# Studies on two-bubble energy transfer model with radiantreceiver structure 

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#### Abstract

A two-bubble model with radiate ( $\alpha$ bubble)-receive ( $\beta$ bubble) structure is constructed to study the energy transfer from one bubble to another. The influence of the non-dimensional distance $d$ and the initial energy ratio $\psi$ on the energy transfer rate is investigated via numerical simulation. The relative received energy $\varepsilon$, relative jet energy $J$, and energy transfer rates $\eta$ are defined to quantify energy transfer. Results show that the energy transfer rate decreases with the increase of $d$ and $\psi$ when the two bubbles' initial radius is identical. With the increase of $d$, the interactions between two bubbles are weakened, and the relative received energy satisfies the law of $\varepsilon \propto 1 / d^{2}$. With the increase of $\psi$, the maximum inner pressure of the $\beta$ bubble increase first and then decreases, while the jet energy of bubble $\beta$ changes with the law of $J \propto \psi$. It is found that the energy storage capacity increases with the bubble radius by simulating different bubble radius ratios.


Bubble, Energy transfer, Radiant-receiver structure, Volume of fluid method

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## 1. Introduction

Bubble collapse is an indispensable feature of cavitation, which occurs when the local pressure is lower than saturated pressure and the liquid water changes into vapour [1]. Bubble collapse is essential for high-speed underwater vehicles, playing a central role in erosion [2-4]. Highpressure pulse and liquid jets caused by bubble collapse can result in structure vibration and material destruction. On the other hand, bubble dynamics is also used in the fields of water treatment, surface cleaning, and material processing [5-7]. Therefore, the profound study of bubble dynamics is of great significance. Rayleigh [8] deduces the simplified equation of bubble motion by assuming the liquid is incompressible. Based on the Rayleigh equation, Plesset

[^0]includes the influence of viscosity and surface tension. It is found that viscosity delays the collapse strength and surface tension affects the collapse time [9]. The velocity of the bubble boundary is comparable to the sound speed of water while the bubble collapse violently. In such a situation, the compressibility can not be ignored. Therefore, a series of bubble dynamics equations of compressible is established by Keller et al. [10] and Prosperetti [11].

The compressibility causes the bubble to produce pressure waves, making the bubble radiate more energy and reducing collapse pressure [12, 13]. Yang et al. [14] use highprecision numerical methods to study the shock waves interacting with a heavy elliptical bubble. The results show that the gas densities and shock intensities significantly affect the shock-bubble interaction. Bempedelis et al. [15] computationally investigate the energy concentration during the shock-induced collapse of a focusing triangular three-bubble
array. The result shows that collapse pressure is 55 times the initial pressure. Moreover, the liquid jet produced by nonspherical bubbles collapse is another hotspot in bubble dynamics. The Kelvin impulse can determine the direction of a jet and predict the bubble's collapse cycle [16]. The collapse of a bubble near a solid wall will produce a liquid jet toward the wall [17]. However, the collapse of a bubble near the free surface will produce a liquid jet away from the surface [18-20]. When the interaction between a bubble and free surface is strong, a fast free-surface spike will rapidly form [21]. For bubble-boundary interaction, visualizations and pressure measurements suggest that non-dimensional distance is an essential factor determining the contribution of the pressure wave and liquid jet in the damage [22-24].

In the study of bubble clusters, researchers find that the collapse propagates inward with apparent geometrical focusing, generating impulsive pressures near the cluster centre. Besides, bubbles' arrangement strongly influences the collapse pressures. Higher initial pressure inside bubbles will lead to more powerful collapse pressure and a longer cycle [25, 26]. To simplify bubbles interaction, researchers study bubbles dynamics from the view of energy. Considering the heat transfer and phase change, Yang et al. [27] derives energy conservation of bubble based on the Navier-Stokes equations. Then based on Hamiltonian mechanics, Benjamin [28] and Ilinskii et al. [29] proposed a multi-bubble model to describe the pulsation and translation of any number of the spherical bubble. Luo et al. [30] study the mechanism of interaction between a cavitation bubble and an air bubble, the relative distance, and the relative size effect that the air bubble exerted on the shock wave produced from the first collapse of the cavitation bubble. Ice-breaking caused by a pair of interacting collapsing bubbles is studied by an experimental approach by Cui et al. [31] They find the bubble-induced shock waves turn out to be crucial to the fracturing of the ice. Liang et al. [32] study non-dimensional parameters on the motion of two bubbles.

Prominent approaches to bubble simulation are the volume of fluid method (VOF), the smoothed particle hydrodynamics (SPH), the boundary element method (BEM), and the eulerian finite-element method (EFEM). After years of development, these methods can solve the non-spherical collapse of bubbles, the transmission of pressure waves, and jets' formation well. The VOF is widely employed to capture the interface in multi-fluid flow problems and is a flexible method for addressing complex free boundary problems. Only one memory location is required for one mesh cell, which is also applicable to other variables. Tomiyama et al. [33] and Garoosi et al. [34] studied the rise of bubbles based on VOF. The SPH is suitable and conservative for simulating dynamic phenomena with large deformation, moving boundary and
multiphase mixing. Zhang et al. [35] and Cheng et al. [36] simulated the bubble rising problem with SPH , the results of which match the experimental ones well. The BEM improves efficiency and decreases the numerical error generated by discretization. BEM method is mainly used to simulate the axisymmetric model early [37]. In recent years, Zhang et al. [38] used the density potential method based on BEM to simulate the collapse of three dimensional (3D) bubbles. The EFEM is a practical approach to computational fluid dynamics which has been widely used in the hydrocodes. Liu et al. $[39,40]$ study the interaction of two underwater explosion bubbles, and the pressure characteristics satisfy the experimental result.

This paper adopts the VOF method and constructs a two-bubble energy transfer model with a radiation-receiving structure based on the Navier-Stokes equations and energy conservation law. The effects of non-dimensional parameters on the motion of two bubbles and energy transfer rate are studied via the numerical method.

## 2. Two bubble energy transfer model

Firstly, the relationship between the bubble's energy and the work done by the bubble is analyzed. Then, to analyze the energy transferred from one bubble to another, the radiate-receive bubble energy model is constructed in Fig. 1. The non-dimensional parameters and energy transfer are proposed in later.

The two bubbles are set in infinite domain of water. The pressure within receiving bubble $\beta$ is equal to the ambient pressure, while the internal pressure of the radiation bubble $\alpha$ is not. The $\beta$ bubble can only passively react to the influence of the $\alpha$ bubble.

Figure 2 is the schematic of a bubble system, take the fluid within the bubble and spherical boundary $(r \gg R)$ as the control volume $\Omega$. The liquid is considered incompressible when motion is of low-speed. The adiabatic assumption is valid, and the parameter of air such as $C_{p}$, dynamic viscosity et al. are constant. For simplicity, gravity and inertia are neglected as well.


Figure 1 Schematic of the two-bubble energy transfer model. The $\beta$ bubble on the left and the $\alpha$ bubble on the right.


Figure 2 Schematic of bubble system energy conversion. $R(t), p_{b}(t), \dot{R}$, $\dot{r}, p_{\infty}$ denotes bubble radius, bubble pressure, bubble boundary velocity, fluid velocity at $r$, and far-field pressure, respectively.

The energy of the bubble is potential energy, and the energy of the water is kinetic mainly. The continuity equation is shown in Eq. (1):
$\nabla \cdot \boldsymbol{u}=0$.
In spherical coordinates, $u_{\theta}=0, u_{\phi}=0$, Eq. (1) can be simplified to
$\frac{1}{\left(r^{*}\right)^{2}} \frac{\partial\left[\left(r^{*}\right)^{2} \cdot u_{r}\right]}{\partial r^{*}}=0$.
The differential equation is solved as
$u_{r^{*}} \cdot\left(r^{*}\right)^{2}=$ const.
The relationship between the two boundaries of the control body is
$\left.u_{r^{*}}\right|_{r^{*}=R}=\frac{\mathrm{d} R}{\mathrm{~d} t},\left.u_{r}\right|_{r^{*}=r}=\frac{\mathrm{d} r}{\mathrm{~d} t}$.
Therefore, the fluid velocity passing through the two boundaries of the control body satisfies that
$4 \pi R^{2} \cdot \frac{\mathrm{~d} R}{\mathrm{~d} t}=4 \pi r^{2} \cdot \frac{\mathrm{~d} r}{\mathrm{~d} t}$.
The work done by $p_{\infty}$ and $p_{b}(t)$ is as follows:

$$
\begin{align*}
\mathrm{d} W_{k}(t) & =4 \pi R^{2} \cdot p_{b}(t) \cdot \mathrm{d} R-4 \pi r^{2} \cdot p_{\infty} \cdot \mathrm{d} r \\
& =\left[p_{b}(t)-p_{\infty}\right] \mathrm{d} V(t), \tag{6}
\end{align*}
$$

where $V(t)$ is bubble volume. $W_{k}(t)$ will be converted into the kinetic energy of water according to the conservation of energy.

$$
\begin{align*}
W_{k}(t) & =\int_{V_{0}}^{V(t)}\left[p_{b}(t)-P_{\infty}\right] \mathrm{d} V(t) \\
& =\frac{1}{2} \int_{\Omega} m v^{2}(t) \mathrm{d} \varsigma=E_{k}(t) \tag{7}
\end{align*}
$$

where $m$ and $v$ represent the mass and velocity of the fluid microelement.

The potential energy of the bubble at time $t$ is defined as $E_{p}(t)=\frac{1}{1-\gamma} p_{b}(t) V(t)$ [41]. The bubble potential energy at the initial time is indicated as $E_{p}(0)$, the relationship between the work and bubble potential energy is
$\int_{V_{0}}^{V} P_{b}(t) \mathrm{d} V(t)=E_{p}(t)-E_{p}(0)$.
For a multi-bubble system, the contribution of individual bubble to the kinetic energy of water is quantified by its work in this paper.

In order to quantify the energy transfer between bubbles, energy transfer rate $\eta_{k}$, relative received energy $\varepsilon$, and relative jet energy $J$ are defined by Eq. (9).
$\eta=\frac{\max \left|W_{k B}\right|}{\max \left|W_{k A}\right|}, \varepsilon=\frac{E_{R}(t)}{E_{p_{B}}(0)}, J=\frac{E_{j}(t)}{E_{p_{B}}(0)}$,
where $W_{k A}$ and $W_{k B}$ represent the total work. $E_{p_{B}}(0), E_{R}$ and $E_{j}(t)$ represent the initial potential energy, receive energy and jet energy of the $\beta$ bubble.

In Fig. $1, D, p_{A}(0), p_{B}(0), R_{A}(0)$, and $R_{B}(0)$ denote the initial distance between the two bubbles, the initial pressure of the $\alpha$ bubble, the initial pressure of the $\beta$ bubble, the initial radius of the $\alpha$ bubble and the initial radius of the $\beta$ bubble, respectively. These variables affect the two bubble motions. Moreover, air density $\rho_{a}$, water density $\rho_{l}$, air viscosity $\mu_{a}$, water viscosity $\mu_{l}$, and adiabatic coefficient $\gamma$ are also related to bubbles motion. By selecting $p_{A}(0), \rho_{l} D$ as the basic quantity and considering that the physics parameters of water and air are constant, two non-dimensional parameters can be obtained: the non-dimensional distance between two bubbles $d$ and the initial energy ratio of the two bubbles $\psi$.
$d=\frac{D}{R_{A \max }+R_{B}(0)}, \psi=\frac{p_{A}(0) R_{A}^{3}(0)}{p_{B}(0) R_{B}^{3}(0)}$,
where the $R_{A \text { max }}$ is calculated by $p_{A}(0)$ and $R_{A}(0)$. The effects of $d$ and $\psi$ on the motion of two bubbles and energy transfer rate are studied in the following parts.

## 3. Numerical method and analysis

### 3.1 Numerical method

Ignoring heat, mass transfer and turbulence effect, the governing equations of the flow field are
$\frac{\partial \rho}{\partial t}+\nabla \cdot(\rho \boldsymbol{U})=0$,
$\frac{\partial(\rho \boldsymbol{U})}{\partial t}+\nabla \cdot(\rho \boldsymbol{U} \boldsymbol{U})=-\nabla p+\nabla \cdot \boldsymbol{\tau}+\boldsymbol{f}_{\boldsymbol{\sigma}}$,
$c_{p}\left[\frac{\partial}{\partial t}(\rho T)+\nabla \cdot(\rho \boldsymbol{U} T)\right]=\nabla \cdot(k \nabla T)+\frac{D p}{D t}+\dot{q}_{V}$,
where $\tau$ is the viscous stress tensor which is calculated as $\boldsymbol{\tau}=\mu\left[\nabla \boldsymbol{U}+\nabla \boldsymbol{U}^{\mathrm{T}}-\frac{2}{3}(\nabla \cdot \boldsymbol{U}) \boldsymbol{I}\right] . \quad \mu, \boldsymbol{I}$, and $\boldsymbol{U}$ denote dynamic viscosity, unit tensor, and velocity, respectively. $f_{\sigma}$ represents the source term caused by surface tension [42,43]. $\dot{q}_{V}, c_{p}$, and $k$ represent the rate of heat source or sink within the material volume per unit volume, specific heat at constant pressure, and the thermal conductivity of the substance. $\rho$ is the mixture density of water and air, which is calculated as $\rho=\alpha_{l} \rho_{l}+\alpha_{a} \rho_{a}$, where $\alpha_{l}$ and $\alpha_{a}$ represent the volume fraction of the liquid and air phases.

The VOF model is used in a compressible two-phase flow problem, and the volume fraction is solved with the transport equation [44]:

$$
\begin{align*}
& \frac{\partial \alpha_{l}}{\partial t}+\nabla \cdot\left(\alpha_{l} \boldsymbol{u}\right)+\nabla \cdot\left[\alpha_{l}\left(1-\alpha_{l}\right) \boldsymbol{U}_{\boldsymbol{r}}\right] \\
& =\alpha_{l}\left(1-\alpha_{l}\right)\left(\frac{1}{\rho_{a}} \frac{D \rho_{a}}{D p_{a}}-\frac{1}{\rho_{l}} \frac{D \rho_{l}}{D p_{l}}\right) \frac{D p_{l}}{D t}+\alpha_{l} \nabla \cdot \boldsymbol{u}, \tag{14}
\end{align*}
$$

where $\boldsymbol{u}, \boldsymbol{U}_{\boldsymbol{r}}, p_{l}$, and $p_{a}$ represent the velocity field, the relative velocity between the two phases, the pressure of the liquid phase, and the pressure of the gas phase, respectively.

The state equations of gas and pure water are given in Eqs. (15) and (16).
$p_{a} V_{a}=n R T_{a}$,
where $V_{A}, T_{a}, n$, and $R$ represent gas volume, gas temperature, the amount of substance of gas, and ideal gas constant.

$$
\begin{align*}
\frac{1}{\rho_{l}}= & 0.001278-2.1055 \times 10^{-6} T_{l}+3.9689 \times 10^{-9} T_{l}^{2}  \tag{16}\\
& -4.3772 \times 10^{-13} p_{l}+2.0225 \times 10^{-13} p_{l} T_{l},
\end{align*}
$$

where $T_{l}$ denotes liquid temperature.
Parameters used in the numerical are shown in Table 1.
At the inlet and outlet boundary of computational domain, which is located far from the bubble, the velocity satisfies a pressureInletOutletVelocity boundary condition, which

Table 1 Parameters used in the numerical

| Parameters | Air | Water | Mix |
| :---: | :---: | :---: | :---: |
| Specific heat capacities $C_{p}(\mathrm{~J} /(\mathrm{mol} \mathrm{K}))$ | 1007 | - | - |
| Specific heat capacities $C_{v}(\mathrm{~J} /(\mathrm{mol} \mathrm{K}))$ | - | 4195 | - |
| Dynamic viscosity $\mu(\mathrm{Pa} \mathrm{s})$ | $1.84 \times 10^{-5}$ | $8.55 \times 10^{-4}$ | - |
| Prandtl number $P_{r}(1)$ | 0.7 | 7.0 | - |
| Heat of formation $H_{f}(\mathrm{~J} / \mathrm{mol})$ | 0 | 0 | - |
| Surface tension coefficient $\sigma(\mathrm{N} / \mathrm{m})$ | - | - | 0.07 |
| Ideal gas constant $R(\mathrm{~J} /(\mathrm{mol} \mathrm{K}))$ | - | - | 8.31 |
| Gravity acceleration $g\left(\mathrm{~m} / \mathrm{s}^{-2}\right)$ | - | - | 0 |

allows the velocity to adjust its component normal to the boundary freely. For the pressure field, the pressure satisfies a waveTransmissive boundary condition with the atmospheric pressure $p_{\infty}$ as mean value, i.e., an approximate nonreflecting boundary condition with linear relaxation to the atmospheric pressure $p_{\infty}$ [45]. The Euler, Gauss linear, and Gauss upwind discrete methods are adopted in partial derivative, gradient term, and convection term, respectively.

In this paper, the compressibleInterFoam solver of OpenFOAM framework is used to solve Eqs. (11), (12), and (14) by using the pressure-correction and PIMPLE loop.

### 3.2 Verification

The verification simulation is the motion of a single bubble in the free field with a two-dimensional axisymmetric model. The initial bubble radius is 0.6 mm in the core area, and the initial bubble pressure is 0.6 MPa .

Figure 3 shows the schematic diagram of computational mesh, and the $z$-axis is the axis of symmetry. Four mesh resolutions are tested in Table 2. The only difference is the number of the nodes inside the initial radius of the bubble in the simulation. For each mesh, the resolution near the bubble is high enough to simulate the motion.

Figure 4 is the result of verification. In the first cycle, the evolution of bubble radius is consistent with the KellerHerring equation. Considering the calculation accuracy and efficiency, the resolution with 60 nodes inside the initial radius of the bubble is adopted in this paper.

## 4. Results and discussion

Energy transfer is affected by the deformation and the states of two bubbles. Two different typical motions of the $\beta$ bubble are focused. The two bubbles oscillate when the interaction


Figure 3 Schematic diagram of the computational mesh [46]. The whole calculation domain is $45 \mathrm{~mm} \times 90 \mathrm{~mm}$, and the maximum aspect ratio outside is $1: 10$. The core region is $1.8 \mathrm{~mm} \times 3.6 \mathrm{~mm}$ in the centra of the mesh, and the aspect ratio of the grid in the core area is $1: 1$. The red semicircle indicates the position of the bubble.

Table 2 Number of the nodes inside the initial radius of the bubble and number of cells

| Mesh resolution | Number of the nodes inside the initial radius of the bubble | Number of cells | Error (\%) |
| :---: | :---: | :---: | :---: |
| Coarse | 10 | 45000 | 9.97 |
| Medium | 20 | 180000 | 6.25 |
| Fine | 60 | 1620000 | 4.02 |
| Hyperfine | 80 | 2880000 | 4.02 |

is weak, and the $\beta$ bubble collapses violently when the interaction is strong.

The white curve in Fig. 5 represents the shape and position of the two bubbles. Overall, the pressure wave generated by the $\alpha$ bubble is the medium of energy transfer, which indirectly changes the surface pressure of the $\beta$ bubble and makes $\beta$ bubbles deform. The red curve in Fig. 5 indicates the shape and position of the two bubbles at the initial time. It can be seen that the $\beta$ bubble contracts first in


Figure 4 Comparison between the numerical solutions and the theoretical solution. $R_{\max }$ denotes the maximum radius of the theoretical solution, and $t_{\tau}$ denotes the cycle of the theoretical solution.


Figure 5 Motion of the two bubbles is oscillation. The calculation conditions are $\psi=3, d=2.0$. The upper part of each picture is the pressure diagram, and the lower part is the velocity field. Both bubbles are in a state of oscillation.
$0 \mathrm{~ms}<t<0.06 \mathrm{~ms}$ and moves away from the $\alpha$ bubble. Then, when $0.08 \mathrm{~ms}<t<0.14 \mathrm{~ms}$, the $\beta$ bubble expands and moves toward the $\alpha$ bubble. The volume history of the two bubbles is shown in Fig. 6. The numerical results will be analyzed in detail in Sect. 4.1.

When the interaction between the two bubbles is strong, the motion is shown in Fig. 7. The white curve represents the shape and position of the two bubbles. When $t=0.02$ ms , the $\alpha$ bubble affect the environment by expanding. High


Figure 6 Volume history of the two bubbles $(\psi=3, d=2.0) . V_{0}$ and $t_{\tau}$ represent bubbles' initial volume and the cycle of the $\alpha$ bubble.


Figure $7 \beta$ bubble prodcues a jet. The calculation conditions are $\psi=$ $6, d=1.2$. The upper part of each picture is the pressure diagram, and the lower part is the velocity field. The $\beta$ bubbles produce a jet away from the $\alpha$ bubble.
pressure generated by the $\alpha$ bubble appears on the $\beta$ bubble's boundary, causing the $\beta$ bubble's boundary to compress and increasing the velocity up to $10 \mathrm{~m} / \mathrm{s}$ between two bubbles. When $0.04 \mathrm{~ms}<t<0.10 \mathrm{~ms}$, the non-spherical expansion of the $\beta$ bubble further develops the jet. The $\beta$ bubble collapse near $0.10 \mathrm{~ms}<t<0.12 \mathrm{~ms}$ and expand to its maximum volume near $t=0.12 \mathrm{~ms}$. Then, when $t>0.14 \mathrm{~ms}$, the $\beta$ bubble contracts again. The displacement of the geometric centroid of the $z$ axis is used to characterize the translation of the bubble. The deformation and displacement history of the two bubbles is shown in Fig. 8, the red curve represents the displacement of the two bubbles in the $z$ direction, and the blue curve represents the volume change of the two bubbles. The solid line indicates the state of the $\alpha$ bubble, and the dotted line indicates the state of the $\beta$ bubble. When $0<t<0.6$ ms , the $\beta$ bubble first closes to the $\alpha$ bubble and then moves away. This phenomenon is similar to the study about a bubble and free surface by Robinson et al. [47]. They found that the bubble is away from the free surface when it expands and approaches the free surface when it contracts.

When the two bubbles are close, the $\alpha$ bubble produces a strong pressure wave near the $\beta$ bubble. Driven by the pressure wave, the $\beta$ bubble deforms and produces a jet. The evaluation of jet energy will be analyzed in detail in Sect. 4.2.

### 4.1 The effect of non-dimensional distance on bubble behavior and the energy transfer

The simulation configuration in this section is that the initial radius of the two bubbles is 0.5 mm , and the initial energy ratio $\psi$ is set to 3.0. When $d>5.0$, the interaction between


Figure 8 Deformation and displacement history of the two bubble $(\psi=$ $6, d=1.2$ ). $V_{0}, R_{0}, t_{\tau}$, and $T$ represent bubbles' initial volume, initial radius, the cycle of the $\alpha$ bubble, and the displacement of bubbles, which is calculated by $T=z_{c}-z_{c_{0}}$, where $z_{c}$ and $z_{c_{0}}$ represent the bubble's coordinate at time $t$ and the initial coordinate. The red curve represents the displacement of the two bubbles in the $z$ direction, and the blue curve represents the volume change of the two bubbles. The solid line indicates the state of the $\alpha$ bubble, and the dotted line indicates the state of the $\beta$ bubble.
the two bubbles is weak, and bubbles are in oscillation. If the displacement of bubbles is ignored, two bubbles will merge when $d<1.0$, so this paper mainly focuses on the situation when non-dimensional distance $d$ ranges from 1.0 to 5.0.

Figure 9 shows the evolution of the $\beta$ bubble' motion with $d$. From Fig. 9a to b, the $\beta$ bubble produces a jet away from the $\alpha$ bubble, and the $\alpha$ bubble is in the state of oscillation. With the increase of $d$, the collapse time of $\beta$ bubble is prolonged, and the deformation of $\alpha$ bubble decreases.

There is a critical state between Fig. 9b and c, when the $\beta$ bubble rebounds, the jet generated by the $\beta$ bubble gradually disappears. In Fig. 9 c and d , the $\beta$ bubble still has large non-spherical deformation. However, the non-spherical deformation disappear when $d>1.5$. Both bubbles are in a state of oscillation.

The change of the energy transfer rate with $\psi$ is shown in Fig. 10. When $d<1.5$, the interaction between the two bubbles is strong, and $d$ is sensitive to the motion. The kinetic energy transfer rate decreases rapidly. However, when $d>1.5$, the interaction between the two bubbles is weak, and $d$ is not sensitive to the motion. The energy transfer rate decreases slowly.


Figure 9 Evolution of the $\beta$ bubble' motion with $d$. Calculation results under several typical $d$ : a $d=1.0 ; \mathbf{b} d=1.1 ; \mathbf{c} d=1.2 ; \mathbf{d} d=1.3 ; \mathbf{e} d=1.4$; $\mathbf{f} d=1.5$. a and $\mathbf{b}$ take the moment when the $\beta$ bubble is about to collapse, and $\mathbf{c - f}$ take the moment when the $\beta$ bubble is about to rebound.


Figure 10 Change of energy transfer rate with $d$.

It is assumed that the $\alpha$ bubbles radiate energy by spherical pressure waves, and the $\beta$ bubbles can fully receive energy. In addition, the $\beta$ bubble has been receiving energy during contracting. The relationship between relative received energy and distance $D$ is

$$
\begin{equation*}
\varepsilon=\frac{E_{R}(t)}{E_{p_{B}(0)}}=\overbrace{\underbrace{\frac{\max \left|E_{\alpha}\right|}{4 \pi D^{2}}}_{\text {Energy density produced by } \alpha \text { bubble at } D} \cdot \pi R_{B}^{2}}^{\text {Energy received by } \beta \text { bubble }} \cdot \frac{1}{E_{p_{B}(0)}} . \tag{17}
\end{equation*}
$$

$\max \left|E_{\alpha}\right| \approx E_{p_{A}(0)}$ can be obtained when the $\alpha$ bubble radiates its potential energy completely. Equation (17) is simplified to
$\varepsilon=\psi \cdot \frac{R_{B}^{2}}{4\left[R_{B}(0)+R_{A \max }\right]^{2}} \cdot \frac{1}{d^{2}}$.
The change of bubbles volume is coincided with Fig. 6 when $d>1.4$. So, it can be found that $R_{A \max } \approx 1.5 R_{B}(0)$, and the average radius of the $\beta$ bubble $\bar{R}_{B}$ is nearly $0.9 R_{B}(0)$ during contracting. So Eq. (18) is simplified to
$\varepsilon=\frac{0.097}{d^{2}}$.
Equation (19) as the theoretical solution of $\varepsilon-d$ is shown in Fig. 11. It is assumed that the received energy from the $\alpha$ bubble is converted into the potential energy of the $\beta$ bubble, namely $E_{R}(t)=E_{p}(t)-E_{p}(0)$. The result obtained in this way is used as the numerical solution of $\varepsilon-d$.

The fitting curve is
$\varepsilon^{*}=\frac{a}{d^{2}}+b$,
where $a=0.131$ and $b=0.033$. The relationship between relative energy and non-dimensional distance satisfies the law of $\varepsilon \propto \frac{1}{d^{2}}$. The change of bubble potential energy is bigger than the energy transferred by the pressure wave. This section ignores the energy transferred to the $\beta$ bubble from other energy, such as liquid kinetic energy. Therefore, there is an error between the numerical and theoretical solutions.


Figure 11 Theoretical solutions, numerical solutions, and the fitting curve change with $d$.

### 4.2 The effect of the initial energy ratio on bubble behavior and energy transfer

In order to obtain the detailed effect of the initial energy ratio on the energy transfer rate, the two bubbles' initial radius is retained as 0.5 mm , and the value of $d$ is set to 1.2 .

Figure 12 shows the variation of the motion of two bubbles with $\psi$. When $1.5<\psi<3.5$ (Fig. 12a-e), the non-spherical deformation increase with $\psi$. When $\psi<3.5$ (Fig. 12f-h), the $\beta$ bubble produce a liqure jet. With the increase of $\psi$, the $\beta$ bubble collapses faster and the strength of the jet higher.

Figure 13 indicates the energy transfer rate decreases with the increase of $\psi$, but the energy received by the $\beta$ bubble increases with $\psi$. When $\psi<5.0$, compare the increase of $\alpha$


Figure 12 Variation of motion with $\psi$. Calculation results under several typical $\psi: \mathbf{a} \psi=1.5 ; \mathbf{b} \psi=2.0 ; \mathbf{c} \psi=2.5 ; \mathbf{d} \psi=3.0 ; \mathbf{e} \psi=3.5 ; \mathbf{f} \psi=4.5$; $\mathbf{g} \psi=5.0 ; \mathbf{h} \psi=5.5$. a-d take a moment when the $\beta$ bubble is about to rebound, and e-h take a moment when the $\beta$ bubble is about to collapse.


Figure 13 Variation of energy transfer rate and maximum transferred energy with $\psi$. The energy received by the $\beta$ bubble increases with the increase of $\psi$, but there is a limit close to $10 \times 10^{-5} \mathrm{~J}$.
bubble energy, the transferred energy increases slowly with $\psi$. When $\psi>5$, the transferred energy is almost at its limit. Therefore, the energy transfer rate is decreased with $\psi$.

Jet energy is an essential part of the bubble system energy when the interaction between two bubbles is strong. It is assumed that the jet energy is equivalent to the liquid kinetic energy of the cylinder in Fig. 14. $r_{\mathrm{jet}}, L, p_{b}^{a}$, and $p_{b}^{\infty}$ indicate the radius of the cylinder, the length of the cylinder, the $\beta$ bubble' surface pressure near the $\alpha$ bubble, and the $\beta$ bubble' surface pressure away from the $\alpha$ bubble, respectively. The equivalent radius of the $\beta$ bubble is $R_{B}^{*}$ during collapse, and $r_{\mathrm{jet}}=a R_{B}^{*}$. Jet energy can be evaluated by
$E_{j}=\left(p_{b}^{a}-p_{\infty}\right) \cdot \pi r_{\text {jet }}^{2} \cdot 2 R_{B}^{*}$.
The relative jet energy can be evaluated by
$J=\frac{E_{j}}{E_{p_{B}}(0)}=\frac{\left(p_{b}^{a}-p_{\infty}\right) \pi r_{\mathrm{jet}}^{2} \cdot 2 R_{B}^{*}}{\frac{1}{\gamma-1} p_{B}(0) V_{B}(0)}$.
The motion of the two bubbles is similar to Fig. 8, and the cycle of the $\alpha$ bubble is more than 5 times than collapse time of the $\beta$ bubble. So, it is assumed that $p_{b}^{a}$ is constant during the generation of the jet. The relationship of $p_{b}^{a}$ and the boundary velocity of $\alpha$ bubble $\dot{R}_{\mathrm{A}}$ can be derived from Bernoulli's theorem, namely
$p_{b}^{a}=\frac{\rho_{l}}{2} \dot{R}_{A}^{2}+p_{\infty}$.
Brennen [48] proved that when $R \gg R_{0}$, the relationship between initial pressure $p_{A}(0)$ and boundary velocity $\dot{R}_{A}$ is
$\dot{R}_{A}=\left(\frac{2}{3} \frac{p_{A}(0)-p_{\infty}}{\rho_{l}}\right)^{2}$.
Combining Eqs. (22) and (23), the relationship between relative jet energy and initial energy ratio is
$J=\frac{2 a^{2} \pi(\gamma-1)}{3} \cdot \frac{\left(R_{B}^{*}\right)^{3}}{R_{A}(0)^{3}}\left\{\psi-\left[\frac{R_{A}(0)}{R_{B}(0)}\right]^{3}\right\}$.
When $\psi>3.5$, the volume history is similar to Fig. 8, and $\left(R_{B}^{*} / R_{A}(0)\right)^{3} \approx 0.5$. When $a=0.1$, Eq. (24) is simplified to $J=0.0021(\psi-1)$.


Figure 14 Theoretical schematic diagram of jet energy.

Equation (25) as the theoretical solution of $J-\psi$ shows in Fig. 15. Numerically, jet energy can be calculated directly.

However, the actual shape of the liquid jet is a cone, not a cylinder. In addition, the assumption that $p_{b}^{a}$ is constant is also a reason for the error. The in-depth analysis of the two problems will be discussed.

Figure 16 shows the change of pressure and jet energy of $\beta$ bubble. The maximum inner pressure of the $\beta$ bubble appears at $2.5<\psi<5.0$. However, the jet energy increases with $\psi$. It can be inferred that for a determined size of a bubble, its energy storage capacity, measured by the energy transfer, is limited in the two-bubble energy transfer model.

In order to study the influence of the initial energy ratio on energy transfer in detail, the relationship between energy transfer and $\psi$ under different initial radius ratios is studied. The three types are as follows:
(1) The $\alpha$ bubble initial radius to 0.5 mm and the $\beta$ bubble initial radius as 0.6 mm .
(2) The $\alpha$ bubble initial radius to 0.5 mm and the $\beta$ bubble initial radius as 0.7 mm .
(3) The $\alpha$ bubble initial radius to 0.5 mm and the $\beta$ bubble initial radius as 0.8 mm .

Figure 17 shows the relative potential energy of the $\beta$ bub-


Figure 15 Theoretical solutions, numerical solutions, and the fitting curve change with $\psi$


Figure 16 Average pressure and jet energy history of the $\beta$ bubble under different $\psi$.


Figure 17 Effect of different radius ratios of two bubbles on $\psi-\varepsilon . R_{a} / R_{b}=$ $5: 5, R_{a} / R_{b}=5: 6, R_{a} / R_{b}=5: 7$, and $R_{a} / R_{b}=5: 8$ indicate that the initial size of $\alpha$ bubble is 0.5 mm , and the initial size of $\beta$ bubble are 0.5 mm , $0.6 \mathrm{~mm}, 0.7 \mathrm{~mm}$, and 0.8 mm , respectively.
ble $\varepsilon$ with the initial radius of the $\beta$ bubble. It can be concluded that the larger the ratio of bubble $\beta$ to $\alpha$, the more energy it transferred.

## 5. Conclusion

This study contributes to assessing the amount of energy transferred from one bubble to another during bubble interaction. In this paper, the radiant-receiver energy bubble model is constructed. The effects of non-dimensional distance and the initial energy ratio on the two bubble's motion are focused. The effect of the initial radius of the $\beta$ bubble on the motion is also analyzed.

When the initial radius is identical, the effect of $d$ on the motion can be observed: the two bubbles are in the state of oscillation when $d>1.5$ and the motion of the $\beta$ is changeable when $1.0<d \leqslant 1.5$. In addition, the $\beta$ bubble almost does not produce a jet when $d=1.2$. The received energy satisfies the law of $\varepsilon \propto 1 / d^{2}$ when $d \geqslant 1.4$.

Subsequently, the relationship between the motion of two bubbles and $\psi$ is analyzed while $d=1.2$. With the increase of $\psi$, the motion of two bubbles becomes intense. The $\beta$ bubble gradually produces a jet when $\psi>3.0$. The jet energy is higher with $\psi$ and satisfies the law of $J \propto \psi$. In addition, the maximum inner pressure of the $\beta$ appears in $2.5<\psi<5.0$.

Finally, the influence of $\beta$ bubble radius on motions is analyzed. It is found that the larger the initial radius of the $\beta$ bubble, the higher its energy storage capacity.

Author contributions Zhendong Bian and Tezhuan Du designed the research. Zhendong Bian and Tezhuan Du completed formal analysis and data curation. Zhendong Bian wrote the first draft of the manuscript. Tezhuan Du, Rundi Qiu, Yongjiu Wang, Jingzhu Wang, and Bo Yin advanced some helpful proposal on the first draft of the manuscript. Rundi Qiu, Yongjiu Wang, Jingzhu Wang, and Bo Yin helped organize the manuscript. Zhendong Bian and Tezhuan Du revised and edited the final version.

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# 辐射－接收结构的双气泡能量传递模型 

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摘要 为了研究从一个气泡传递到另一个气泡的能量，本文构建了辐射－接收结构的双气泡能量模型。基于数值仿真，研究了无量纲距离 $(d)$ 和初始能量比 $(\psi)$ 对气泡能量传递的影响。定义相对接收能量 $(\varepsilon)$ ，相对射流能量 $(J)$ 以及能量传递率 $(\eta)$ 来量化气泡的能量传递。结果表明，当两泡初始半价完全相同时，能量传递率随着无量纲距离，初始能量比的增加而减小。随着无量纲距离的增加，两泡的相互作用减弱，相对接收能量满足关系：$\varepsilon \propto 1 / d^{2}$ 。当初始能量比增加时，接收泡的最大内压先增加后减小，但接受泡的射流能量始终增加，满足规律：$J \propto \psi$ 。通过模拟不同的气泡半径比，结果表明：气泡的储能能力随气泡半径的增大而增大．


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