A Tensorial Archie's Law for Water Saturation Evaluation in Anisotropic Model

Chen Guo, *Member, IEEE*, Zhenzhen Fan, *Member, IEEE*, Bowen Ling, and Zhifang Yang

***Abstract*— In oil and gas exploration, formation water (or hydrocarbon) content estimation is essential for reservoir evaluation, development, and production. Archie's law, which associates the formation resistivity and water saturation, has been widely adopted for reservoir assessment. However, the accuracy of the scalar-based Archie's law falls when the formation exhibits strong heterogeneity (e.g., fractured shales), as the electrical anisotropy is neglected in the scalar model. In this letter, we propose a tensorial Archie's law based on the effective resistivity tensor of the formation. We construct numerical experiments of idealized three-phase formation geometries that contain ellipsoidal inclusions. The resistivity tensor is calculated from the simulation results and used in the newly proposed Archie's law to calculate the water saturation of the formation, and the model is validated by comparing the predicted saturation with the calculated value from the known geometries. The results show that the tensorial Archie's law captures the anisotropy of the formation by including all tensor elements of the resistivity, thus improving the predictability.**

***Index Terms*— Anisotropy, Archie's law, resistivity, tensorial, water saturation.**

I. Introduction

THE inclining attention on the unconventional reservoir exploration requires reliable estimation of the formation water content (i.e., water saturation), which has been an important topic in petroleum engineering and geophysical research in recent decades [1], [2]. Unlike in conventional reservoirs where isotropic and homogeneous formations dominate, the unconventional reservoir (e.g., shale) contains complex topological features such as layered media, fracture network, and multiscale geometries, which lead to strong heterogeneity and anisotropy. These properties challenge the reliability of traditional formation evaluation techniques that are scalar based. For instance, the accuracy in water saturation estimation drops significantly in some fractured media or shales [3] as the electromagnetic well logging uses scalar parameters (e.g., resistivity and permittivity) for computation.

Analytic models estimate the water saturation by effective resistivity, which are widely used to evaluate the reservoir water content [4]–[6]. Among these models, Archie's law [7] is recognized as a classic model that calculates the water saturation through the measured resistivity, porosity, and other parameters. Studies have shown the success of Archie's law in various conventional reservoir applications, examples include but are not limited to, well log evaluation technique which permits the calculation of water saturation in sedimentary units [8] and multiphase systems with conducting phases [9]. Recently, Cai *et al.* [10] presented a review of various electrical conductivity models of porous media. Cook *et al.* [11] applied Archie's law in fractured formation filled with gas hydrate and observed a discrepancy between the prediction and the measurements. Attempts have been made to improve the prediction accuracy of Archie's law in anisotropic formations. Ellis *et al.* [12] and Friedman and Jones [13] proposed models with geometric factor (or anisotropy factor) that take the directional dependencies into account. Yet, these modifications remained the scalar form, which showed limitations in media or formations with strong anisotropy. On the other hand, tensor representation of the electrical properties was suggested [14] to model the anisotropic formation to adapt to the development of highly deviated and horizontal drilling applications, where the reservoir anisotropy can no longer be ignored.

The resistivity is defined by the electromagnetic constitutive relation and represented by a second-order tensor which consists of nine components that quantify electric resistance in different directions (e.g., x-, y-, and z-direction in Cartesian coordinates). In classic electric field theory, such tensor form can fully characterize a general media with or without anisotropy. However, the information with directional dependency will be lost if only the scalar conductivity or resistivity is computed. As a result, in shale or fractured formations with strong anisotropy, the inversion results of water saturation are less convincible by using scalar-based models such as Archie's law. Studies have been conducted by using electrical tensor to calculate the water saturation

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in anisotropic formation. Kennedy *et al.* [14] and Klein [15] have proposed to use eigenvalue equation to calculate water saturation and the results were analyzed for the case of a horizontal well penetrating the heterogeneous medium. However, the method is not widely used in reservoir evaluation due to the difficulties in calculating the saturation with a complicated tensor matrix; Schoen *et al.* [16] proposed to use tensorial electrical parameters to predict water saturation in heterogeneous shale. However, it needs to install a receiver or transmitter coil on the original logging device, which is less efficient in practical engineering applications. In this study, we propose a tensorial modification of Archie's law aiming at: 1) characterization of anisotropic formation by using resistivity tensor and 2) simplifying the tensor calculation in field measurements by introducing the tensor invariants.

The letter is structured as follows: In Section II, we discuss the theory of electromagnetic computation and the tensorial modify method in Archie's law. In Section III, we introduce the numerical simulation in detail. The model-fitting results will be analyzed in Section IV, and a tensorial Archie's law is proposed through the validation of various anisotropic geometries. By comparing all the results, we draw conclusions, which are summarized in Section V.

II. Theoretical Method

A. Resistivity Tensor

In Electromagnetics, the constitutive relations between the electric field strength and the current density are

$$\vec{J} = \sigma \vec{E} \tag{1}$$

$$\vec{D} = \varepsilon \vec{E} \tag{2}$$

where $\boldsymbol{E}$ is the electric field strength [V / m], $\boldsymbol{J}$ is the conductive current density [A/m^2], $\boldsymbol{D}$ is the electric displacement field [C/m^2], σ is the conductivity [S/m], and ε is the permittivity [F/m]. When we consider the dynamic Maxwell's equation in the phasor domain, the current density is contributed by both conductive and displacement parts, and the conductivity can be included in a complex dielectric permittivity ε^*

$$\vec{J} + j\omega\varepsilon\vec{E} = j\omega\left(\varepsilon + \frac{\sigma}{j\omega}\right)\vec{E} = j\omega\varepsilon^*\vec{E}. \tag{3}$$

As the reciprocal of conductivity σ, resistivity ρ [$\Omega \cdot$ m] is more applicable to be measured in most applications, therefore, we conduct the discussion using resistivity in this letter with no further specification.

When the composite is isotropic, one common practice is treating both σ and ε as scalars [17]. To resolve the anisotropic nature of the fractured media, we preserve the tensor form of the effective properties in our study. In this study, we assume that the effective medium containing the host (rock matrix) and the inclusion satisfies electric symmetry. Based on such assumption, the 3×3 resistivity tensor contains six independent elements, with $\rho_{ij} = \rho_{ji}$. The tensor of the resistivity is determined by (4) in the Cartesian coordinate

$$\begin{pmatrix} \rho_{xx} & \rho_{xy} & \rho_{xz} \\ \rho_{yx} & \rho_{yy} & \rho_{yz} \\ \rho_{zx} & \rho_{zy} & \rho_{zz} \end{pmatrix} = \begin{pmatrix} E_{xx} & E_{xy} & E_{xz} \\ E_{yx} & E_{yy} & E_{yz} \\ E_{zx} & E_{zy} & E_{zz} \end{pmatrix} \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix}^{-1} \tag{4}$$

where ρ_{xx} denotes the resistivity component exhibited in the x-direction while the electric field strength E_{xx} applies from the same direction. E_{xx} denotes the x component of the vector E_x (the electric field strength applied along the x-direction).

B. Tensorial Archie's Law

Modeling the relations between formation electrical behavior and porosity [18]–[20] has become one of the key subjects in petroleum engineering to predict the oil and water content quantitatively. An analytical model was proposed by Archie and improved by Winsauer [21] and Glover [22]. Given the effective resistivity of most rock samples is dominated by the fluids in the pore space [23], the resistivity-porosity relation proposed by Archie is often employed to predict the water saturation in sedimentary formations [24]. Archie's law is an empirical law that relates the water saturation, the resistivity of formation water, and the effective resistivity of the formation, with its simplified format as follows:

$$S_w = \left[\frac{a\rho_w}{\phi^m \rho_t}\right]^{\frac{1}{n}} \tag{5}$$

where S_w is the water saturation, $\rho_t[\Omega \cdot \mathrm{m}]$ is the measured formation resistivity, $\rho_w[\Omega \cdot \mathrm{m}]$ is the resistivity of water, ϕ is the porosity, a is the tortuosity factor, m is the cementation exponent, and n is the saturation exponent.

Inspired by the concept of tensor invariants in linear algebra, we propose to use the matrix invariants [25], [26] of the resistivity tensor to analyze the data. The generalized tensor invariants are able to represent the resistivity anisotropy information. Let $\boldsymbol{\rho}$ be the effective resistivity tensor matrix, the three orders of tensor invariance are defined as follows:

$$I_1 = \mathrm{tr}(\boldsymbol{\rho}) \tag{6}$$

$$I_2 = \frac{1}{2}\left((\mathrm{tr}(\boldsymbol{\rho}))^2 - \mathrm{tr}\left(\boldsymbol{\rho}^2\right)\right) \tag{7}$$

$$I_3 = \det(\boldsymbol{\rho}) \tag{8}$$

where I_1, I_2, and I_3 are the first-, second-, and third-order invariants, respectively, and tr denotes trace, det denotes determinant. The tensor invariants (I_1, I_2, and I_3) contain both isotropic and anisotropic information of the resistivity tensor, and remain invariant in spite of the observation coordinate system altering.

In this letter, we propose a tensorial Archie's law through data fitting based on the simulation results of two-phase fully water-saturated fractured models

$$\rho_e = \frac{I_3^{1/3}}{\rho_w} \tag{9}$$

where ρ_e is the effective resistivity of formation.

The formulation is further validated with 224 numerical experiments of three-phase fracture models. Through testing, we get the saturation factor n as a polynomial function of the second-order invariant I_2. We have normalized all quantities and specifically, I_2 is normalized by ρ_{rock}^2 (ρ_{rock} is the resistivity of rock frame). The tensorial Archie's law equation of the three-phase model is proposed as follows:

$$S_w = \left[a\phi^{-m}\frac{\rho_w}{I_3^{1/3}}\right]^{\frac{1}{1.17I_2^2-6.36I_2+8.69}}. \tag{10}$$

The detailed data analysis and validation are discussed in Section IV.

III. Numerical Simulation

In this letter, a finite-element method (FEM) solver, COMSOL Multiphysics, is applied to simulate the Electromagnetic wave propagation in the fractured rock model. The COMSOL radio frequency (RF) module is employed for physics field simulation and the tensor computation is post-processed with MATLAB. The perfect electric boundary condition is applied to the model for simulation. When the numerical model is geometrically complex, FEM has the advantage of flexibility in dealing with boundary conditions and mesh generation parameters. Considering the geometry of the ellipsoidal model analyzed in this letter is irregular with fine structure, both convergence speed and accuracy need to be taken into account in mesh gridding.

A. Construction of Anisotropic Fracture Rock Models

In this letter, the idealized fracture model proposed by Anderson *et al.* [27] and Amos [28] is employed, in which the fracture is simplified as an ellipsoid. According to the common fracture shape, the ellipsoidal fracture can be further divided into two types for discussion: prolate (needle-like) fracture and oblate (disk-like) fracture. The idealized fracture models are simulated in two cases: 1) rock-water two-phase model: the background is rock matrix and the ellipsoid is fracture-filled with water and 2) rock-water-oil three-phase model: the shell of the ellipsoid is water and the core is oil, i.e., oil-in-water, as shown in Fig. 1. The two-phase model is used as the calibration of water saturation evaluation using Archie's law and the three-phase model is used to predict water saturation with Archie's law.

Fig. 1 shows the two-phase and three-phase fracture model constructed by COMSOL Multiphysics. SF-1 and SF-2 are the three-phase single fracture (SF) models. CF-1 and CF-2 are the three-phase crossing fracture models. The side length of the rock matrix is $L = 1$ cm, and the x-, y-, and z-directions corresponding to the three semiaxes of the ellipsoid are b, c, and a, respectively. Define the axial ratio of ellipsoidal fracture: $r = b : a$. In our study, fractures are allowed to overlap each other, and the model geometry is parameterized, thus the model can be generalized for both ellipsoids and spheres. For the three-phase model, the oil and water ellipsoids are confocal. By varying the axial ratio, the proportion of oil/water, and the rotation angles of the ellipsoids, a number of models with different geometries and porosities can be obtained.

The material property setting for each phase is shown in Table I.

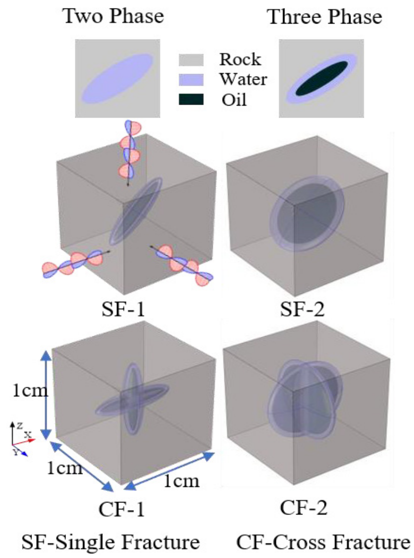

Fig. 1. Idealized anisotropic fracture models. Gray indicates the rock matrix, blue represents the water content, and black is the oil in the three-phase model. Two fracture arrangements are considered: SF and CF.

TABLE I
MATERIAL PROPERTY SETTING OF FRACTURE MODEL

Values	Permittivity(real)/ (F·m^{-1})	Permittivity(image)/ (F·m^{-1})	conductivity(S·m^{-1})
water	80	10	1
oil	2	0.1	1e-4
rock	4	1	1e-3

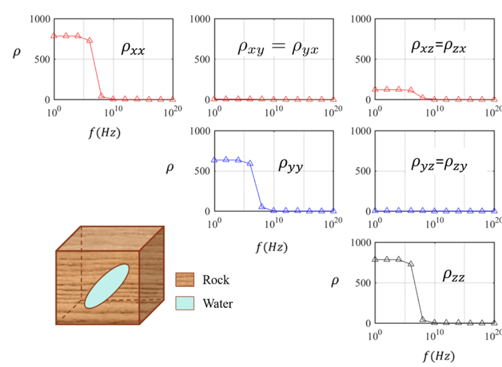

Fig. 2. Resistivity tensor of an anisotropic rock model.

B. Effective Resistivity Tensor Calculation

We run the RF module to simulate the resistivity tensor of the anisotropic fracture model in a wide range of frequency spectrums. We apply the electric field in each direction of the Cartesian coordinates to the fracture model so the full tensor of effective resistivity $\boldsymbol{\rho}$ can be determined, with

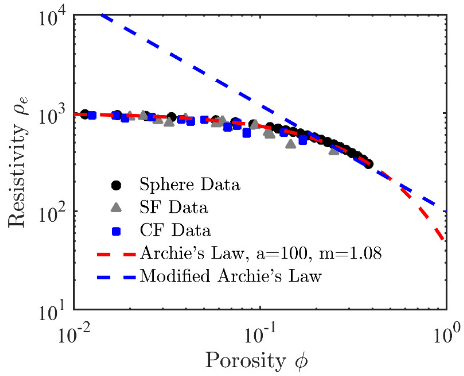

Fig. 3. Resistivity tensorial fitting results of two-phase fracture model. The black dot is the tensorial sphere data, the gray triangle indicates the tensorial SF data, and the blue square represents the tensorial CF data.

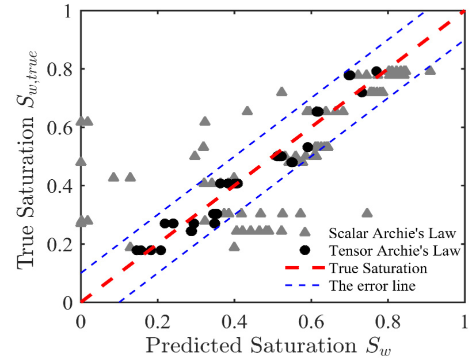

Fig. 4. Comparison results for the three-phase SF model. The red dashed line is the true water saturation value, and the two blue dashed lines are the ±10% absolute error bars.

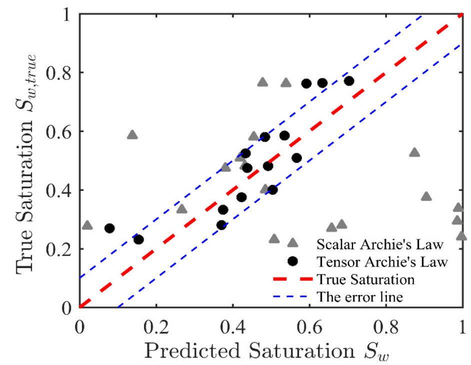

Fig. 5. Comparison results for the three-phase CF model. The red dashed line is the true water saturation value, and the two blue dashed lines are the ±10% absolute error bars.

an example shown in Fig. 2 (only the real part of $\boldsymbol{\rho}$ is considered in this letter). Fig. 2 shows that the anisotropy information of rock models with ellipsoidal fracture can be fully captured in the form of the resistivity tensor. Among these results, we choose the data at a frequency of 10 kHz, which approximates the measurement frequency of most well logging applications [29], to conduct analysis for Archie's Law modification.

IV. Analysis of Simulation Results

A. Water Saturation Prediction for Two-Phase Fracture Model

The two-phase model only contains rock matrix and water. When $S_w = 1$, i.e., fully water-saturated, the resistivity data under different porosity is obtained by simulating the isotropic spherical inclusion model, which is used as the calibration of water saturation evaluation. Fig. 3 shows the results of tensorial resistivity ρ_e and porosity ϕ in the log-log coordinates for spherical inclusion, SF as well as cross fractures (CF) model.

In Fig. 3, a total of 273 sets of data are calculated, including different fracture geometries and rotation angles. The blue dotted line is the result of calibration in the log-log coordinates by the traditional scalar-form Archie's law, $a = 100$, $m = 1.08$ and the red dotted line is the result of tensorial Archie's law. It can be seen from the result, whether the fracture is an isotropic model (e.g., sphere) or anisotropic model (e.g., ellipsoid); the fitting results of the tensorial Archie's s law are much better than the traditional scalar Archie's law.

B. Water Saturation Prediction for Three-Phase Fracture Model

On the application basis, the tensorial Archie's law is expected to be able to predict the water saturation from the measured resistivity. In this section, we demonstrate the predictability of the tensorial Archie's law. The results for the three-phase SF model are shown in Fig. 4. A total of 224 sets of data are calculated, including different fracture geometries and rotation angles.

Fig. 4 shows the comparison results of the data. The water saturation is normalized by the volume of porosity. The gray triangles and the black dots are the evaluated water saturation data through the scalar Archie's law and the tensorial Archie's law, respectively. It can be seen that there is a large error in the calculation of water saturation for the anisotropic model using the scalar form of Archie's law. The black dots indicate the revised results introducing tensor invariants, the red dashed line represents the true water saturation values, and the two blue dashed lines represent ±10% of the absolute error between the predicted value and the true value. It can be seen from Fig. 4 that by introducing the tensor invariants the prediction accuracy of Archie's law is improved noticeably.

The water saturation of the three-phase CF model calculated using the tensorial Archie's law is illustrated as follows.

Fig. 5 shows the water saturation prediction of the three-phase CF model using both the tensorial Archie's law and the scalar Archie's law. It can be seen from Fig. 5 that the prediction results of the tensorial Archie's law perform better than that of the scalar one. Yet a few predicting values of the tensorial law deviate from the ±10% error bars. One possible reason is that in the numerical simulation, the electric field intensity distribution at the sharp edges of the ellipsoids can be extremely strong, which would have an overwhelming impact on the computation of the effective tensor resistivity. In addition, a relatively strong electric polarization would occur when the two ellipsoid ends get close to each other as the crossing angle decreases, which would affect the calculation of the electrical parameters as well. Overall, the water saturation prediction using the proposed tensorial Archie's law achieves a satisfactory fitting for the three-phase CF model.

V. Conclusion

In this letter, a tensorial Archie's law is proposed for evaluating the water saturation in both isotropic and anisotropic formations. The two-phase and three-phase idealized ellipsoidal fracture models with different geometries and porosities are numerically simulated, and the effective resistivity tensors of these models are computed and analyzed to revise the scalar Archie's law in two aspects: the formation resistivity ρ_t and the saturation exponent n. Through a large amount of test data, it is found that the tensorial Archie's law can predict the water saturation for both two-phase and three-phase anisotropic fracture models with better accuracy than the scalar Archie's law does, which verifies the necessity of introducing the tensor modification into a conventional Archie's law. The current model is expected to be applicable for formation with simple fracture geometries. More complex fracture geometries with arbitrary alignment will be discussed in our further study.

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