# Capillary Phenomena in Tubes Under Microgravity 

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#### Abstract

Propellant tanks and propulsion systems are essential for space missions. Studying capillary phenomena of the liquid in tubes with varying cross-section under microgravity can provide the theoretical foundation for separating gas from liquid, saving propellant, and so on. In this study, a mathematical model for profiles of the liquid in tubes with varying cross-section under microgravity is obtained. With the given liquid contact angle and geometry of the tube, a model based on the shooting method is developed to predict profiles of the liquid. The length, interface area, and volume of the liquid ring can all be obtained according to the mathematical model. The theory is verified by numerical simulation in which, different geometries of the tube and different volumes and contact angles of the liquid are taken into account.


Keywords Capillary phenomena $\cdot$ Microgravity $\cdot$ Tubes $\cdot$ Shooting method

## 1 Introduction

In the microgravity environment, surface tension plays a major role, and the liquid can be spread on the tube's wall no matter how much its diameter is. Propellant tanks and propellant systems consist of plenty of tube structures. Studying behaviors of the liquid in tubes can provide the theoretical basis for separating gas from liquid, saving propellant, and so on.

Capillary phenomena from the static to the dynamic point of view have been widely studied. The famous LucusWashburn equation was obtained by adopting a balance between the capillary-driven force, the friction force and the gravity forces [1,2]. Since then, capillary-driven flows in cylindrical tubes with uniform or nonuniform cross-section [3-7], oval tubes [8], noncircular tubes [9], complex containers [10], axisymmetric geometries [11], and concentric annuli [12] have been deeply analyzed, and the dynamic equations of flows in these containers have been proposed.

[^0]Capillary-driven flows in corners were also comprehensively studied, including flows in the sharp inner corner [13-15], the corner formed by plates with varying wettability [16], the rounded interior corner [17], the corner formed by a plate and a rounded wall [18], and the curved interior corner [19]. Capillary rise of liquid between parallel plates was analyzed by Dreyer et al. [20] and was extended to the problems of liquid penetration in metal wire mesh between parallel plates [21] and the flow between plates with varying crosssection [22]. Theoretical analysis, drop tower experiments and numerical simulation of capillary driven flows have been used to optimize the propellant management device (PMD) and analyze liquid behaviors in tanks [23-26].

Static capillary surfaces in containers and corners were studied and the well-known Concus-Finn condition was proposed [27]. Carroll [28] derived theoretical expressions of profiles of liquid drops on cylindrical fibers. And it was extended to liquid drops on fibers with varying cross-sections by Chen et al. [29]. Michielsen et al. [30] explored liquid drops on conical fibers and proposed a method based on minimizing the Gibbs free energy to predict profiles of liquid drops. Profiles of liquid drops at the bottom of cylindrical fibers standing on flat substrates [31] and at the tips of cylindrical fibers [32] were obtained by Du et al. Liquid bridges were widely studied, such as liquid bridges between parallel plates, between two spheres, or between a plane and a cylindrical wall [33-39]. Capillary surfaces in cubes [40], polyhedral containers [41] and truncated-cone-shaped
containers [42] were analyzed too. The hydrodynamic stability of constrained capillary surfaces was comprehensively analyzed by Bostwick and Steen [43].

However, profiles of liquid drops in tubes with varying cross-section under microgravity have not been obtained yet. In this study, through theoretical derivation, the mathematical model for the profiles of liquid drops is obtained. With the given liquid contact angle and geometry of the tube, profiles of liquid drops can be predicted with the shooting method. Numerical simulations are performed with the volume of fluid (VOF) method in Fluent by considering different diameters of the tube and different volumes and contact angles of the liquid. Comparisons of $r_{2}$ and $V_{1}$ between theoretical and numerical results are presented and they are quite consistent with each other. Moreover, influences of the contact angle and the volume of liquid on the profiles are discussed in detail.

## 2 Derivation of the Theoretical Model

Figure 1 shows the vertical cross-sectional view of the model. The blue region represents the liquid and the black curves represent the tube's wall. The liquid is spread around the tube's wall. The cylindrical coordinate is used for analysis in this study. The $z$ axis is the axis of symmetry of the tube and the $r$ axis is perpendicular to the $z$-axis. The liquid profile is symmetric with respect to both of the $z$ and $r$ axes, so the analysis is conducted in the first quadrant of the model. The two end points of the profile curve in the first quadrant are Point $A\left(r_{1}, z_{1}\right)$ and Point $B\left(r_{2}, 0\right)$. Point $A$ locates on the tube's wall and Point $B$ locates on the $r$ axis.


Fig. 1 Vertical cross-sectional view of the model
$\varphi$ is the angle between the positive direction of the $z$ axis and the normal to the profile curve of the liquid. $L$ is the length of the liquid drop. The liquid contact angle on the wall of the tube is $\theta$. The profile of the tube's wall can be expressed as a function of $r, f(r)$. When $z=0$, the radius of the tube is $r_{0}$.

A continuous, differentiable curve at any reference point can be described by its local tangent and its curvature $K=1 / R$. A smooth surface can be described at any reference point by its tangential plane and the two principal curvatures $K_{1}=1 / R_{1}$ and $K_{2}=1 / R_{2}$, written as
$K_{1}=-\frac{d}{d r} \frac{d z / d r}{\sqrt{1+(d z / d r)^{2}}}$
$K_{2}=-\frac{1}{r} \frac{d z / d r}{\sqrt{1+(d z / d r)^{2}}}$
According to the Young-Laplace equation $K_{1}+K_{2}=$ Constant. The expression of the profile curve of the liquid can be obtained by adding up the two main curvatures, which leads to
$\frac{1}{r} \frac{d}{d r} \frac{r d z / d r}{\sqrt{1+(d z / d r)^{2}}}=C_{1}$
where $C_{1}$ is a constant. The radial component of the normal to the surface equals the sine of the angle $\varphi$. Multiplying Eq. (3) by $r$, the first integral of Eq. (3) can be obtained as below
$\frac{r d z / d r}{\sqrt{1+(d z / d r)^{2}}}=r \sin \phi=\frac{C_{1}}{2} r^{2}+C_{2}$
where $C_{2}$ is also a constant. The boundary conditions are

$$
\begin{gather*}
r=r_{1}, \cdots \varphi=\pi+\theta-a, a=\arctan f_{r=r_{1}}  \tag{5}\\
r=r_{1}, \cdots \varphi=\pi / 2
\end{gather*}
$$

where $f$ means $\mathrm{d} f / \mathrm{d} r$. By combining Eqs. (4) and (5), we can determine $C_{1}$ and $C_{2}$ as follows:
$C_{1}=-\frac{2\left[r_{1} \sin (\theta-\alpha)+r_{2}\right]}{r_{1}^{2}-r_{2}^{2}}$
$C_{2}=\frac{r_{1}^{2} r_{2}+r_{1} r_{2}^{2} \sin (\theta-\alpha)}{r_{1}^{2}-r_{2}^{2}}$
For the gradient of the profile curve of the liquid, $\mathrm{d} z / \mathrm{d} r$, there exists a relationship as below
$-\frac{d z}{d r}=\tan \phi=\frac{\sin \phi}{-\left(1-\sin ^{2} \phi\right)^{0.5}}$

[^1]The denominator on the right hand side of Eq. (8) is negative because $\varphi$ is greater than $\pi / 2$. We combine Eqs. (4)-(8) to obtain the expression of the gradient of the profile curve, which is a function of $r$, and after simplification, we have the expression as below

$$
\begin{equation*}
\frac{d z}{d r}=-\frac{r^{2}\left[r_{1} \sin (\theta-\alpha)+r_{2}\right]+r_{1} r_{2}\left[r_{1}+r_{2} \sin (\theta-\alpha)\right]}{\left\{r^{2}\left[r_{1} \sin (\theta-\alpha)+r_{2}\right]^{2}\left(r_{2}^{2}-r^{2}\right)-r_{1}^{2}\left[r_{1}+r_{2} \sin (\theta-\alpha)\right]^{2}\left(r_{2}^{2}-r^{2}\right)\right\}^{0.5}} \tag{9}
\end{equation*}
$$

Writing
$\mathrm{a}=-\frac{r_{1}+r_{2} \sin (\theta-\alpha)}{r_{1} \sin (\theta-\alpha)+r_{2}}$
Equation (7) can be transformed into the form
$\frac{d z}{d r}=-\frac{r^{2}+a r_{1} r_{2}}{\left[\left(r_{2}^{2}-r^{2}\right)\left(r^{2}-a^{2} r_{1}^{2}\right)\right]^{0.5}}$
If $r_{1}$ and $r_{2}$ are both known, Eqs. (9) and (11) can be solved numerically with the ode45 method or the Runge-Kutta method and the expression of the profile can be obtained.

The length $L$ of the liquid ring and the volume of liquid $V_{1}$ are the major concerned parameters. They can both be obtained according to the theoretical solutions proposed above. When the coordinate values of Point A are obtained, $L$ will be determined. The volume of liquid is expressed as
$V_{1}=V_{t}-V_{g}$
where $V_{1}$ is the volume of liquid, $V_{\mathrm{t}}$ is the volume of the tube with a height of $L, V_{\mathrm{g}}$ is the volume of gas, whose expressions are written as follows:
$V_{t}=2 \pi \int_{0}^{z_{1}} r^{2} d z$
$V_{g}=2 \pi \int_{0}^{z_{1}} r^{2} d z=-2 \pi \int_{r_{2}}^{r_{1}} r^{2} \frac{r^{2}+a r_{1} r_{2}}{\left[\left(r_{2}^{2}-r^{2}\right)\left(r^{2}-a^{2} r_{1}^{2}\right)\right]^{0.5}} d r$

## 3 Numerical Simulation

Numerical simulation is performed in Fluent with the VOF method. Tubes with different sizes and a kind of silicone oil named by its kinematic viscositiy ( 2 cSt ) are used to

Table 1 Liquid properties ( $25^{\circ} \mathrm{C}$ )

| Liquid | $\mu$ <br> $(\mathrm{kg} /(\mathrm{m} * \mathrm{~s}))$ | $\rho$ <br> $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ | $\sigma$ <br> $(\mathrm{N} / \mathrm{m})$ | $v$ <br> $\left(10^{-6} \mathrm{~m}^{2} / \mathrm{s}\right)$ |
| :--- | :--- | :--- | :--- | :--- |
| 2 cSt | 0.001746 | 873 | 0.0183 | 2 |



Fig. 2 3D Mesh model of a tube with varying cross-section
establish the model in our study. The properties of 2 cSt are shown in Table 1. Figure 2 shows a mesh model of the tube with varying cross-section. Grid independence verification has been carried out and the appropriate number of grids is chosen. Boundary layers are established near the tube's wall and the expansion ratio between two adjacent layers is 1.2 .

Numerical settings are shown in Table 2. As the $R e$ number during the re-orientation process of liquid is much smaller than 2000, laminar flow is assumed. The time-step size is 0.0005 s and the simulation result is saved automatically every 1500 steps. The calculation time is long enough to make sure that the liquid in the tube has reached the equilibrium state. The calculation process is very stable because the Courant number is much smaller than 1.

At initial time, the liquid forms a liquid column in the tube, as shown in Fig. 3, in which the red region represents the liquid. If the volume of liquid is within a certain range,

Table 2 Numerical settings

| Equations | Settings |
| :--- | :--- |
| Pressure-velocity coupling equation | SIMPLEC |
| Spatial discretization of the pressure <br> equation | Body force weighted |
| Spatial discretization of the volume <br> fraction equation | Geo-reconstruct |
| Spatial discretization of the momen- <br> tum equation | Second-order upwind scheme |
| Spatial discretization of the gradient <br> equation | Least square cell |



Fig. 3 Initial liquid distribution
the liquid will form a section of liquid ring and spread around the tube's wall at equilibrium, as shown in Fig. 4a, b , in which the yellow surface stands for the liquid-gas interface at equilibrium. In this case, the liquid-gas interface is similar to that of the liquid bridge. If the volume of liquid is too big, the liquid will appear as a liquid column and form a liquid seal at equilibrium. In this case, the liquid-gas interface is part of a sphere. If the volume of liquid is not big enough, the liquid will form one or some liquid drops and attach to the tube's wall at equilibrium. Unfortunately, the critical volumes of liquid between these three liquid phases are not found in this paper. The other
kinds of liquid-gas interfaces except the liquid-ring interface are not discussed further in this paper.

## 4 Results and Analysis

The geometry of the tube, the volume of liquid and the liquid contact angle are given at first in the simulation. We measure the coordinate values of points $A$ and $B$ from the numerical results and compare them with the theoretical predictions. We measure two sets of data of points on the profile curves on two orthogonal cross-sections that pass the axis of the tube respectively at a moment when the liquid is at equilibrium, and take the average values of these two sets of data as the final results. Theoretical predictions are obtained by using the shooting method combined with Eqs. (11)-(14). When $V_{l}$ is known, $r_{1}$ is firstly estimated to be in a given range and $r_{2}$ is assumed an estimated value. We calculate the volume of liquid based on $r_{1}$ and $r_{2}$ iteratively according to Eqs. (11)-(14) until the difference between the calculated volume and the given value of $V_{l}$ is less than $1 \mathrm{e}^{-5}$. During this process, $r_{1}$ is guaranteed to locate on the tube's wall. When $r_{2}$ is known, it is substituted into the mathematical model at first. Then $r_{1}$ is adjusted within a given range until it locates on the tube's wall. After $r_{1}$ is determined through the shooting method, theoretical predictions of the profile and the volume of liquid are obtained.

Numerical and theoretical results are shown in Table 3. The ratios of theoretical results to numerical results of $V_{1}$ and $r_{2}$ are also presented in Table 3. The values of ratios are mostly between 95 and $105 \%$. To give a more intuitive observation, these ratios are plotted in Fig. 5.

Fig. 4 Liquid-gas interfaces at equilibrium. a $\theta=30^{\circ}$, $f=80 / 60^{0.5 *}(r-40)^{0.5} \mathrm{~mm} . \mathbf{b}$ $\theta=40^{\circ}, f=2.5(r-40) \mathrm{mm}$

Table 3 Model parameters, numerical results and theoretical predictions based on the shooting method

|  | Parameters |  |  |  | Numerical results |  |  | Theoretical predictions for given $V_{l}$ |  |  |  | Theoretical predictions for given $r_{2}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No | $r_{0}$ [mm] | Expressions of tube wall's profile [mm] | $\theta\left[{ }^{\circ}\right]$ | $V_{l}\left[\mathrm{~mm}^{3}\right]$ | $r_{1}[\mathrm{~mm}]$ | $z_{\text {l }}[\mathrm{mm}]$ | $r_{2}[\mathrm{~mm}]$ | $r_{l}[\mathrm{~mm}]$ | $z_{l}[\mathrm{~mm}]$ | $r_{2}[\mathrm{~mm}]$ | Ratios of theoretical to numerical results of $r_{2}$ [\%] | $r_{1}[\mathrm{~mm}]$ | $z_{l}[\mathrm{~mm}]$ | $V_{l}\left[\mathrm{~mm}^{3}\right]$ | Ratios of theoretical to numerical results of $V_{l}[\%]$ |
| \#1 | 20 | $r=20$ | 30 | 22,619 | 20 | 21.37 | 11.86 | 1 | 21.50 | 12.25 | 103.3 | 1 | 22.00 | 24,015 | 106.2 |
| \#2 | 20 | $r=20$ | 40 | 22,619 | 20 | 18.72 | 10.56 | 1 | 18.56 | 10.85 | 102.7 | 1 | 18.81 | 23,420 | 103.5 |
| \#3 | 20 | $r=20$ | 50 | 20,106 | 20 | 15.61 | 10.66 | 1 | 15.55 | 10.42 | 97.75 | / | 15.36 | 19,539 | 97.18 |
| \#4 | 40 | $r=40$ | 30 | 201,060 | 40 | 44.91 | 21.60 | 1 | 44.76 | 23.02 | 106.6 | / | 46.07 | 218,420 | 108.6 |
| \#5 | 40 | $r=40$ | 50 | 150,800 | 40 | 31.13 | 22.05 | 1 | 30.26 | 21.84 | 99.05 | 1 | 30.07 | 148,740 | 98.63 |
| \#6 | 40 | $r=40$ | 70 | 100,530 | 40 | 19.85 | 23.50 | 1 | 19.95 | 23.02 | 97.96 | 1 | 19.57 | 96,712 | 96.20 |
| \#7 | 60 | $r=60$ | 40 | 452,390 | 60 | 48.20 | 39.19 | 1 | 48.89 | 38.28 | 97.68 | 1 | 49.03 | 459,510 | 101.6 |
| \#8 | 60 | $r=60$ | 50 | 452,390 | 60 | 42.78 | 35.37 | 1 | 43.08 | 35.12 | 99.29 | 1 | 42.81 | 446,610 | 98.72 |
| \#9 | 60 | $r=60$ | 60 | 339,290 | 60 | 34.83 | 37.30 | 1 | 33.51 | 37.10 | 99.46 | 1 | 33.30 | 335,290 | 98.82 |
| \#10 | 80 | $r=80$ | 30 | 1,407,400 | 80 | 85.50 | 50.15 | 1 | 84.99 | 49.72 | 99.14 | 1 | 84.38 | 1,383,800 | 98.32 |
| \#11 | 80 | $r=80$ | 40 | 1,206,400 | 80 | 69.43 | 47.86 | 1 | 68.74 | 48.44 | 101.2 | 1 | 69.47 | 1,236,100 | 102.5 |
| \#12 | 80 | $r=80$ | 70 | 804,250 | 80 | 38.73 | 47.07 | 1 | 39.88 | 46.04 | 97.81 | / | 39.06 | 770,390 | 95.79 |
| \#13 | 20 | $z=4(r-20)$ | 30 | 25,260 | 25.50 | 22.05 | 14.14 | 25.39 | 21.60 | 13.91 | 98.37 | 25.31 | 21.23 | 24,196 | 95.79 |
| \#14 | 20 | $z=4(r-20)$ | 45 | 19,177 | 23.79 | 15.20 | 13.29 | 23.78 | 15.11 | 13.02 | 97.97 | 23.70 | 14.78 | 18,206 | 94.94 |
| \#15 | 40 | $z=2.5(r-40)$ | 40 | 174,547 | 53.30 | 33.30 | 28.05 | 53.12 | 32.80 | 28.11 | 100.2 | 53.14 | 32.86 | 175,531 | 100.6 |
| \#16 | 40 | $z=2.5(r-40)$ | 35 | 174,547 | 54.43 | 36.11 | 30.04 | 54.37 | 35.93 | 29.89 | 99.50 | 54.27 | 35.69 | 171,498 | 98.25 |
| \#17 | 20 | $z=8 * 5^{0.5}(r-20)^{0.5}$ | 30 | 20,241 | 21.74 | 20.20 | 13.01 | 21.21 | 19.67 | 12.70 | 97.62 | 21.16 | 19.24 | 19,153 | 94.62 |
| \#18 | 20 | $z=8 * 5^{0.5}(r-20)^{0.5}$ | 40 | 17,683 | 21.30 | 16.87 | 12.63 | 20.81 | 16.13 | 12.32 | 97.55 | 20.78 | 15.75 | 16,727 | 94.59 |
| \#19 | 40 | $z=5^{*} 10^{0.5}(r-40)^{0.5}$ | 20 | 174,224 | 49.33 | 44.10 | 25.87 | 47.89 | 44.41 | 26.44 | 102.2 | 48.10 | 45.01 | 182,306 | 104.6 |
| \#20 | 40 | $z=5^{*} 10^{0.5}(r-40)^{0.5}$ | 40 | 174,224 | 45.26 | 32.07 | 21.47 | 44.31 | 32.84 | 21.04 | 98.00 | 44.25 | 32.59 | 170,139 | 97.66 |
| \#21 | 40 | $z=80 / 60{ }^{0.5}(r-40)^{0.5}$ | 20 | 156,182 | 53.42 | 37.81 | 26.60 | 53.51 | 36.96 | 27.28 | 102.3 | 53.94 | 38.56 | 165,726 | 106.1 |
| \#22 | 40 | $z=80 / 60{ }^{0.5}(r-40)^{0.5}$ | 30 | 156,182 | 49.97 | 32.42 | 24.80 | 49.88 | 32.46 | 24.97 | 99.68 | 49.66 | 32.10 | 153,852 | 98.51 |
| \#23 | 40 | $z=80 / 60{ }^{0.5}(r-40)^{0.5}$ | 40 | 134,182 | 47.15 | 27.30 | 24.43 | 46.73 | 26.79 | 24.12 | 97.73 | 46.62 | 26.58 | 130,925 | 97.57 |
| \#24 | 40 | $z=80 / 600^{0.5}(r-40)^{0.5}$ | 50 | 123,379 | 45.03 | 22.76 | 24.16 | 44.94 | 22.96 | 22.98 | 94.10 | 44.64 | 22.26 | 112,685 | 91.33 |
| \#25 | 60 | $z=120 / 90^{0.6}(r-60)^{0.6}$ | 20 | 478,665 | 85.37 | 54.96 | 46.41 | 86.33 | 57.40 | 46.25 | 99.66 | 86.14 | 57.15 | 471,841 | 98.57 |
| \#26 | 60 | $z=120 / 90^{0.6}(r-60)^{0.6}$ | 30 | 478,665 | 78.60 | 46.21 | 42.65 | 78.91 | 47.07 | 41.60 | 97.54 | 78.98 | 47.16 | 462,253 | 96.57 |
| \#27 | 60 | $z=120 / 90^{0.6}(r-60)^{0.6}$ | 40 | 426,929 | 73.51 | 38.74 | 40.09 | 73.60 | 38.62 | 39.25 | 101.3 | 73.77 | 38.91 | 436,499 | 102.2 |



Fig. 5 Ratios of theoretical predictions to numerical results

The abscissa represents the number of conditions and the ordinate represents the ratio values. The black squares stand for the ratio values calculated for $r_{2}$ and the red dots represent the ratio values calculated for $V_{1}$. It can be seen that the most of the ratios are around $100 \%$, which indicates that numerical results are in good agreement with theoretical ones.

A tube with uniform cross-section is the most common condition. We take it as an example for further analysis. To compare profiles of the liquid obtained through numerical simulations and theoretical predictions more comprehensively and intuitively, we take the coordinate values of a series of points on the simulated liquid surface and plot them together with theoretical results in Fig. 6. The figure shows profiles of the liquid in the tube with $r_{0}=80 \mathrm{~mm}$. Condition numbers, $\# 10, \# 11, \# 12$, are labeled on the topleft corner of the figure, which are consistent with those in Table 3. The horizontal and vertical axes represent $r$ and $z$ respectively. The curves are theoretical profiles and the dots stand for numerical results. It shows that numerical results of the profiles are in good agreement with theoretical predictions.

For tubes with uniform cross-section, Eq. (11) can be transformed into
$\frac{d z}{d r}=-\frac{r^{2}+a r_{1} r_{2}}{\left[\left(r_{2}^{2}-r^{2}\right)\left(r^{2}-a^{2} r_{1}^{2}\right)\right]^{0.5}}$
where $a=\left(r_{1}-r_{2} \cos \theta\right) /\left(r_{1} \cos \theta-r_{2}\right)$.
For further analysis, Eq. (15) is recast in the dimensionless form


Fig. 6 Comparison between theoretical predictions and numerical results. The condition No. is \#10, \#11 and \#12 in Table 3
$\frac{d \bar{z}}{d \bar{r}}=-\frac{\bar{r}^{2}+\overline{a r}_{2}}{\left[\left(\bar{r}_{2}^{2}-\bar{r}^{2}\right)\left(\bar{r}^{2}-\bar{a}^{2}\right)\right]^{0.5}}$
where $\bar{a}=\frac{1-\bar{r}_{2} \cos \theta}{\cos \theta-\bar{r}_{2}}, \bar{r}_{2}=r_{2} / r_{1}$ and $\bar{z}_{1}=z_{1} / r_{1}$.
The dimensionless volume of liquid is expressed as follows:
$\bar{V}_{l}=\bar{V}_{t}-\bar{V}_{g}$
$\bar{V}_{t}=2 \pi \bar{z}_{1}$
$\bar{V}_{g}=2 \pi \int_{\bar{r}_{2}}^{1} \bar{r}^{2} \frac{\bar{r}^{2}+\overline{a r}_{2}}{\left[\left(\bar{r}_{2}^{2}-\bar{r}^{2}\right)\left(\bar{r}^{2}-\bar{a}^{2}\right)\right]^{0.5}} d \bar{r}$
When $\bar{r}_{2}$ is constant, the dimensionless length $\bar{L}$ and volume $\bar{V}_{l}$ vs the contact angle of the liquid are shown in Fig. 7a, b, respectively. The values of $\bar{r}_{2}$ are designed to be $0.5,0.6$, and 0.7 . When $r_{2}$ is kept constant, both of the dimensionless length and volume of liquid decrease monotonically with the increasing contact angle of the liquid.

Profiles of the liquid under given values of $\bar{r}_{2}$ and $\bar{V}_{l}$ are shown in Fig. 8a, b, respectively. Curves with different colors stand for profiles of the liquid under different contact $\underline{a n g l e s}^{2}$ of the liquid. In Fig. 8a, $\bar{r}_{2}$ is 0.5 and, in Fig. 8b, $\bar{V}_{l}$ is 3. When $\bar{r}_{2}$ is kept constant, the smaller the contact angle, the higher position the profile curve will have, which

Fig. 7 a Dimensionless length $\bar{L}$ vs contact angle and $\mathbf{b}$ dimensionless volume $\bar{V}_{l}$ vs contact angle, with $\bar{r}_{2}$ constant.

this study, and the tube with uniform cross-section is analyzed in detail as an example. When $r_{2}$ is kept constant, the length of the liquid ring $L$ and the volume of liquid $V_{l}$ keep increasing as the liquid contact angle decreases. When $V_{l}$ is kept constant, $L$ and $r_{2}$ keep increasing as the liquid contact angle decreases. The mathematical model and the method proposed in this study will be helpful for liquid management in space, which can provide the theoretical foundation for separating gas from liquid, saving propellant, and so on.

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Data Availability The data presented in this study are available on request from the corresponding author.

## Declarations

Ethics Approval Not applicable, no experiment has been conducted on human or animals.

Conflict of Interest The authors declare no competing interests.

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