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EIGENFREQUENCY LOCI CROSSINGS, VEERINGS AND MODE SPLITTINGS OF OVERHANGED CANTILEVERS

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Eigenfrequency loci veering, which indicates strong mode coupling and sometimes mode localization, is a much sought-after property in the applications of mass sensing and opto/electromechanics. A weak physical coupling is the mechanism responsible for the eigenfrequency loci veering and overhang is a widely used structure to realize such mechanism. A continuum model, which is more accurate and straightforward as compared with the discrete models, is presented for the structure of two overhanged cantilevers.The eigenvalue problem formulation based on this continuum model leads to a direct computation of the eigenfrequencies, which does not involve any numerical discretization procedure. A comprehensive study on the eigenfrequency loci veerings and mode splittings of the overhanged structure is presented. The influences of various parameters on the eigenfrequency loci crossing and veerings are also systematically studied. An efficient optimum design tool for the eigenfrequency loci veering of an overhanged structure is provided by the continuum model together with a direct computation method.

Keywords: Eigenfrequency, crossing, veering, mode splitting

1. Introduction

Due to its catastrophic nature, a tiny introduction of disorder, such as adsorption of an analyte, can cause rather significant variations of the eigenfrequencies and mode shapes of a coupled structure, which is the sensing mechanism of many mass resonators [1, 2, 3]. In an overhanged mass resonator based on the mode shape sensing mechanism, the effect of mode localization needs to be enhanced to increase its sensitivity [1]. The enhancement mechanism is to reduce the coupling stiffness [1, 2]. For the resonator with two overhanged cantilevers, the physical coupling stiffness k_c and the cantilever effective stiffness k are with the following relation [2]

$$\frac{k_c}{k} = \frac{\omega_2^2 - \omega_1^2}{2\omega_1^2}. \tag{1}$$

Where ω_2 and ω_1 are the two (measured) adjacent eigenfrequencies and $\omega_2 > \omega_1$. The difference between two eigenfrequencies ($\omega_2 - \omega_1$) is termed the frequency splitting [4]. Clearly, according to the above equation, the smallest frequency splitting results in the smallest kc for a given k. This smallest frequency splitting is also called veering neck or veering width. Veering signifies the strongest mode coupling [4]. An essential information conveyed by Eq. (1) is that a weak physical coupling (k_c) results in an eigenfrequency loci veering, which corresponds to the strongest mode coupling.

As shown in Fig. 1, an overhang, which is variously called a shared mechanical ledge or shuttle mass, connects two cantilevers. Overhang provides a simple and direct coupling mechanism for (sub)structures, which has been used in various applications [1-3]. The discrete models with the lumped parameters have been used to study the overhanged structure [1, 2]. In this study, we provide a continuum mechanics model to study the two overhanged cantilevers. Our continuum model presents a systematic and comprehensive study on the impacts of various parameters on the eigenfrequencies. For example, the geometric parameters, such as the length, width and thickness (which implicitly includes the separation distance) and material property (Young’s modulus) of an overhang together with the various differences such as mass, stiffness and length between two cantilevers. The previous discrete models [1, 2] in essence only offer some qualitative explanations on the experimental findings rather than providing an optimization tool. With this continuum model, an efficient and more accurate optimization tool is presented.

2. Model development

In the schematic diagram of an overhanged cantilever structure in Fig. 1, one end of the overhang sis connected with two beams and the other is clamped. Here b, h and Lo are the width, thickness and length of the overhang, respectively. The corresponding parameters for beam 1 and 2 are b_1, h_1 L_1 and b_2, h_2 L_2. For the succinctness reason, the governing equation is written as follows

$$\begin{cases} m\dfrac{\partial^2 w}{\partial t^2} + EI\dfrac{\partial^4 w}{\partial x^4} = 0, \quad 0 \le x \le L_o \\ m_1\dfrac{\partial^2 w_1}{\partial t^2} + E_1 I_1\dfrac{\partial^4 w_1}{\partial x^4} = 0, \quad L_o \le x \le L_1 \\ m_2\dfrac{\partial^2 w_2}{\partial t^2} + E_2 I_2\dfrac{\partial^4 w_2}{\partial x^4} = 0, \quad L_1 \le x \le L_2 \end{cases}. \tag{2}$$

Here m, m_1 and m_2 are the mass per unit length of the overhang, beam 1 and beam 2, respectively.

And E, E_1, E_2 and I, I_1, I_2 are the Young's moduli and the second moments of area of the overhang and two beams, respectively. The following dimensionless quantities are introduced :

$$\tau=\sqrt{\frac{E_1 I_1}{m_1 L_1^4}},\ \xi=\frac{x}{L},\ \xi_o=\frac{L_o}{L_1},\ \xi_2=\frac{L_2}{L_1},\ W=\frac{w}{L_1},\ W_i=\frac{w_i}{L_1}. \quad (3)$$

Now Eq. (2) is nondimensionalized as follows:

$$\begin{cases}\alpha\dfrac{\partial^2 W}{\partial\tau^2}+\gamma\dfrac{\partial^4 W}{\partial\xi^4}=0, \quad 0\le x\le\xi_o\\ \dfrac{\partial^2 W_1}{\partial\tau^2}+\dfrac{\partial^4 W_1}{\partial\xi^4}=0, \quad \xi_o\le x\le 1\\ (1+\Delta_1)\dfrac{\partial^2 W_2}{\partial\tau^2}+(1+\Delta_2)\dfrac{\partial^4 W_2}{\partial\xi^4}=0, \quad \xi_o\le x\le\xi_2\end{cases} \quad (4)$$

The dimensionless quantities in Eq. (4) are defined as follows

$$\alpha=\frac{m}{m_1},\ \gamma=\frac{EI}{E_1 I_1},\ \Delta_1=\frac{m_2}{m_1}-1,\ \Delta_2=\frac{E_2 I_2}{E_1 I_1}-1. \quad (5)$$

Physically, α and γ are the dimensionless mass per unit length and bending stiffness of the overhang; Δ_1 and Δ_2 are the beam 2 dimensionless deviations of the mass per unit length and bending stiffness from those of beam 1, respectively. There are six boundary conditions equations are for the cantilevered overhang and another six equations are to ensue the continuity of displacement, slope, moment and shear force at x = Lo. Therefore, there are total 12 equations for us to carry out the eigenfrequency computation for Eq. (4). And again, for the succinctness reason, we just give some results without presenting the detailed computation procedures.

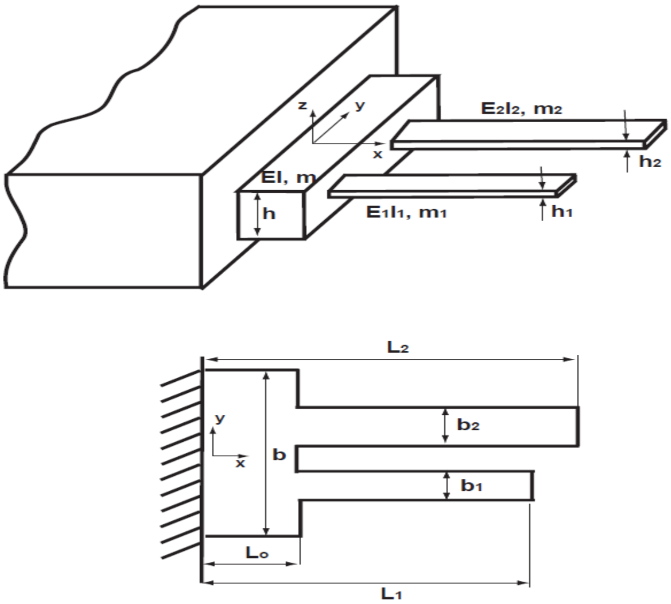

Figure 1: The schematic diagrams of an overhanged cantilever structure with two beams and their dimensions.

3. Results and discussion

Because it is customary to present the square root of the (dimensionless) eigenfrequency, the variations of the first ten β_n s ($\beta_n = \sqrt{\omega_n}$, n = 1 to 10) as the functions of ξ_o are presented in Fig. 2. The eigenfrequency square roots of a uniform beam (f_i s) are also presented for comparison. For the convenience of statement, we call β_n and f_i eigenfrequency hereafter.

In Fig. 2, there are only five eigenfrequencies (f_i s, i = 1 to 5) whose value is less than 16 and the values of these first five eigenfrequencies (f_i s) [41] are presented in Table 1. In comparison, in the same range there are ten eigenfrequencies in the overhanged structure. Because the overhang couples the two beams, there are newly emerging eigenfrequencies o β_{2i} is with even subscript numbers and marked as dashed lines in Fig. 2. At ξ o = 0, the two beams are uncoupled. Therefore, they are independent and show the characteristics of a uniform beam, i.e., $\beta_{2i} = \beta_{2i-1} = f_i$ at ξ_o = 0. At ξ_o = 0, the eigenfrequencies of β_{2i-1} and β_{2i} arise as a pair. As ξ_o increases, β_{2i-1} and β_{2i} begin to separate: β_{2i-1} stays unchanged as f_i; while, β_{2i} increases rapidly to approach f_{i+1}. As the result, the mode associated with β_{2i} also experiences the same emerging and then separating process. The same scenario also occurs in a circular graphene membrane [5] and overhanged beams tuned by an electrostatic force. This new β_{2i}-mode generating and separating phenomenon is called mode splitting [5]. The emergence of new modes is due to the mixing of modes [5]. As shown later, the mode shape associated with β_{2i} is the mixing of the modes associated with f_i and f_{i+1}. A mode splitting is equivalent to an transparency [5]. In opto/electromechanics, an eletromagnectically induced transparency means that the system is switched from reflecting to transmitting the optical/electromagnetic waves [6].

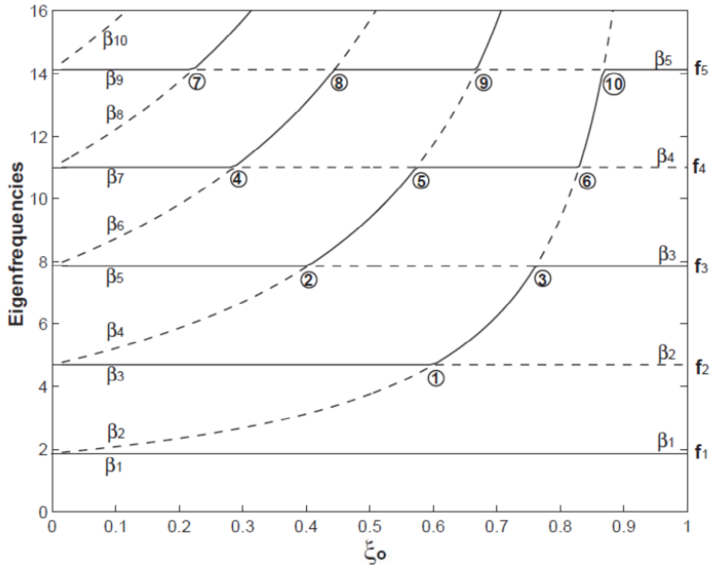

Figure 2: The first ten eigenfrequencies (β_n , n = 1 to 10) as the functions of the overhang length (ξ_o) for two identical beams,

4. Conclusion

A continuum model on the overhanged two-cantilever structure, which can be easily extended to an array with multiple cantilevers, is proposed. Unlike a discrete model which only describes two-mode interaction, this continuum model includes the interactions of all modes. The eigenvalue formulation leads to a direct computation on the system eigenfrequencies without any discretization procedure. With this continuum model and a direct computation method, the eigenfrequency loci veering are presented by simply computing the eigenfrequencies as the functions of the overhang length/thickness and differences of the beam length, stiffness and mass. The overhang couples the two cantilevers, which leads to the mode splitting and then veering. The mode splitting, which generates a new mode, results from the mixing of two adjacent modes of a uniform beam. With the increase of the overhang length, the rapid eigenfrequency increase of this new mode results in its separation from that of a lower mode, which then leads to its veering with that of a higher mode. The overhang length is the most important parameter determining the veering loci; the length difference of the two beams can produce more veering loci. By presenting the eigenfrequency spectra, a more comprehensive study is provided, which can be useful in the optimum design of the overhanged structures. In the structure of two overhanged cantilevers, the veering indicates the strong mode coupling, while, the mode localization can be small or even none. The veering locus of two adjacent eigenfrequencies also corresponds to the locus of two other adjacent eigenfrequencies with the largest separation. Physically, this means that the weak and strong physical couplings can be easily alternated by exciting two different modes, which may hold some potential applications for mass sensing or opto/electromechanics.

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