqs. (A1) could be changed into the following forms:

\[
\log(\sigma - \sigma_0) = \log(H_S^{(S)} - \sigma_0) + n_S^{(S)} \log(p_S)
\]

\[
\log(\sigma - \sigma_0) = \log(H_R^{(R)} - \sigma_0) + n_R^{(R)} \log(p_R)
\]

(A1)

Based on true stress vs true strain curves in Fig. (a), the relationships between \(\log(H_S^{(S)} - \sigma_0)\) and \(\log(p_S)\) are shown in Fig. (b). By means of linear fitting of experimental data in Fig. (b), the slope and intercept of linear fitting equation are calculated. Then, strain hardening modulus \(H_0\) and hardening exponent \(n_0\) can be obtained. The calculated average values of plastic parameters are: \(H_0^{(S)}\approx 50\, \text{MPa}, \ n_0^{(S)}\approx 0.6, \ H_0^{(R)}\approx 360\, \text{MPa}, \) and \(n_0^{(R)}\approx 0.6.\)

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