An alternative numerical method for calculating C^* parameter under mode I loading utilizing rate-dependent energy release rate

Xianhe Du^{b,c}, Yanwei Dai^d, Qiang Han^b, Yinghua Liu^c, Peng Cao^{a,*}, Guian Qian^{e,f,*}, Filippo Berto^g

^a College of Architecture and Civil Engineering, Beijing University of Technology, Beijing 100124, China

^b School of Civil Engineering & Transportation, South China University of Technology, Guangzhou 510641, China

^c Department of Engineering Mechanics, AML, Tsinghua University, Beijing 100084, China

^d Institute of Electronics Packaging Technology and Reliability, Beijing University of Technology, Beijing 100124, China

^e State Key Laboratory of Nonlinear Mechanics (LNM), Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China

^f School of Engineering Science, University of Chinese Academy of Sciences, Beijing 100049, China

^g Department of Mechanical and Industrial Engineering, Norwegian University of Science and Technology (NTNU), Richard Birkelands vei 2b, 7491 Trondheim, Norway

ARTICLE INFO

Keywords:

Power-law creep
Energy release rate
Virtual crack closure technique (VCCT)
 C^* -integral
Equivalent domain integral

ABSTRACT

In a fracture analysis at elevated temperature, the determination of fracture parameter C^* under steady-state creep is a crucial point. This paper proposes a rate-dependent energy release rate $\dot{G}$, by analogy with the definition of creep fracture parameter C^* under steady-state creep. The relationship between $\dot{G}$ and C^* is deduced after a brief review of the definitions of parameters in fracture mechanics. Aimed at calculating $\dot{G}$ and C^* , a finite element formulation based on the virtual crack closure technique (VCCT) is established, which is called the extended VCCT in this paper. The numerical algorithm for this proposed formulation is presented and implemented into commercial finite element codes. The results obtained by the proposed numerical method are compared with those by finite element contour integral method and from different engineering estimation methods for SENT, MT and CT specimens. Good agreements of the C^* results show that the extended VCCT is a reliable and efficient numerical method to calculate C^* under steady-state creep.

1. Introduction

Fracture is a significant problem in engineering applications. Meanwhile, how to determine the reasonable fracture parameter for characterizing the crack tip stress fields accurately is one of the most important aspects of fracture mechanics. There are three common fracture parameters in fracture mechanics at room temperature which are energy release rate G , stress intensity factor (SIF) K and J -integral.

Conventionally, based on the energy balance principle, energy release rate G is originally used to determine the fracture resistance in elastic materials and is typically referred to the foundational work of Griffith [1,2], then extended to plastic materials by Irwin [3,4] and Orowan [5]. The theoretical foundation of stress intensity factor K is established by Irwin [6] which is acceptable for brittle materials or ductile materials only within small scale yielding (SSY) region near a crack tip [7]. Later, energy release rate G or stress intensity factor K can be considered as an effective fracture parameter in the linear elastic fracture mechanics (LEFM).

For elasto-plastic materials, the path-independent J -integral

presented by Rice [8], can be used as a characterization parameter in fracture mechanics, and it has been widely accepted in the elasto-plastic fracture mechanics (EPFM), and also extended in many fields [9–12]. However, the J -integral is only valid for material that can be treated as a hyperelastic material. Namely, J -integral is based on the assumption that elasto-plastic material is under monotonic loading and the crack doesn't propagate in the process [13]. Moreover, the numerical method called the equivalent domain integral (EDI) method has been used to calculate the value of K and J -integral [14,15], and the singular elements must be assigned around the crack tip.

To facilitate the use of the Griffith theory, Kfoury and Rice [16] have calculated the energy release rate at very small crack extensions in elasto-plastic materials. Sun and Wang [17] extended Griffith fracture energy balance concept to elasto-plastic materials, and a finite element procedure is employed to calculate the variation of energies during crack extension in a plane stress problem. Based on the energy balance principle for crack growth, Fan [18] et al. proposed the energy derivative technique (EDT) to calculate G for elasto-plastic cases. However, as an intrinsic problem for the total energy approach, the EDT method

* Corresponding authors at: College of Architecture and Civil Engineering, Beijing University of Technology, Beijing 100124, China (Peng Cao).

E-mail addresses: caopeng518888@126.com (P. Cao), qianguan@imech.ac.cn (G. Qian).

cannot separate the fracture modes involved in the crack propagation. Recently, the fracture of soft solids using the notion of energy release rate has been discussed by Williams [19]. Xiao [13] et al. defined a surface-forming energy release rate G_s to propose the fracture criterion for crack propagation in elasto-plastic materials, and the EDI method is employed to calculate G_s . However, the surface-forming energy release rate can just be obtained at specified crack length.

Based on Irwin's crack closure integral [20], the numerical method called the virtual crack closure technique (VCCT) is originally proposed by Rybicki and Kanninen [21] to calculate G value. Then the numerical forms of VCCT with high-order and singular finite elements are presented by Raju [22]. Though this approach has mainly been utilized to obtain the energy release rate in elastic analyses, recently, Yang [23] et al. and Xie [24,25] et al. presented the numerical method to determine energy release rate for adhesively bonded joints using VCCT, respectively. The VCCT, EDT and J -integral have been used and discussed in elasto-plastic fracture mechanics analyses for middle-crack tension specimen by Xiao et al. [26]. Numerical results show that the fracture energy is always overestimated by using EDT, and the VCCT could be used for cracked specimen within plastic material. Das and Pradhan [27] presented a numerical simulation of three-dimensional structure with epoxy adhesive joints which are elasto-plastic material, and two crack modes of energy release rate are obtained via VCCT. Jokinen [28] et al. has studied the applicability of VCCT for crack growth analyses of adhesively bonded joints when the material is ductile, and numerical results agree to the double cantilever beam (DCB) experimental data very well. The overview of the development of VCCT and a discussion of corresponding applications in engineering has been presented by Krueger [29].

Several components such as power-plants, chemical reactors and gas turbines operate at an elevated temperature where creep deformation is inevitable. A different parameter, C^* , is proposed by Landes and Begley [30] analogously to the J -integral for rate-independent material behavior [31] and may also be interpreted as an energy release rate analogously to the energy definition of J [32]. As an energy rate line integral in time-dependent fracture mechanics (TDFM), C^* can characterize the crack tip stress field and strain rate field for materials following a nonlinear steady-state creep law. To characterize the crack tip stress fields under small scale creep (SSC), $C(t)$ -integral is proposed by Bassani and McClintock [33]. Generally, EDI is always used to calculate the value of $C(t)$, then C^* can be determined by using this numerical method in ABAQUS [34] since the value of $C(t)$ -integral is equal to C^* under steady-state creep condition. On the other hand, many engineering estimation methods were proposed to estimate C^* , which are aiming conveniently for engineering applications. The widely used estimation methods are the GE/EPRI solutions [35] and the reference stress method (RSM) which is originally proposed by Ainsworth [36].

Heretofore, the relationship between J and G has only been discussed within elastic and elasto-plastic materials. Generally, C^* -integral is defined analogously to the J -integral within the full plastic state. However, there are few investigations for establishing the relationship between G and C^* under the steady-state creep condition.

For providing a rational relationship between G and C^* in the time-dependent fracture mechanics, a rate form of energy release rate, $\dot{G}$, which is called rate-dependent energy release rate, reported in this paper, has been proposed by analogy measurement with the rate form of $\dot{J}$. Furthermore, the relationship between rate-dependent energy release rate $\dot{G}$ and time dependent fracture parameter C^* has also been proposed under the steady-state creep conditions. For numerical evaluation of G based on FE analysis, the EDI methods usually offer better accuracy than VCCT [37]. However, the VCCT is a promising approach for investigating various kinds of fracture cases including elastic, elasto-plastic and multi-materials due to lower computation cost and simpler implementation within FE analysis [38]. Thus, this study develops a numerical method called extended VCCT method with the user-defined element (UEL) provided by ABAQUS software to calculate $\dot{G}$, and three

different specimens including SENT, MT and CT with power-law creep materials are investigated herein. The paper is structured as follows. Section 2 briefly reviews relationships among fracture parameters used in room temperature (K , J and G) and in high temperature (C^* and J). The relationship between rate-dependent energy release rate $\dot{G}$ and C^* is also obtained, and a numerical method to determine C^* value through $\dot{G}$ based on extended VCCT for creep materials is also proposed in Section 2. Section 3 describes the extended VCCT method used to calculate $\dot{G}$ and the numerical procedure of UEL in the present work. Section 4 compares and discusses the results from the proposed method with existing FE results and engineering estimation results. Furthermore, the relationship between $\dot{G}$ and J has also been investigated and discussed in Section 4. The conclusions of the present work are summarized in Section 5.

2. Theoretical background

2.1. Power law creep constitutive equations

The elastic nonlinear viscous behavior of material under creep conditions under uniaxial state follows the relation

$$\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^c = \frac{\dot{\sigma}}{E} + A\sigma^n \quad (1)$$

where $\dot{\epsilon}$ denotes total strain rate, and $\dot{\epsilon}^e$ and $\dot{\epsilon}^c$ are elastic strain rate and creep strain rate respectively which are given by [39]

$$\begin{aligned} \dot{\epsilon}^e &= \frac{\dot{\sigma}}{E} \\ \dot{\epsilon}^c &= A\sigma^n \end{aligned} \quad (2)$$

where σ is applied stress under uniaxial state, and A and n denote creep coefficient and exponent of material constants, respectively. A dot over the quantities represent the temporal derivative and E denotes Young's modulus.

Based on the J_2 flow theory of plasticity, Eq.(1) can be generalized to multi-axial stress states as follows [40]

$$\dot{\epsilon}_{ij} = \frac{1+\nu}{E} \dot{S}_{ij} + \frac{1-2\nu}{3E} \dot{\sigma}_{kk} \delta_{ij} + \frac{3}{2} A \sigma_{eq}^{n-1} S_{ij} \quad 3$$

where ϵ_{ij} are the components of strain tensor, S_{ij} are the components of the deviatoric stress tensor, σ_{kk} is the trace of the stress tensor, σ_{eq} denotes the von Mises equivalent stress defined by $\sigma_{eq} = \sqrt{3(S_{ij}S_{ij})/2}$, ν is Poisson's ratio and δ_{ij} is the Kronecker delta.

2.2. J -integral and G

The J -integral proposed by Rice [8] is used to discuss the potential of path-independent integral for characterizing fracture essence of nonlinear-elastic materials. The J -integral can be expressed as

$$J = \int_{\Gamma} W dy - T_i \left(\frac{\partial u_i}{\partial x} \right) ds = \int_{\Gamma} W dy - \sigma_{ij} n_j \left(\frac{\partial u_i}{\partial x} \right) ds \quad (4)$$

$$W = \int_0^{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij} \quad (5)$$

$$\sigma_{ij} = \frac{\partial W}{\partial \epsilon_{ij}} \quad (6)$$

where W and u_i are strain-energy density and displacement respectively, σ_{ij} and ϵ_{ij} are components of stress tensor and strain tensor respectively, T_i is the traction vector defined by the outward normal, n_j , and here Γ is the contour which encloses the crack tip and starts and finishes on opposite faces of crack as shown in Fig. 1.

Another expression of J -integral for cracked bodies relies on the relationship between potential energy Π and J , which is given by [8]

$$J = - \frac{d\Pi}{B_1 da} \quad (7)$$

Fig. 1. Schematic of a stationary contour surrounding the crack tip.

where B_t is the thickness of a body and a is the crack length. Thus, J can be interpreted as the rate of potential energy change of the body with respect to crack length.

Actually, energy release rate, G , is an alternative for evaluating the fracture behavior of nonlinear material, it has been extended by Irwin [4] referred to the foundational work of Griffith [1], which can be defined as follows

$$G = -\frac{d\Pi}{B_t da} = -\lim_{\Delta a \rightarrow 0} \frac{\Delta\Pi}{B_t \Delta a} \quad (8)$$

$$\Pi = U - W_{\text{ext}} \quad (9)$$

$$U = U_e + W_p \quad (10)$$

where U is the total internal energy which includes reversible elastic energy U_e and irreversible energy W_p , W_{ext} is total work done by the external force. It can be found that J -integral has the same form as well as energy release rate (i.e. Eq. (7) and Eq. (8)), which has then provided an alternative way to calculate the energy release rate, or work/energy per unit fracture surface area [38]. For linear elasticity, J -integral, G and K can be given by

$$J = G = \frac{(1-\nu)^2 K^2}{E} \quad (11)$$

where $\nu = 0$ for plane stress state.

2.3. C^* and $C(t)$ -integral

Under the high temperature, stresses in the vicinity of the crack begin to relax due to creep deformation and the size of the creep zone increases with time if the crack is assumed to remain stationary. Neither K nor J can be an appropriate parameter to uniquely characterize the crack tip stress relaxation behavior within the creep zone because the creep deformation is not considered in their theories [41].

Recognizing the analogy between the power-law descriptions of plasticity and creep, Landes and Begley [30], and Nikbin, Webster and Turner [32] independently defined an integral analogously to Rice's J -integral which is called C^* -integral and defined in the following manner

$$C^* = \int_{\Gamma} W^* dy - T_i \left(\frac{\partial u_i}{\partial x} \right) ds \quad (12)$$

$$W^* = \int_0^{\dot{\epsilon}_{ij}} \sigma_{ij} d\dot{\epsilon}_{ij} \quad (13)$$

$$\sigma_{ij} = \frac{\partial W^*}{\partial \dot{\epsilon}_{ij}} \quad (14)$$

where W^* is the strain-energy rate density. Here Γ is the contour which encloses the crack tip similar to the J -integral method.

Using the analogy with the Hutchinson-Rice-Rosengren (HRR) crack

tip fields [42,43], the singular crack tip stress field and strain rate field can be characterized by C^* for power-law creep materials as follows [44,45]

$$\begin{aligned} \sigma_{ij} &= \left(\frac{C^*}{A I_n r} \right)^{\frac{1}{n+1}} \bar{\sigma}_{ij}(\theta, n) \\ \dot{\epsilon}_{ij}^c &= A \left(\frac{C^*}{A I_n r} \right)^{\frac{n}{n+1}} \bar{\epsilon}_{ij}(\theta, n) \end{aligned} \quad (15)$$

where r and θ are polar coordinates located as the crack tip. The dimensionless constant I_n and the θ -variation functions of $\bar{\sigma}_{ij}(\theta, n)$ and $\bar{\epsilon}_{ij}(\theta, n)$ depend on the creep exponent n . Note that it is complicated to accurately calculate C^* with the method presented by [30].

Another expression of the C^* -integral is proposed by Nikbin, Webster and Turner [32] through the energy release rate, which is given by

$$C^* = \dot{J} = -\frac{d\dot{\Pi}}{B_t da} \quad (16)$$

where $\dot{\Pi}$ is the potential energy rate.

Through the analogy between extensive creep and the fully plastic state, C^* can be estimated from the GE/EPRI solutions [35], which is generally defined as follow

$$C^* = A(W' - a) \left(\frac{a}{W'} \right) h_1 \left(\frac{a}{W'}, n \right) \left(\frac{P \sigma_0}{P_0} \right)^{n+1} \quad (17)$$

where h_1 is a dimensionless function of geometry parameters $\frac{a}{W'}$ and creep exponent n . σ_0 is reference stress, and P_0 is an appropriate reference load.

Based on the reference stress method, Ainsworth [36] proposed an approach to estimate C^* for steady-state creep conditions, which is given by

$$C^* = \frac{K^2(1-\nu^2)}{E} \frac{E \dot{\epsilon}_{\text{ref}}}{\sigma_{\text{ref}}} \quad (18)$$

$$\sigma_{\text{ref}} = \frac{P}{P_L} \sigma_{0.2} \quad (19)$$

where σ_{ref} is reference stress, $\dot{\epsilon}_{\text{ref}}$ is the creep strain rate at the reference stress. P_L is the plastic limit load of the cracked component obeying perfectly plastic law, and $\sigma_{0.2}$ denotes the 0.2% proof stress or the stress at 0.2% inelastic strain. For plane stress state, $\nu = 0$.

To characterize the intensity of the near-tip stress field and strain rate field in elastic nonlinear viscous materials under small scale creep conditions, $C(t)$ -integral has been presented by Bassani and McClintock [33] in the same sense as C^* -integral, which is evaluated by

$$C(t) = \int_{\Gamma \rightarrow 0} \left(\frac{n}{n+1} \sigma_{ij} \dot{\epsilon}_{ij} dy - T_i \left(\frac{\partial u_i}{\partial x} \right) ds \right) \quad (20)$$

where n is the exponent of the material constant of the Norton creep model. Thus, the singular crack tip stress and strain rate fields can be characterized by $C(t)$ -integral under small scale creep conditions as follows [44,45]

$$\begin{aligned} \sigma_{ij} &= \left(\frac{C(t)}{A I_n r} \right)^{\frac{1}{n+1}} \bar{\sigma}_{ij}(\theta, n) \\ \dot{\epsilon}_{ij}^c &= A \left(\frac{C(t)}{A I_n r} \right)^{\frac{n}{n+1}} \bar{\epsilon}_{ij}(\theta, n) \end{aligned} \quad (21)$$

It can be found in Eq. (20) that the contour integral definition of $C(t)$ applied only for sufficiently small contours, which indicates that $C(t)$ -integral will be path-dependent in the general case unless steady-state creep conditions have been attained [33]. When creep time is sufficiently long, $C(t)$ -integral is equal to C^* which is shown as below

$$C^* = \lim_{t \rightarrow \infty} C(t) \quad (22)$$

Based on Eq. (22), C^* can be calculated by transforming Eq. (20) into the domain integrals form in the FE method.

Time characterizing the transition from transient to steady-state creep conditions is given by the redistribution time t_{red} defined as follows [44,45]

$$t_{\text{red}} = \frac{K^2(1 - \nu^2)}{EC^*} \quad (23)$$

2.4. Rate-dependent energy release rate $\dot{G}$

As illustrated in Sections 2.2 and 2.3, the fracture properties of a solid within elasto-plastic material could be characterized by J -integral, which is related to the energy release rate G . Furthermore, the fracture parameter C^* -integral which is used to characterize the near-tip stress field and strain-rate field under steady-state creep condition is defined analogously to J -integral and has the rate form $\dot{J}$ [30,32]. In the same way, we propose a rate form of energy release rate, $\dot{G}$, the analogy to Eq.(8), which is given by

$$\dot{G} = -\frac{d\dot{I}}{B_1 da} = -\lim_{\Delta a \rightarrow 0} \frac{\Delta \dot{I}}{B_1 \Delta a} \quad (24)$$

where $\dot{G}$ is called rate-dependent energy release rate in this paper.

According to the definition of J -integral and G in Eq. (7), Eq. (8) and Eq. (11), the relationship between J and G can be obtained for elastic and elasto-plastic materials

$$J = G \quad (25)$$

Taking the time derivative of Eq.(25), then

$$\dot{J} = \dot{G} \quad (26)$$

Considering the definition of C^* with $\dot{J}$ in Eq. (16) at long times $t \rightarrow \infty$, then we have

$$C^* = \dot{G} \quad (27)$$

Therefore, we can propose another way to obtain C^* through calculating $\dot{G}$ based on Eq. (24) and Eq. (27) with FE method for steady state creep. However, different from the traditional numerical solution of C^* with EDI method, an extended virtual crack closure technique is proposed and used to determine $\dot{G}$, which is much simpler and described in the next section. Generally, this novel method is faster and more convenient for the calculation of C^* utilizing the traditional Finite element method, especially for I and II types fracture mode in complex loading conditions.

3. Computational technique for $\dot{G}$

3.1. Extended virtual crack closure technique

It has been claimed by Irwin that if a crack extends by a small amount Δa , the energy absorbed in the process is equal to the work required to close the crack to its original length [20]. For fracture analysis of mode I crack, the energy release rate also can be written as [38]

$$G_1 = \lim_{\Delta a \rightarrow 0} \frac{1}{2B_1 \Delta a} \int_0^{\Delta a} \sigma_y(\Delta a - r, 0) u_y(r, \pi) dr \quad (28)$$

where σ_y is the normal stress along the closure crack surfaces, and u_y is the relative opening displacement between points on the upper and bottom surfaces of the crack. Although the mode I case is only considered here, however, this method can be extended to a mixed I/II case by incorporating the mode II contribution to the total energy release rate of a mixed one. For three-dimensional crack front, the computational scheme varies differently depending on the characteristics of the crack front type, e.g. curvature of the three-dimensional crack front. For cracks under the three-dimensional condition, the scheme can be adopted similarly presented here by meshing the crack front with numbers of two-dimensional conditions.

However, Eq. (28) is not convenient to implement within FE

Fig. 2. Schematic of VCCT with FE method.

method, then through replacing the stress integral form to nodal force form, Eq. (28) is discretized to formula as following [21]

$$G_1 \cong \frac{F_{y,i}(a) u_{y,i-1}(a)}{2B_1 \Delta a} \quad (29)$$

where $F_{y,i}(a)$ is the nodal force at the node i in the y -direction, $u_{y,i-1}(a)$ is the crack opening displacement at the node $i-1$ in the y -direction behind the crack tip at $a + \Delta a$ as shown in Fig. 2. Consequently, the energy release rate can be approximated conveniently through Eq. (29) and this effective method is called a virtual crack closure technique.

To monitor the energy release rate via VCCT within the numerical process, the super element with dummy nodes presented by Xie and Biggers [46] has been implemented in this work within ABAQUS employing a user-defined element subroutine UEL [34]. Each super element has five nodes and ten degrees of freedom $\{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4, u_5, v_5\}$ as shown in Fig. 3.

In order to calculate the nodal forces at the crack tip, two stiff springs are placed between nodes "1" and "2". The strain energy within the springs is computed as below:

$$U = \frac{1}{2} k_x (u_1 - u_2)^2 + \frac{1}{2} k_y (v_1 - v_2)^2 \quad (30)$$

Variation of Eq. (30) is

$$\delta U = (\delta u_1 - \delta u_2) k_x (u_1 - u_2) + (\delta v_1 - \delta v_2) k_y (v_1 - v_2) = \delta u^T K u \quad (31)$$

where

$$u = \{u_1, v_1, u_2, v_2\}^T \quad (32)$$

$$\delta u = \{\delta u_1, \delta v_1, \delta u_2, \delta v_2\}^T \quad (33)$$

$$K = \begin{bmatrix} K_x & -K_x \\ -K_x & K_x \end{bmatrix} \quad (34)$$

$$K_s = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \quad (35)$$

Fig. 3. Schematic of super element with dummy nodes.

where K is the stiffness matrix for the super element, k_x and k_y are the stiffness of spring along x and y direction respectively, u is the displacement vector related to nodes “1” and “2”. Therefore, the nodal forces at the crack tip are

$$F = Ku \quad (36)$$

$$F = \{F_x \ F_y \ -F_x \ -F_y\}^T \quad (37)$$

In UEL, residual force vector R should be defined and saved in an array containing the contributions of this element to the right-hand-side (RHS) vectors of the overall system of equations and R should approach to tolerance error for controlling the convergence and defined as

$$R = P - F \quad (38)$$

where P is the external force vector.

All the other nodes in the element are “dummy nodes” and do not contribute to the stiffness matrix. Here, they are illustrated to consider the crack length increment Δa ahead the crack tip and the displacement opening $u_{y,i-1}(a)$ behind the crack tip. By using coordinates of dummy nodes, the crack length increment Δa can be calculated by

$$\Delta a = |x_5 - x_1| \quad (39)$$

Since nodes 3 and 4 are behind the crack tip, the displacement opening $u_{y,i-1}(a)$ is

$$u_{y,i-1}(a) = v_3 - v_4 \quad (40)$$

Substituting Eq. (36), Eq. (39) and Eq. (40) into Eq. (29), the energy release rate G in mode I crack can be calculated by

$$G_1 = \frac{1}{2B_t \Delta a} [F_y (v_3 - v_4)] \quad (41)$$

In order to consider the effect of time, we can take the derivative with respect to Eq. (29), then, the numerical calculation formula for rate-dependent energy release rate $\dot{G}$ in mode I crack can be expressed as

$$\dot{G}_1 = \frac{1}{2B_t \Delta a} (\dot{F}_{y,i}(a) u_{y,i-1}(a) + F_{y,i}(a) \dot{u}_{y,i-1}(a)) \quad (42)$$

Substituting Eq. (36) into Eq. (42), then the following relation is obtained

$$\dot{G}_1 = \frac{1}{2B_t \Delta a} [k_y (\dot{v}_1 - \dot{v}_2)(v_3 - v_4) + k_y (v_1 - v_2)(\dot{v}_3 - \dot{v}_4)] \quad (43)$$

where $\dot{v}_1$ is the nodal displacement rate or nodal velocity in the y -axis direction of node “1”, and all the variables can be conveniently obtained in UEL. Therefore, the function of VCCT is extended by Eq. (43), and $\dot{G}_1$ can be directly calculated through this numerical approach under steady-state creep conditions.

3.2. Numerical procedure

The commercial finite element software ABAQUS is considered to complete the nonlinear analysis. In each increment, the implicit integration scheme is employed in UEL subroutine using a Newton-Raphson iteration method. Fig. 4 displays the procedures of the scheme for extended VCCT in UEL and the details of each step are explained as follows,

- Step 1: The properties B_t , k_x and k_y , nodal displacement and velocity vectors u and $\dot{u}$, and the solution-dependent state variables (SVARS) associated with this element are read into UEL from ABAQUS main program.
- Step 2: The super element stiffness matrix K is calculated based on Eq. (34) and (35).
- Step 3: The nodal force vector F can be determined based on Eq. (36).
- Step 4: Compute the residual force vector R based on Eq. (38).

Fig. 4. Flowchart of UEL subroutine in ABAQUS.

- Step 5: If the norm of the residual force vector is larger than the tolerance error, then, reduce the incremental time and go to Step 1, if not go to Step 6. Meanwhile, the tangent stiffness matrix should be calculated for the Newton-Raphson iteration, which is given by

$$K^{\tan} = -\frac{\partial R}{\partial u} = -\left(\frac{\partial P}{\partial u} - \frac{\partial F}{\partial u}\right) = \frac{\partial F}{\partial u} \quad (44)$$

- where $\partial P / \partial u = 0$ in Eq. (44) because the external force vector P is constant.
- Step 6: The crack length increment Δa and the opening displacement $u_{y,i-1}(a)$ are calculated based on Eq. (39) and Eq. (40).
- Step 7: The energy release rate G_1 and rate-dependent energy release rate $\dot{G}_1$ for Mode I crack can be determined by Eq. (41) and Eq. (43) respectively. And the data of SVARS will be updated to ABAQUS main program at the end of each increment step.
- Step 8: The above process will be repeated in each UEL until the end of the time required.

Fig. 5. The geometry of the single edge crack tension specimen.

4. Numerical results and discussion

4.1. SENT specimen

In order to calibrate the accuracy of Eq. (27) and $\dot{G}$ that determined through extended VCCT for creep material with power-law behavior at elevated temperature, a single edge notched tension specimen (SENT) with plane strain state is considered in this case. The geometry under loading is shown in Fig. 5.

The height and width of the specimen are $L = 460$ mm, $W' = 100$ mm, and crack length $a = 50$ mm respectively. Young's modulus of the material is $E = 175$ GPa, and Poisson's ratio is $\nu = 0.33$. The elastic nonlinear viscous model is given in Eq. (1) with the coefficients $A = 2.13 \times 10^{-18}$ and $n = 7$. The uniform distribution applied loading is $\sigma = 35$ MPa.

Totally, 13,280 linear four-node quadrilateral elements (CPE4) are employed herein, and the representative FE-meshes used for the extended VCCT is shown in Fig. 6. User defined elements (UELs) have been generated ahead of the crack tip through a program developed with Python script for extended VCCT.

Firstly, the validation of the extended VCCT in computing SIF of SENT specimen is given. The empirical expression of SIF for SENT specimen is given by [47]

$$K_I = \sigma \sqrt{\pi a} \left(1.122 - 0.231 \left(\frac{a}{W'} \right) + 10.55 \left(\frac{a}{W'} \right)^2 - 21.71 \left(\frac{a}{W'} \right)^3 + 30.382 \left(\frac{a}{W'} \right)^4 \right) \quad (45)$$

With an initial crack length $a = 50$ mm and specimen width $W' = 100$ mm, the stress intensity factor $K_I = 1241.0$ N·mm^{-3/2} can be obtained through Eq. (45), and energy release rate $G_I = 7.8424$ N/mm can be obtained via Eq. (11). The numerical result of energy release rate via extended VCCT for the same specimen is $G_I^{\text{VCCT}} = 7.8141$ N/mm. The relative error is -0.36% , which indicates that the present extended VCCT can accurately calculate the SIF of SENT specimen, and then Eq.

Fig. 6. Typical FE meshes of extended VCCT for SENT specimen.

(43) is used to calculate $\dot{G}$ with extended VCCT, after that C^* can be obtained with Eq. (27).

For comparison and analysis of the accuracy of the extended VCCT, C^* results obtained with $C(t)$ -integral using EDI method based on Eq. (20) and Eq. (22), GE/EPRI method (EPRI) and the reference stress method (RSM) are compared with that of extended VCCT. For EDI method, the integral contours are fabricated around the crack tip for $C(t)$ -integral calculation, and the maximum and the minimum contours are assigned and shown in Fig. 7. Note that the minimum size of the FE mesh for the crack front is selected as 0.02 mm if it is not specified.

For SENT specimen, C^* had been studied by Saxena using the EPRI method [41], which is given by

$$C^*_{\text{EPRI}} = \frac{A a h_1}{(1 - a/W')^n} \left(\frac{\sigma}{1.455 \eta_1} \right)^{n+1} \quad (46)$$

where $h_1 = 0.514$ and $\eta_1 = 0.414$ are the geometry parameters. Thus, $C^*_{\text{EPRI}} = 0.91$ N/(mm·h) can be obtained.

For RSM, C^* can be estimated with Eq. (18) and Eq. (19). If the material follows the power-law behavior in Eq. (2), C^* can be written as

$$C^*_{\text{RSM}} = A \sigma_{\text{ref}}^{n-1} K^2 (1 - \nu^2) \quad (47)$$

The limit load P_L is given by [48]

$$P_L = \frac{2}{\sqrt{3}} \sigma_{0.2} W' \left[1 - \left(\frac{a}{W'} \right) - 1.232 \left(\frac{a}{W'} \right)^2 + \left(\frac{a}{W'} \right)^3 \right] \quad (48)$$

Then, we have $C^*_{\text{RSM}} = 2.236$ N/(mm·h). All those C^* results from extended VCCT, EDI method, EPRI method and RSM are compared in Fig. 8.

As can be seen in Fig. 8, all the values of $C(t)$ -integral determined from EDI method in the minimum and maximum contours show a decreasing tendency under constant loading with the increase of time because of the stress relaxation at crack tip caused by creep deformation. One noteworthy point is that the contour integral definition of $C(t)$ -integral applies only for sufficiently small contours, and will not be path-independent in the general case (i.e. Eq. (20)) unless steady state creep conditions have been attained. In this case, values of $C(t)$ -integral determined in the minimum and maximum contours become the same constant when $t > 10$ h and $C(t)$ becomes C^* (i.e. Eq. (20) $\rightarrow$ Eq. (12)).

The $\dot{G}$ calculated by extended VCCT is path-independent as no

Fig. 7. Typical FE meshes of EDI method for SENT specimen.

Fig. 8. Comparisons of the C^* results for SENT specimen.

contour is required in this numerical method. As expected, $\dot{G}$ also tends to a constant value when the steady-state creep condition is achieved ($t > 10$ h), and it compares well with the numerical results of C^* via EDI method under the same conditions. It is indicated that Eq. (27) is reasonable to be used under the steady-state creep, and the energy definition of C^* in Eq. (16) can be numerically calculated by $\dot{G}$ through extended VCCT method based on Eq. (43). Another notable point is that the second term in the square bracket in Eq. (43) is always the same order to the first term under transient creep conditions. However, when steady-state creep conditions are achieved, the second term is two orders or even higher than the first term, therefore, C^* are mainly influence by the second term in Eq. (43).

It also can be seen from Fig. 8 that, C^* obtained from the two engineering estimation methods, EPRI method and RSM, do not agree with EDI method and extended VCCT, and EPRI method always

Fig. 9. Schematic of element lengths around the crack tips for EDI and extended VCCT models.

underestimate C^* while RSM will overestimate C^* for SENT specimens.

Furthermore, the efficiency of EDI method and extended VCCT are considered herein. The min contour radius in EDI model is fabricated equally to the element length Δa in extended VCCT for assurance the same element dimension around the crack tip for the two models shown in Fig. 9, and the numbers of elements for these two models are nearly the same. The typical FE meshes of EDI and extended VCCT models are shown in Fig. 10 respectively.

The numbers of elements are 3130 and 3176 for EDI and extended VCCT models respectively. Meanwhile, only one integral contour is fabricated to monitor $C(t)$ results in EDI model, and the same increment time is assigned for both methods in order to keep the consistent computational conditions for two models. When the calculations are completed in the same computer, CPU time for the two numerical methods against different creep time is plotted in Fig. 11 with the semi-logarithm coordinates.

It can be found in Fig. 11 that CPU time is increased with the increment of creep time for both methods, meanwhile, the computational cost of extended VCCT is invariably lower than that of the EDI method in the different creep time, ranging from 279.2 s to 786.4 s for EDI method and from 57.8 s to 382.8 s for extended VCCT. Since the result of $C(t)$ with EDI method needs to compute integral in every element encompassed in contours, however, no contour is required in extended VCCT method and the calculation can be completed only in one user defined element. It is one of the significant reasons for lower computation cost for fracture parameter C^* calculation in this case. To investigate the availability of this numerical method in different geometries, middle crack tension specimen and compact tension specimen are calculated and discussed in the next subsection.

4.2. MT specimen and CT specimen

Middle crack tension specimen and compact tension specimen are investigated in this subsection, and the geometries of MT and CT specimens are shown in Fig. 12.

Both MT specimen and CT specimen are assumed in plane strain state. For MT specimen, the geometry parameters are $W' = 100$ mm and $L = 300$ mm respectively. The crack length is $a = 25$ mm and the applied loading is $\sigma = 70$ MPa. For CT specimen, the specimen width is $W' = 46$ mm and crack length is $a = 23$ mm. The applied loading is $P = 494.5$ N. The materials used in these specimens also follow the power-law behaviors at elevated temperature with constants are $A = 1 \times 10^{-16}$ and $n = 5$. Young's modulus and Poisson's ratio are $E = 200$ GPa and $\nu = 0.3$ respectively. Due to the geometric and load symmetry, only one half FE-model of the MT specimen is created. The numbers of elements for MT and CT models are 13600 and 9050 respectively, and the typical FE-meshes of two specimens have been shown in Fig. 13.

The SIFs for MT specimen and CT specimen are given by Kim et al. [48] by means of analytic solution in Ref. [47]

Fig. 10. Typical FE meshes of EDI and extended VCCT models for efficiency test.

Fig. 11. CPU time of EDI method and extended VCCT.

$$K_I^{MT} = \sigma \sqrt{a} \sqrt{\pi \sec\left(\frac{\pi}{2} \frac{a}{W'}\right)} \left[1 - 0.25 \left(\frac{a}{W'}\right)^2 + 0.06 \left(\frac{a}{W'}\right)^4 \right] \quad (49)$$

$$K_I^{CT} = \frac{P \left(2 + \frac{a}{W'} \right) \left[0.886 + 4.64 \left(\frac{a}{W'}\right) - 13.32 \left(\frac{a}{W'}\right)^2 + 14.72 \left(\frac{a}{W'}\right)^3 - 5.6 \left(\frac{a}{W'}\right)^4 \right]}{B_t \sqrt{W'} \left(1 - \frac{a}{W'} \right)^{\frac{3}{2}}} \quad (50)$$

The SIFs of both specimens obtained with extended VCCT and reference results with Eq. (49) and Eq. (50) are given in Table 1.

By comparison, all the relative errors are within 1%, which indicates that the present extended VCCT can accurately calculate the SIFs of MT and CT specimens. Then, the C^* value of both MT specimen and CT have been calculated by $\dot{G}$ via extended VCCT and compared with $C(t)$ -integral based on EDI method, EPRI method and RSM. The C -integral results are normalized respect to the values of C_{FE}^* which are calculated by Kim et al. [48], being $C_{FE}^* = 0.001706$ N/(mm·h) for MT specimen and for CT specimen respectively. On the other hand, time t is normalized respect to the redistribution time, t_{red} . Based on Eq. (23), the redistribution times for MT and CT specimens are 1083 h and 2256 h respectively. The comparison results for C^* are plotted in Fig. 14.

As can be seen in Fig. 14, $C(t)$ results of the two specimens given by

Kim et al. [48] show a rapidly decreasing tendency and path-dependent effect under transient creep conditions ($t < t_{red}$). Different $C(t)$ values would be obtained in contours by means of EDI method, thus the results always need to be extrapolated back to the crack tip. For steady-state creep conditions ($t \geq t_{red}$), $C(t)$ results of the two specimens show path-independent and the values converge into the C^* .

For the engineering estimation methods, C^* for both MT and CT specimens estimated by the EPRI method compare well with the FE results, however, for the RSM, the C^* results only compare well for MT specimen, but not so reliable for the CT specimen.

As noted, $\dot{G}$ calculated by extended VCCT of the two specimens tend to a constant value when the steady-state creep condition is attained ($t \geq t_{red}$), and the results are good agreement with the Kim's C^* value of MT and CT specimen respectively. It's indicated that C^* can be calculated accurately by the proposed $\dot{G}$ via extended VCCT for all specimens considered herein for the steady-state creep conditions. Another notable point is that the results of $\dot{G}$ calculated by extended VCCT have a different tendency to the $C(t)$ values not only for MT and CT specimens but also for SENT specimen under the transient creep conditions ($t < t_{red}$). That is because the strain rate $\dot{\epsilon}_{ij}$ involved in the definition of $C(t)$ integral (i.e. Eq. (20)) just denotes the creep strain rate, but the elastic strain rate does not feature (no plasticity is assumed). Meanwhile, the stresses do change during the transient creep which is due to stress redistribution or relaxation. Consequently, their time derivatives are not zero which are involved in the definition of $\dot{G}$ (or $\dot{J}$). Therefore, $\dot{G}$ (or $\dot{J}$) is not equal to $C(t)$ at this state. However, if the long-time limit of the secondary creep exists, the stresses become constant in the steady-state creep conditions and the elastic strain rates and time derivatives of stresses vanish leaving only the term equivalent to Eq. (20), and in this limit $C(t)$ becomes C^* . Namely, $\dot{G}$ (or $\dot{J}$) is equal to $C(t)$ (C^*).

Moreover, the efficiency of EDI method and extended VCCT for these specimens are considered here too. The numerical results of CPU time against the normalized creep time are plotted in Fig. 15

The same phenomena to the SENT specimen can be found in Fig. 15, both MT and CT specimens, all the computation cost of extended VCCT is invariably lower than that of the EDI method in the different creep time, being at most 60% and 0.81% of EDI method cost for MT and CT specimen respectively. One reason is no contour required in extended VCCT which is discussed in Section 4.1. On the other hand, $\dot{G}$ is computed only by algebraic expression with Eq. (43) rather than stress

Fig. 12. Geometries of MT and CT specimens.

integral with Eq. (28) which is also helpful to improve the computational efficiency. Meanwhile, the associated calculation only involves the nodal reaction forces, opening displacements and corresponding rate forms which are the general outputs of FEA. Therefore, the implementation of extended VCCT is much simpler within FEM, and the computation cost is always lower than that of EDI method under same conditions.

4.3. Relationship between $\dot{G}$ and $\dot{J}$

As we know that the definition of C^* -integral is similar to the J -integral defined for nonlinear elastic materials (i.e. Eq. (12)) [8,30], and the value of J is path-independent and increases at the constant rate $\dot{J}$ at long times (i.e. Eq. (16)). Namely, it is also suitable to calculate C^* at steady-state creep conditions. However, Eq. (4) shows the path-

Table 1
Comparisons of the K values.

Geometry	K (MPa mm ^{1/2})		
	Ref. [48]	Extended VCCT	Error (%)
MT	637.22	638.88	0.26
CT	704.10	701.96	-0.30

dependent at transient creep [44], which would lead to the different results of the rate-form of J -integral in contours. In order to investigate the relationship between $\dot{G}$ and $\dot{J}$ in different contours, $\dot{J}$ for SENT, MT and CT specimens are calculated by FE software ABAQUS again. The materials and conditions are all the same to the models used for $C(t)$ calculation, and 10 contours are set around the crack tip for each model

Fig. 13. Typical FE-meshes of extended VCCT models for MT and CT specimens.

Fig. 14. Comparison of the C -integral results for MT and CT specimens: a) MT specimen, b) CT specimen.

respectively. The $\dot{G}$ via extended VCCT are compared with $\dot{J}$ through EDI method from the minimum and maximum contours which are plotted in Fig. 16.

It can be found that all the $\dot{J}$ results in the maximum contours for the SENT, MT and CT specimens decrease monotonically at transient creep, then a constant can be attained at long time for each specimen respectively. Whereas, the $\dot{J}$ result in the minimum contour increase sharply at first, then decrease to a constant value at long time which are consistent to $\dot{J}$ results in the maximum contour for each specimen. That is because the path-independence of J is valid only for a crack within a material which can be treated as a hyperelastic material, and hence is restricted to proportional loading condition. In practice, for monotonically increasing loads condition, it is reasonable to believe that J obtained from Eq. (4) exhibits path independent feature. However, the introduction of creep behavior will lead to stress reversals (unloading or relaxation) near the crack tip which is very common in creep as shown in Fig. 17. Consequently, the contour independent of Eq. (4) is not rigorous in creep case, which would lead to the path-dependent of $\dot{J}$ for the short time. However, path independence of J will recur for sufficiently large times due to the stresses become constants if the materials follow power-law creep behaviors which is discussed in Section 4.2.

Moreover, as noted in Fig. 16, the result of $\dot{G}$ obtained via extended VCCT is generally in good agreement with the $\dot{J}$ result in the minimum contour for each specimen. It's worth noting that $\dot{J}$ via EDI method for a CT specimen has been studied by Hellen [49], and result of $\dot{J}$ presented there manifests the similar tendency to the result of $\dot{G}$ obtained via extended VCCT. However, the tendency of $\dot{J}$ results in the maximum contour is exquisitely different from the minimum ones unless the time period is sufficient long, indicating that $\dot{J}$ via EDI method also suffers from the path-dependent and the sufficient small contour is necessary

Fig. 15. CPU time of EDI method and extended VCCT for MT and CT specimens: a) MT specimen, b) CT specimen.

not only for $C(t)$ but also for $\dot{J}$ calculation. Unlike the $\dot{J}$, however, with the advantage of extended VCCT, the path-dependent is not existed coincidentally for $\dot{G}$ calculation in creep, meanwhile, and the result of $\dot{G}$ can always provide reliable results for $\dot{J}$ with lower computational cost in creep as discussed in the previous subsection.

Another advantage of the VCCT is that it is more convenient to separate the crack modes in crack growth process when appropriate vector components are utilized in computation of components of the energy release rates. Meanwhile, extended VCCT is reliable for $\dot{J}$ calculation via $\dot{G}$ not only for steady-state creep conditions but also for transient creep conditions. It reveals that the numerical method proposed in this paper using for C^* calculation can be possibly extended to the transient creep conditions [50].

5. Conclusions

This paper proposes a rate form energy release rate, $\dot{G}$, which is called rate-dependent energy release rate, by analogy with the definition of time dependent fracture parameter C^* . The relationship between $\dot{G}$ and C^* has been deduced and a numerical method using the rate-dependent energy release rate, $\dot{G}$, is used to determine C^* under steady-state creep conditions. The proposed method provides a novel way for calculating the C^* based on the energy definition of C^* which is different from the traditional numerical solution approach using a contour integral definition. Due to the advantages of simplicity and effectiveness within FE analysis, the VCCT method has been performed via user defined element (UEL) and extended to calculate the $\dot{G}$ when the materials follow the power law creep rule.

The C^* results calculated by the proposed numerical method are

Fig. 16. $\dot{G}$ or $\dot{J}$ against time for the SENT, MT and CT specimens.

compared with existing FE results and different engineering estimation methods. Three different geometry patterns including SENT, MT and CT specimens are implemented to calibrate the proposed numerical method. Overall good agreements of the results between the proposed method and the other methods in different specimens provide confidence to determine C^* at elevated temperatures with proposed method. Meanwhile, since there is no contour needed in the proposed method, the path-dependent can be avoided coincidentally for $\dot{G}$ calculation. What's more, $\dot{G}$ is computed only by algebraic expression rather than stress integral, and associated calculation only involves the nodal reaction forces, opening displacements and corresponding rate forms which are the general outputs of FEA. Therefore, the implementation of extended VCCT is convenient in the FEM, and thus the computation cost is regularly lower than that of EDI method under the same conditions. Consequently the extended VCCT is proven to be a reliable, efficient and alternative numerical method to calculate fracture parameter C^* by means of $\dot{G}$ under steady-state creep conditions.

Moreover, the freedom degree of the node can be released conveniently in the extended VCCT, thus this method can be easily

transformed for considering the creep crack growth simulations. More importantly, $\dot{G}$ and $\dot{J}$ as creep fracture resistant capacity parameters can be obtained directly using the method proposed in this paper and provide fracture or failure parameters for the judgement of the crack initialization and propagation phenomena based on the fracture mechanics frame entirely. Through a series of numerical simulation, the accuracy and efficiency for the calculation of C^* are proven using this novel method, especially for I types fracture mode in various loading conditions

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Fig. 17. Distributions of crack-tip stresses along the tangential direction, a) SENT specimen, b) MT specimen and c) CT specimens.

Acknowledgement

Discussions with Prof. Zhuo Zhuang, Prof. Bin Liu and Dr. Si Xiao from Tsinghua University, and Dr. Daniele Barbera from University of Strathclyde are gratefully acknowledged in this paper. This work was also financially supported by the Natural Science Foundation of Qinghai Province in China (2017-ZJ-933Q), the National Natural Science Foundation of China (Grant No. 51769028, 51508137, 11872364, 11902009) and CAS Pioneer Hundred Talents Program.

References

- [1] A.A. Griffith, The phenomena of rupture and flow in solids, *Philos. Trans. Roy. Soc. Lond. A: Math. Phys. Eng. Sci.* 221 (1921) 163–198.
- [2] Griffith AA. The theory of rupture. *Proceedings of the First International Congress of Applied Mechanics*. Delft 1924. p. 55–63.
- [3] G.R. Irwin, *Fracture dynamics. Fracturing of Metals*, American Society for Metals, Cleveland, 1948, pp. 147–166.
- [4] G.R. Irwin, Onset of fast crack propagation in high strength steel and aluminum alloys, *Naval Res. Lab.* 2 (1956) 379–386.
- [5] E. Orowan, *Fracture and strength of solids*, Rep. Prog. Phys. 12 (1949).
- [6] G.R. Irwin, Analysis of stresses and strains near the end of a crack traversing a plate, *J. Appl. Mech.* 24 (1957) 361–364.
- [7] G.R. Irwin, *Fracture*. Encyclopedia of Physics, Springer-Verlag, New York, 1958.
- [8] J.R. Rice, A path independent integral and the approximate analysis of strain concentration by notches and cracks, *J. Appl. Mech.* 35 (1968) 379–386.
- [9] D. Frank, H. Remes, J. Romanoff, J-integral-based approach to fatigue assessment of laser stake-welded T-joints, *Int. J. Fatigue* 47 (2013) 340–350.
- [10] E. Barati, Y. Alizadeh, J.A. Mohandesi, Relationship between J-integral and averaged strain-energy density for U-notches in the case of large control volume under Mode I loading, *Eng. Fract. Mech.* 78 (2011) 1317–1322.
- [11] H. Haftbaradaran, J. Qu, A path-independent integral for fracture of solids under combined electrochemical and mechanical loadings, *J Mech Phys Solids*. 71 (2014) 1–14.
- [12] E. Barati, A. Mohammadi, A new practical equation for evaluation of strain-energy density distribution and J-integral in plates with blunt V-notches under bending loading, *Mater. Des.* 46 (2013) 873–880.
- [13] S. Xiao, H.-L. Wang, B. Liu, K.-C. Hwang, The surface-forming energy release rate based fracture criterion for elastic-plastic crack propagation, *J Mech Phys Solids*. 84 (2015) 336–357.
- [14] B. Moran, C.F. Shih, A general treatment of crack tip contour integrals, *Int. J. Fract.* 35 (1987) 295–310.
- [15] K.N. Shivakumar, I.S. Raju, An equivalent domain integral method for 3-dimensional mixed-mode fracture problems, *Eng. Fract. Mech.* 42 (1992) 935–959.
- [16] H. Fühling, T. Seeger, ICF4 discussion: “Elastic/plastic separation energy rate for crack advance in finite growth steps,” by A. P. Kfourri and J. R. Rice ICF4 discussion: “A contribution to the analysis of quasi static crack growth in sheet materials,” by A. U. de Koning. *Int J Fracture*. 1977;13:728–31.
- [17] C.T. Sun, C.Y. Wang, A new look at energy release rate in fracture mechanics, *Int. J. Fract.* 113 (2002) 295–307.
- [18] C. Fan, P.Y. Ben Jar, J.J. Roger Cheng, Prediction of energy release rates for crack growth using FEM-based energy derivative technique, *Eng. Fract. Mech.* 74 (2007) 1243–1254.
- [19] J.G. Williams, The Griffith Medal lecture: the fracture mechanics of soft solids, *Eng. Fract. Mech.* 149 (2015) 192–198.
- [20] G.R. Irwin, *Fracture*, Handbud der Physik. 1958;6:551.
- [21] E.F. Rybicki, M.F. Kanninen, Finite-element calculation of stress intensity factors by a modified crack closure integral, *Eng. Fract. Mech.* 9 (1977) 931–938.
- [22] I.S. Raju, Calculation of strain-energy release rates with higher order and singular finite elements, *Eng. Fract. Mech.* 28 (1987) 251–274.
- [23] C. Yang, W. Sun, J.S. Tomblin, S.S. Smeltzer, A semi-analytical method for determining the strain energy release rate of cracks in adhesively-bonded single-lap

- composite joints, *J. Compos. Mater.* 41 (2007) 1579–1602.
- [24] D. Xie, J. Chung, A.M. Waas, K.W. Shahwan, J.A. Schroeder, R.G. Boeman, et al., Failure analysis of adhesively bonded structures: from coupon level data to structural level predictions and verification, *Int J Fract.* 134 (2005) 231–250.
- [25] D. Xie, A.M. Waas, K.W. Shahwan, J.A. Schroeder, R.G. Boeman, Fracture criterion for kinking cracks in a tri-material adhesively bonded joint under mixed mode loading, *Eng. Fract. Mech.* 72 (2005) 2487–2504.
- [26] T. Xiao, R.D. Liao, Z.X. Zuo, Computations of energy release rate under monotonic and cyclic loading conditions, *Comput. Mater. Sci.* 46 (2009) 694–699.
- [27] R.R. Das, B. Pradhan, Adhesion failure analyses of bonded tubular single lap joints in laminated fibre reinforced plastic composites, *Int. J. Adhes. Adhes.* 30 (2010) 425–438.
- [28] J. Jokinen, M. Wallin, O. Saarela, Applicability of VCCT in mode I loading of yielding adhesively bonded joints-a case study, *Int. J. Adhes. Adhes.* 62 (2015) 85–91.
- [29] R. Krueger, Virtual crack closure technique: history, approach, and applications, *Appl. Mech. Rev.* 57 (2004) 109–143.
- [30] J.D. Landes, J.A. Begley, *Mechanics of Crack Growth*, ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959, 1976, pp. 128–128-21, , <https://doi.org/10.1520/STP33943S>.
- [31] J.R. Rice, *Mathematical analysis in the mechanics of fracture*, 1968.
- [32] K.M. Nikbin, G.A. Webster, C.E. Turner, Relevance of nonlinear fracture mechanics to creep cracking. *Crack Fract.* 1976;ASTM STP 601:47–62.
- [33] J.L. Bassani, F.A. McClintock, Creep relaxation of stress around a crack tip, *Int. J. Solids Struct.* 17 (1981) 479–492.
- [34] ABAQUS. ABAQUS User's Guide. Version 6.13 ed. USA: Dassault Systèmes; 2013.
- [35] V.Kumar, M.D. German, C.F. Shih, An engineering approach for elastic plastic fracture analysis. EPRI Report NP-1931. Palo Alto, CA: Electric Power Research Institute; 1981.
- [36] R.A. Ainsworth, The assessment of defects in structures of strain hardening material, *Eng. Fract. Mech.* 19 (1984) 633–642.
- [37] L. Banks-Sills, Application of the finite element method to linear elastic fracture mechanics, *Appl Mech Rev.* 63 (1991) 447–461.
- [38] W. Zeng, G.R. Liu, C. Jiang, X.W. Dong, H.D. Chen, Y. Bao, et al., An effective fracture analysis method based on the virtual crack closure-integral technique implemented in CS-FEM, *Appl. Math. Model.* (2015).
- [39] F.N. Norton, *The creep of steel at high temperature*, McGraw-Hill, New York, 1929.
- [40] Y.J. Chao, X.K. Zhu, L. Zhang, Higher-order asymptotic crack-tip fields in a power-law creeping material, *Int. J. Solids Struct.* 38 (2001) 3853–3875.
- [41] A. Saxena, *Nonlinear Fracture Mechanics for Engineers*, CRC Press, Boca Raton, 1998.
- [42] J.W. Hutchinson, Singular behaviour at the end of a tensile crack in a hardening material, *J. Mech. Phys. Solids.* 16 (1968) 13–31.
- [43] J.R. Rice, G.F. Rosengren, Plane strain deformation near a crack tip in a power-law hardening material, *J. Mech. Phys. Solids.* 16 (1968) 1–12.
- [44] R.A. Ainsworth, P.J. Budden, Crack tip fields under non-steady creep conditions-I. Estimates of the amplitudes of the fields, *Fatigue Fract. Eng. M.* 13 (1990) 263–276.
- [45] C.S. Oh, Y.J. Kim, K.B. Yoon, R.A. Ainsworth, K. Nikbin, The relationship between C_t and contour integrals under small-scale transient creep, *Eng. Fract. Mech.* 77 (2010) 565–574.
- [46] D. Xie, S.B. Biggers Jr, Progressive crack growth analysis using interface element based on the virtual crack closure technique, *Finite Elem. Anal. Des.* 42 (2006) 977–984.
- [47] Tada H, Paris PC, Irwin G. *The Stress Analysis of Cracks Handbook*; 1985.
- [48] Y.J. Kim, J.S. Kim, N.S. Huh, Y.J. Kim, Engineering C-integral estimates for generalised creep behaviour and finite element validation, *Int. J. Press. Vessels Pip.* 79 (2002) 427–443.
- [49] T.K. Hellen, The virtual crack extension method for creep fracture, *Fatigue Fract Eng M.* 14 (1991) 627–636.
- [50] X.M. Tan, G.Z. Wang, S.T. Tu, et al., Creep constraint and fracture parameter $C?$ for axial semi-elliptical surface cracks with high aspect ratio in pressurized pipes[J], *Eng. Fract. Mech.* 199 (2018) 358–371.